

# Value of Information

*Decision Analysis and the Value of Information*

Jo Eidsvik

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# My background:

Professor of Statistics at NTNU in Trondheim, NORWAY.

Education:

- MSc in Applied Mathematics, Univ of Oslo
- PhD in Statistics, NTNU

Work experience:

- Norwegian Defense Research Establishment
- Statoil

Research interests:

- Spatial statistics, spatio-temporal statistics,
- Computational statistics,
- Geoscience applications,
- Design of experiments,
- Decision analysis, value of information,

I like tennis, hiking, skiing, etc.

# Plan for course

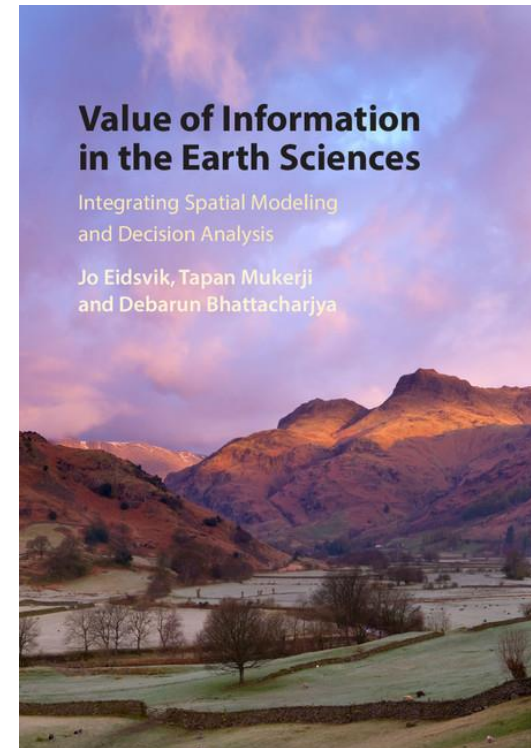
<b>Time</b>	<b>Topic</b>
<b>Lecture 1</b>	Introduction and motivating examples
	Elementary decision analysis and the value of information
	Multivariate statistical modeling, dependence, graphs
	Value of information analysis for dependent models
<b>Lecture 2</b>	Re-cap of VOI and statistical dependence
	Spatial statistics, spatial design of experiments
	Value of information analysis in spatial decision situations
	Examples of value of information analysis in Earth sciences
<b>Lecture 3</b>	Computational aspects of VOI analysis, approximate calculations
	Sequential information gathering
	Examples from Earth sciences

Every day: Small exercises.

# Material:

Relevant background reading :

- Eidsvik, J., Mukerji, T. and Bhattacharjya, D., Value of information in the Earth sciences, Cambridge University Press, 2015.
- Howard R.A. and Abbas, A.E., Foundations of decision analysis, Pearson, 2015.
- Many spatial statistics books:
  - Cressie and Wikle (2011),
  - Chiles and Delfiner (2012),
  - Banerjee et al. (2014),
  - Pyrcz and Deutsch (2014),
  - etc.



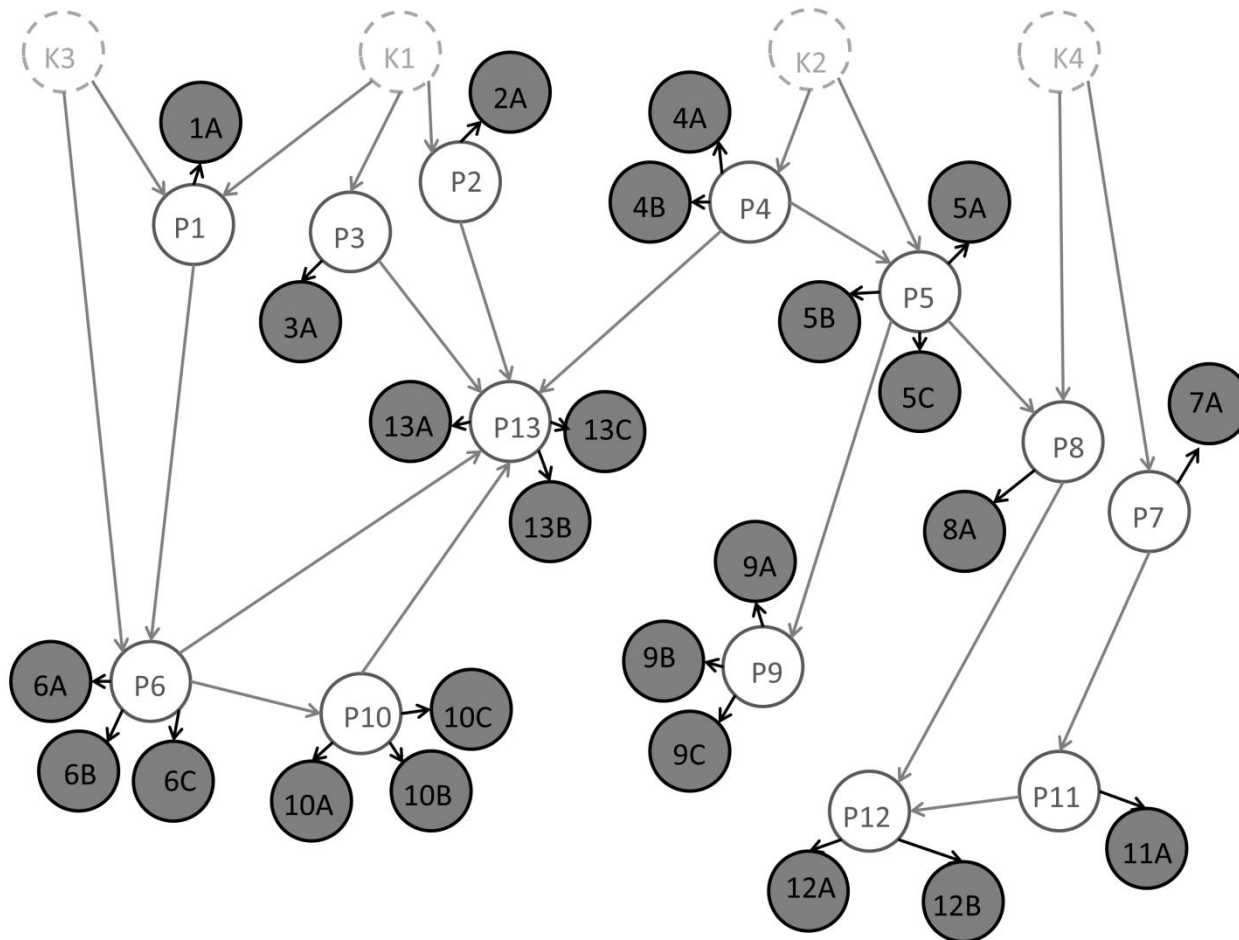
# Motivating examples:

Integration of (spatial) modeling and decision analysis.

Collect data to resolve uncertainties and make informed decisions.

# Motivation (a petroleum exploration example)

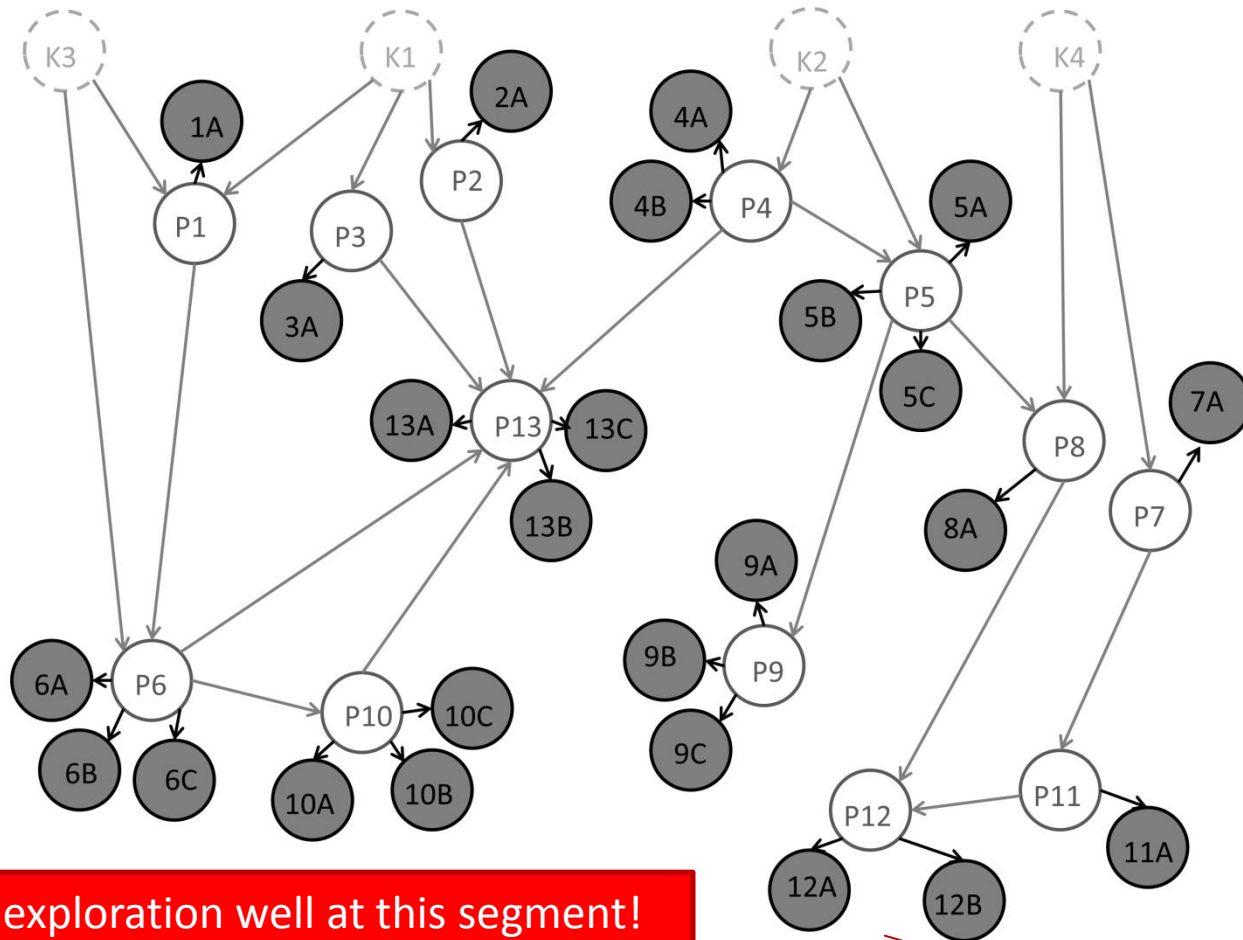
Gray nodes are petroleum reservoir segments where the company aims to develop profitable amounts of oil and gas.



# Motivation

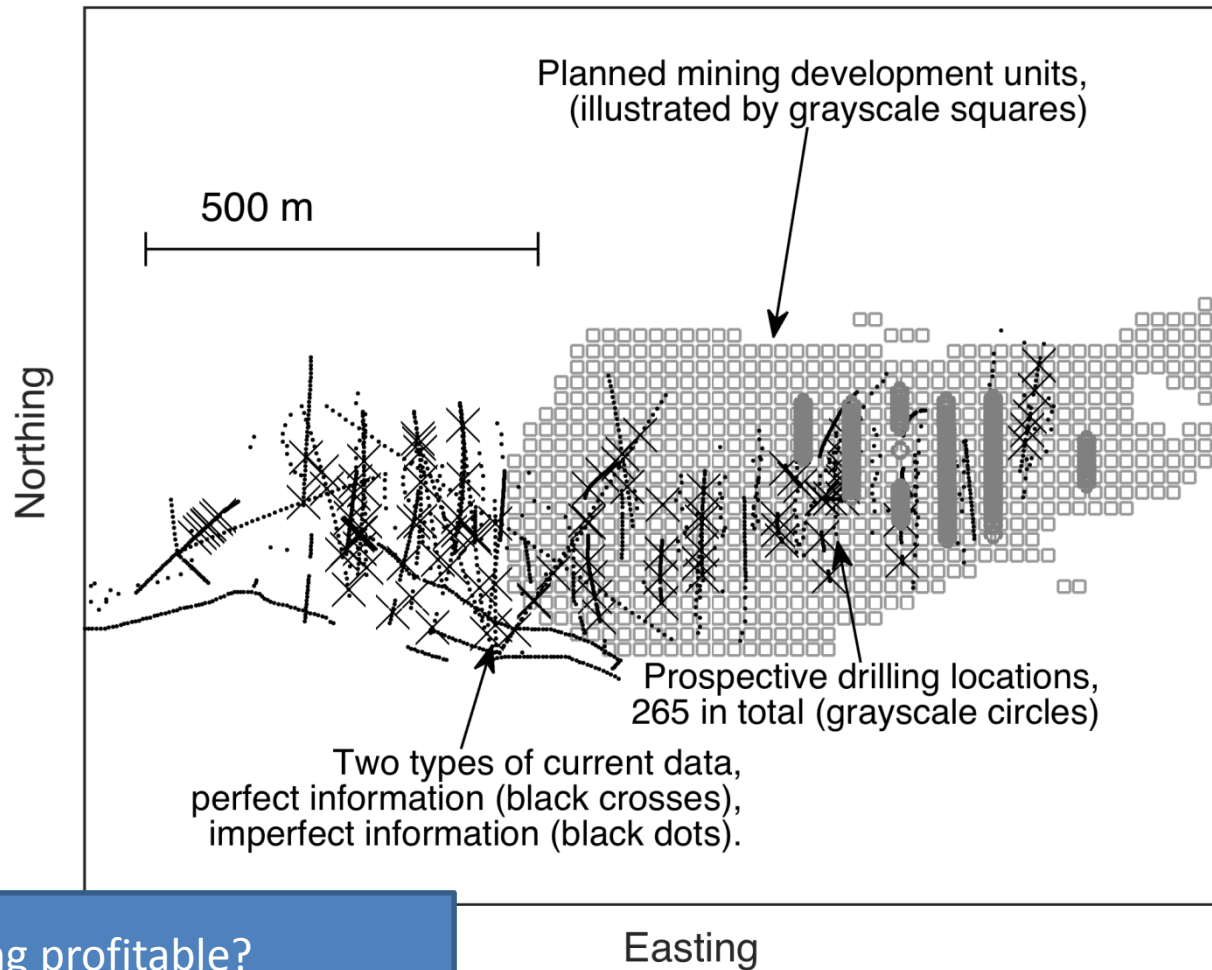
## (a petroleum exploration example)

Gray nodes are petroleum reservoir segments where the company aims to develop profitable amounts of oil and gas.



Drill the exploration well at this segment!  
The value of information is largest.

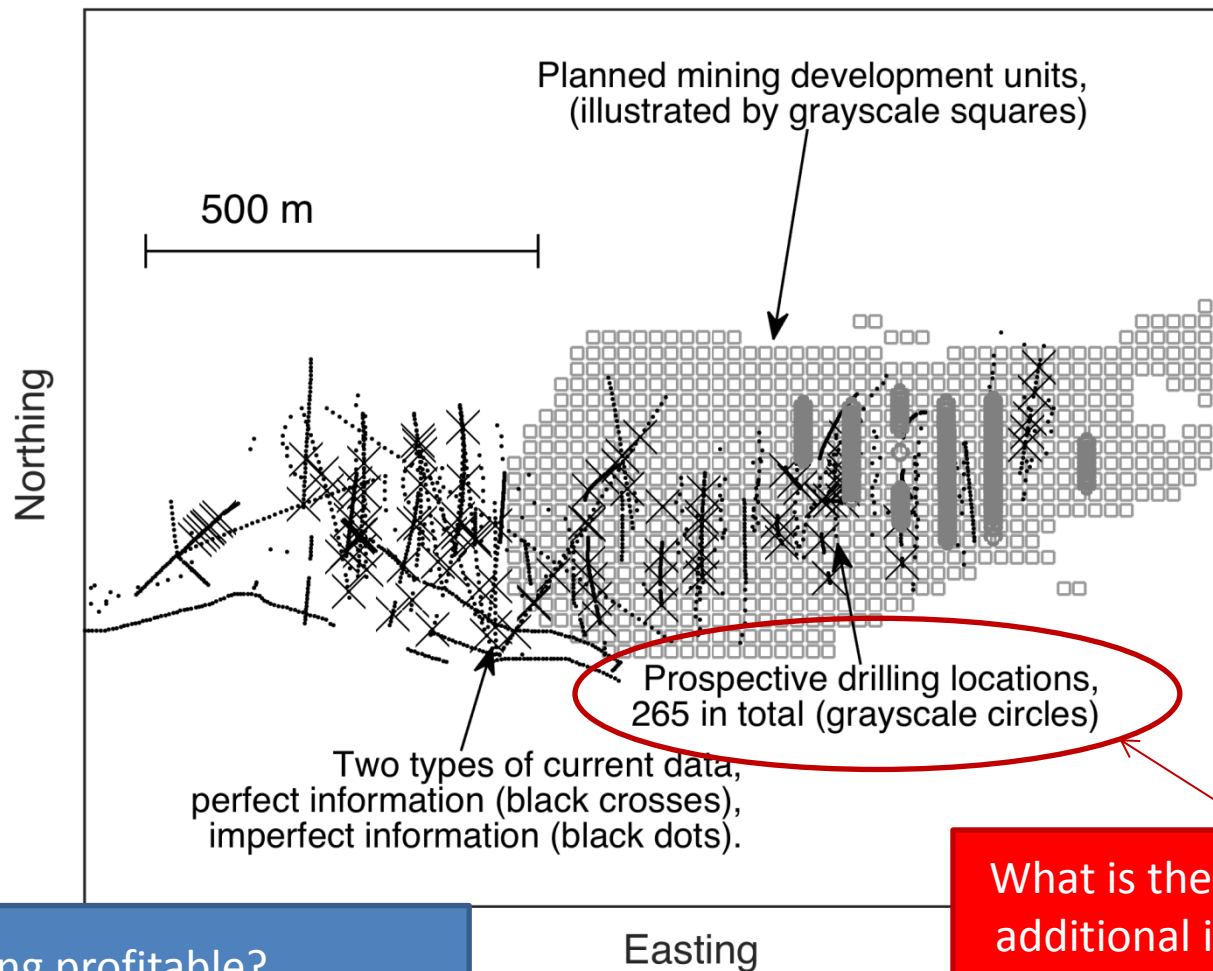
# Motivation (an oxide mining example)



Is mining profitable?

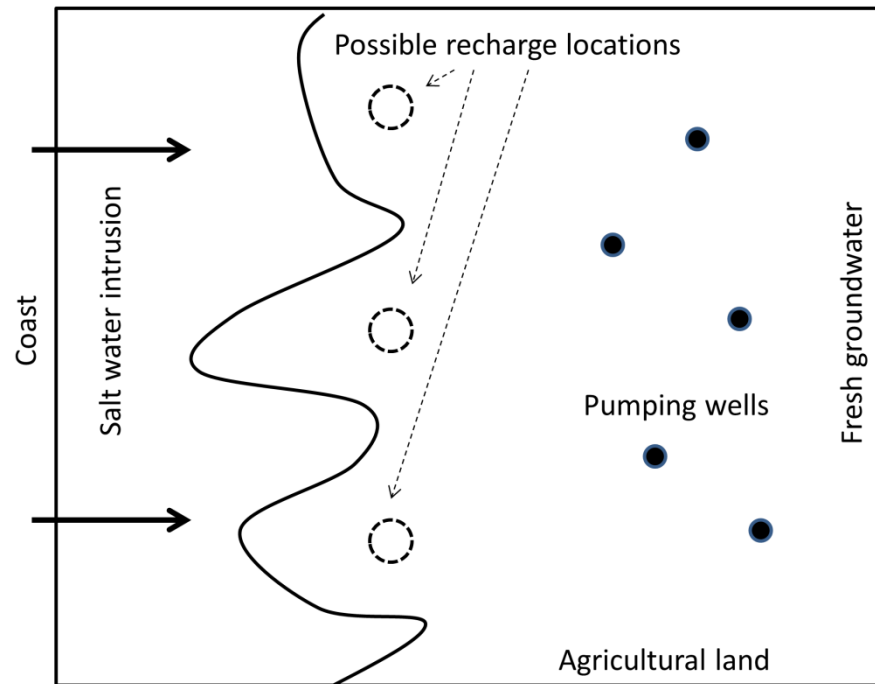


# Motivation (an oxide mining example)



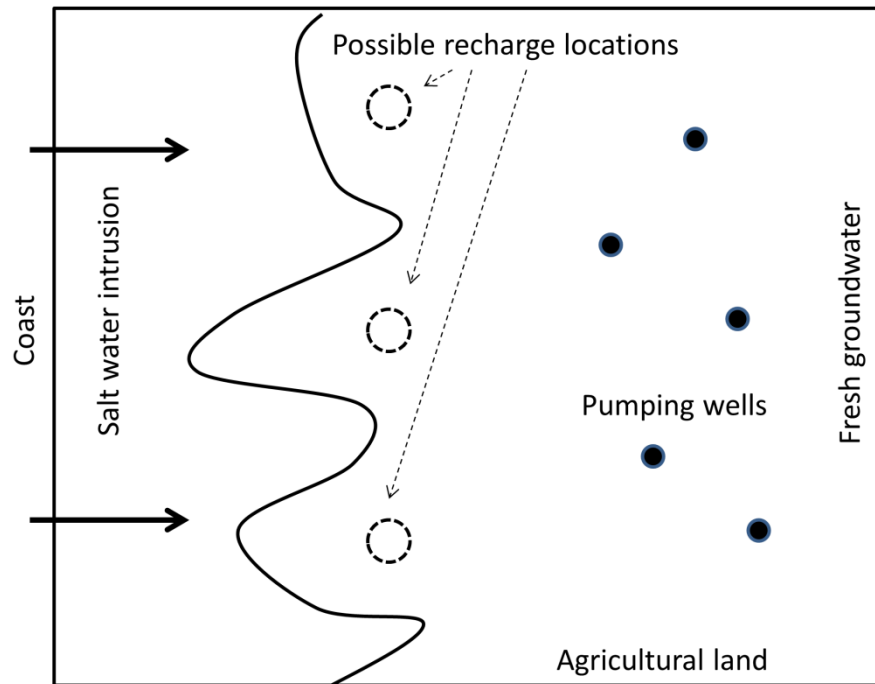
# Motivation (a groundwater example)

Which recharge location is better to prevent salt water intrusion?



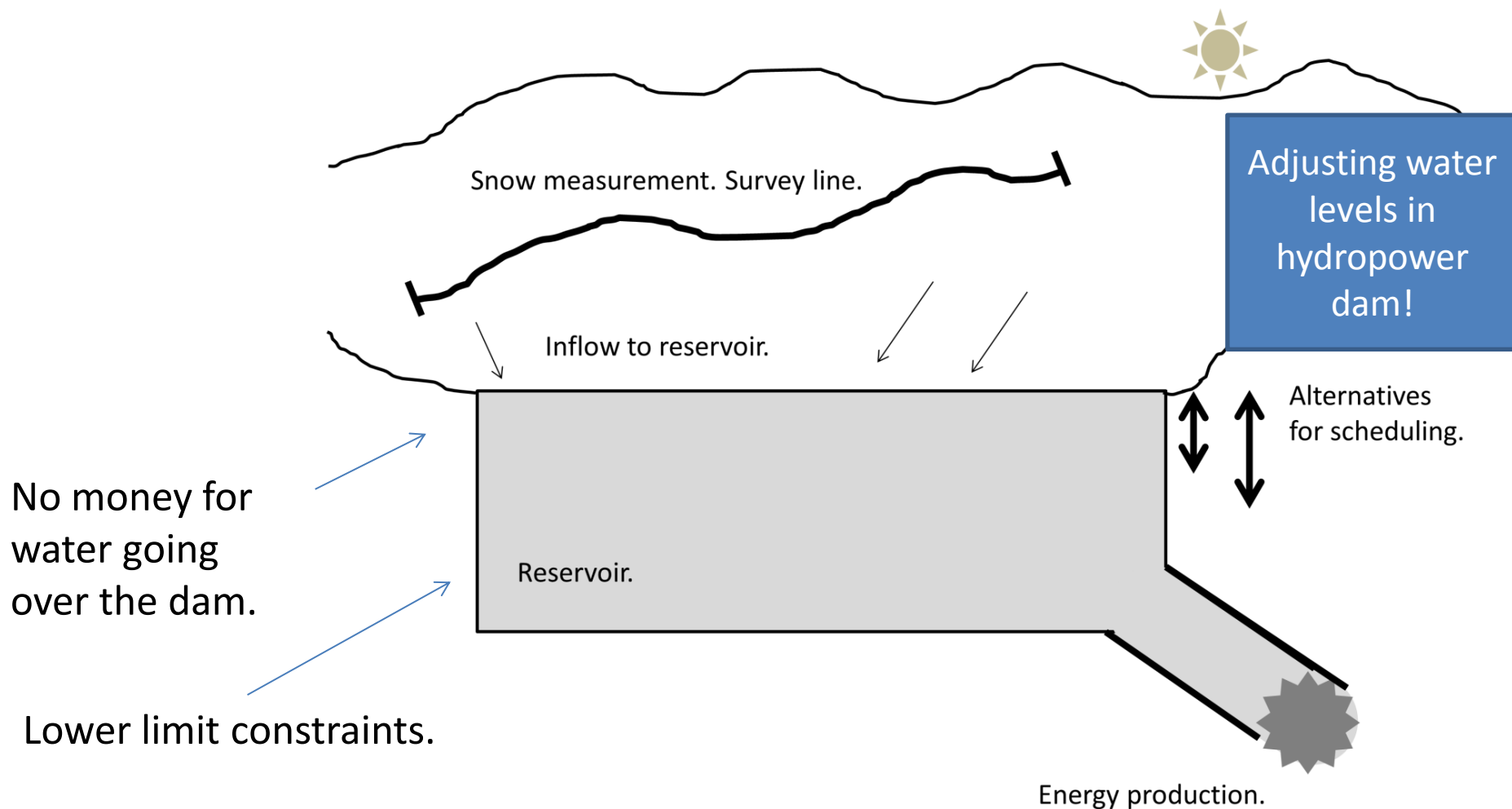
# Motivation (a groundwater example)

Which recharge location is better to prevent salt water intrusion?

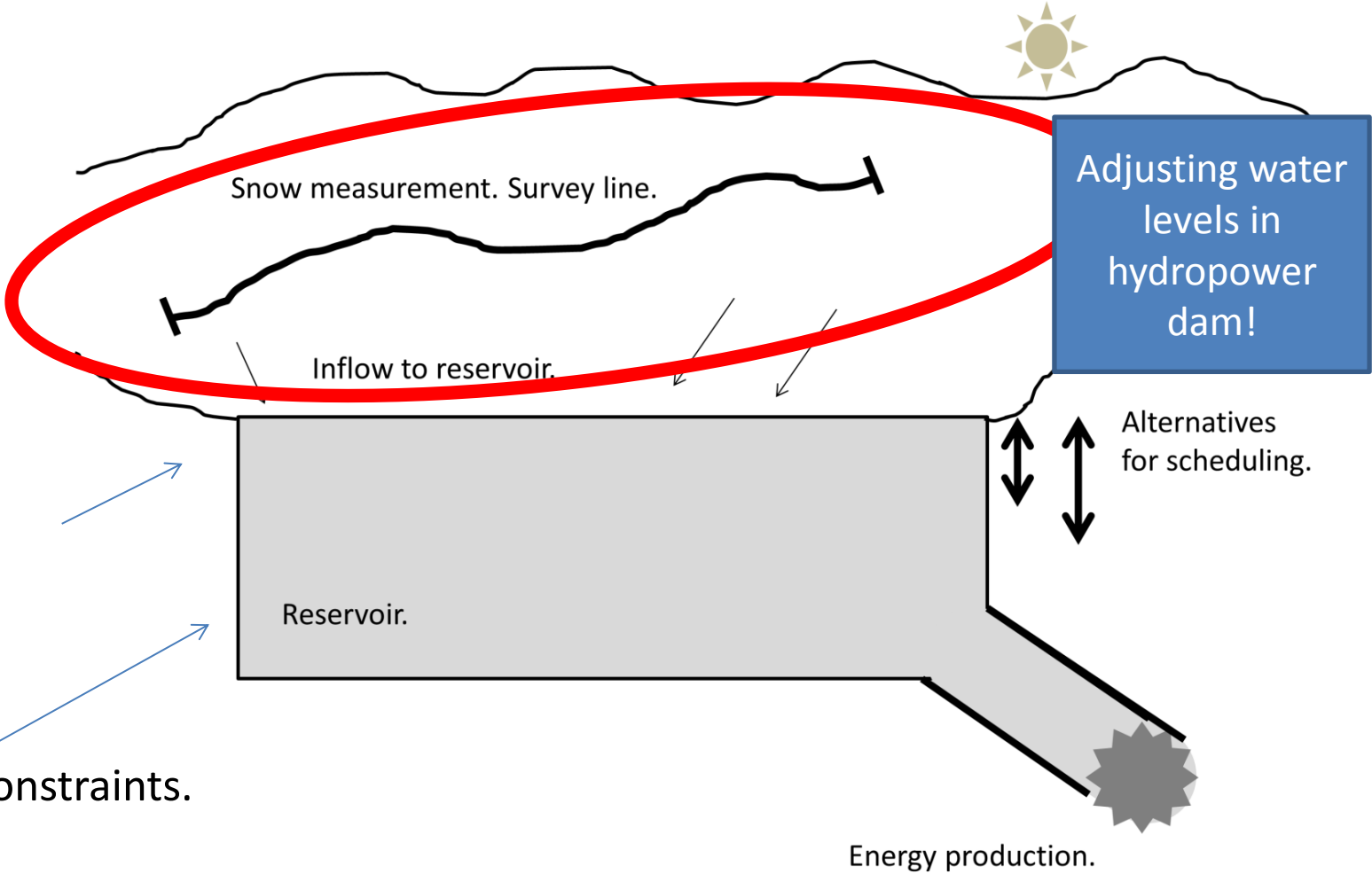


Is it worthwhile to acquire electromagnetic data before making the decision about recharge?

# Motivation (a hydropower example)



# Motivation (a hydropower example)



Acquire snow measurements?

# Other applications

- Farming and forestry – how to set up surveys for improved harvesting decisions?
- Biodiversity – where to monitor different biological variables for sustainability?
- Environmental – how monitor where pollutants are, to minimize risk or damage?
- Robotics - where should drone (UAV) or submarine (AUV) go to collect valuable data?
- Industry reliability – how to allocate sensors to ‘best’ monitor state of system?
- Internet of things – which sensors should be active now?



# Which data are valuable?

Five Vs of big data:

- Volume
- Variety
- Velocity
- Veracity
- **Value**

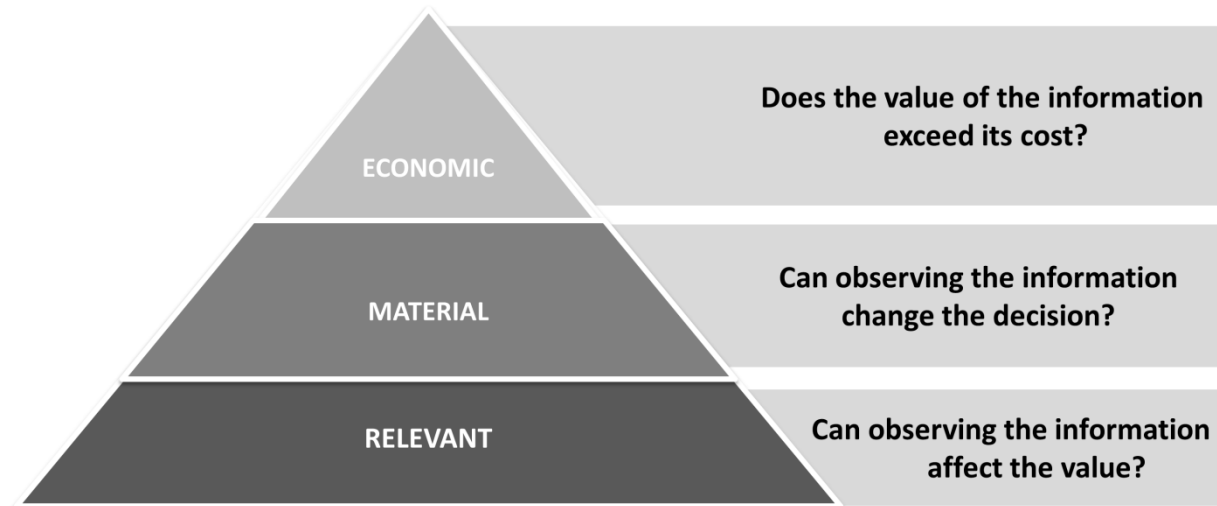


We must acquire and process data that has value!  
Data should help us answer a key question.

# Value of information (VOI)

We often consider purchasing more data before making difficult decisions under uncertainty.

The value of information (VOI) is useful for quantifying the value of the data, before it is acquired and processed.

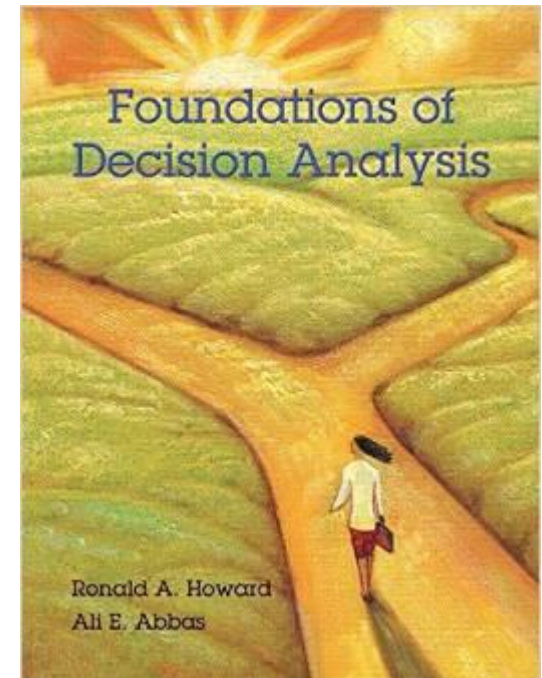


This pyramid of conditions - VOI is different from other information criteria (entropy, variance, prediction error, etc.)



# Decision analysis (DA)

*Decision analysis attempts to guide a decision maker to clarity of action in dealing with a situation where one or more decisions are to be made, typically in the face of uncertainty.*



Howard, R.A. and Abbas, A., 2015, *Foundations of Decision Analysis*, Prentice Hall.

# Framing a decision situation

**Rules of actional thought.** (Howard and Abbas, 2015)

- Frame your decision situation to address the decision makers true concerns.
- Base decisions on maximum expected utility.

‘...systematic and repeated violations of these principles will result in inferior long-term consequences of actions and a diminishes quality of life...’

*(Edwards et al., 2007, Advances in decision analysis: From foundations to applications, Cambridge University Press.)*

# Pirate example

(For motivating decision analysis and VOI)



# Pirate example

- **Pirate example:** A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).



Pirate makes decision based on preferences and maximum **utility** or **value**!

- Digging cost.
- Revenues if he finds the treasure .

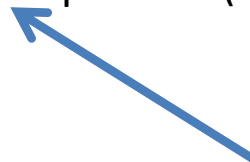
# Pirate example

- **Pirate example:** A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).

$$a \in \{0,1\}$$



$$x \in \{0,1\}$$



Pirate makes decision based on preferences and maximum **utility** or **value**!

- Digging cost.
- Revenues if he finds the treasure .



$$\max_{a \in \{0,1\}} \{E(v(x,a))\}$$

# Mathematics of decision situation:

- **Alternatives**

$$a \in \{0,1\} = A$$

- **Uncertainties (probability distribution)**

$$x \in \{0,1\} = \Omega \quad p(x=1) = 0.01$$

- **Values**

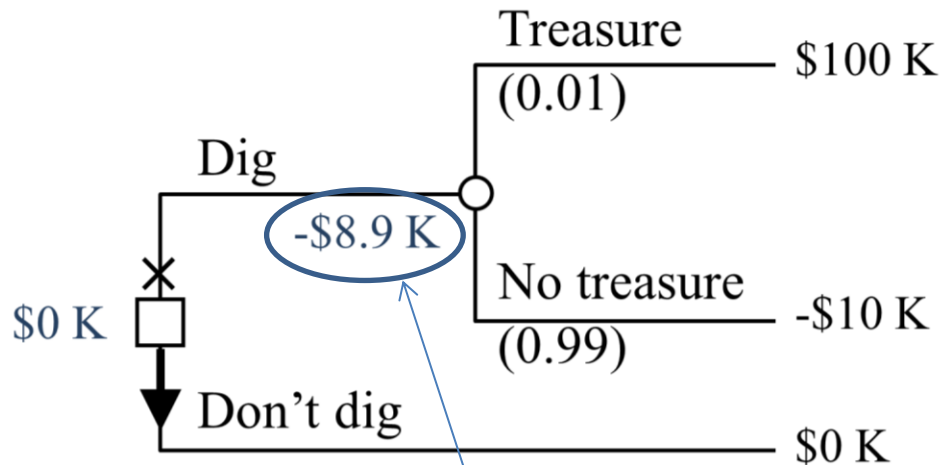
$$v = v(x, a)$$

$$v(x=0, a=1) = -10000 \quad v(x=1, a=1) = 100000 \quad v(x, a=0) = 0$$

- **Maximize expected value**

$$a^* = \arg \max_{a \in A} \{E(v(x, a))\}$$

# Pirate's decision situation



Risk neutral!

$$E(u(v_{dig})) = E(v_{dig}) = 0.01(100000) + 0.99(-10000) = -8900$$

# Pirate example



- **Pirate example:** A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).
- Pirate can collect **data** before making the decision, if the experiment is worth its price!



- Perfect information.  
Clairvoyant!



- Imperfect information.  
Detector!



# Value of information (VOI)

- VOI analysis is used to compare the additional value of making informed decisions with the price of the information.
- If the VOI exceeds the price, the decision maker should purchase the data.

$$\text{VOI} = \text{Posterior value} - \text{Prior value}$$

# VOI – Pirate considers clairvoyant

$$PV = 0 = \$0K$$



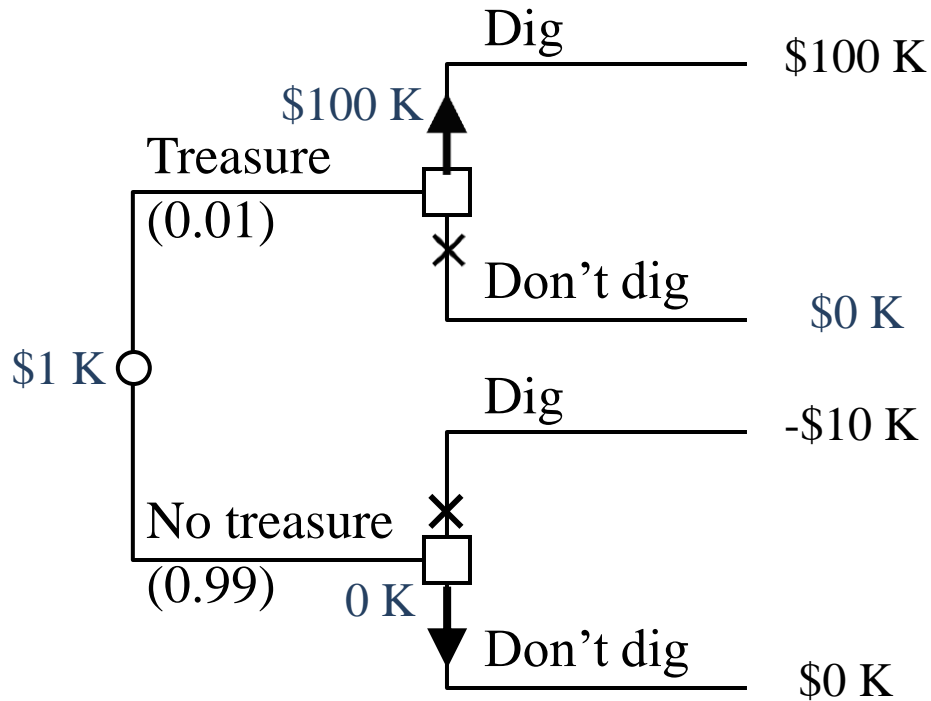
$$PoV(x) = \sum_x \max_{a \in A} \{v(x, a)\} p(x)$$

$$= \left(0.01 \cdot \max\{0, 100\}\right) + \left(0.99 \cdot \max\{0, -10\}\right) = \$1K$$

$$VoI(x) = PoV(x) - PV = 1 - 0 = \$1K$$

Conclusion: Consult clairvoyant if (s)he charges less than \$1000.

# PoV – decision tree, perfect information



# Pirate example - detector



- **Pirate example:** A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).
- Pirate can collect **data** before making the decision, if the experiment is worth its price!




Pirate makes decision based on preferences and maximum expected **value!**


- Digging cost.
- Revenues if he finds the treasure .


# Pirate example - detector



- **Pirate example:** A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).
- Pirate can collect **data** with a detector before making the decision, if this experiment is worth its price!

$$a \in \{0,1\}$$



$$x \in \{0,1\}$$


$$y \in \{0,1\}$$




Pirate makes decision based on preferences and maximum expected **value!**

- Digging cost.
- Revenues if he finds the treasure .

$$\max_{a \in \{0,1\}} \{E(v(x,a) | y)\}$$


# Detector experiment

Accuracy of test:

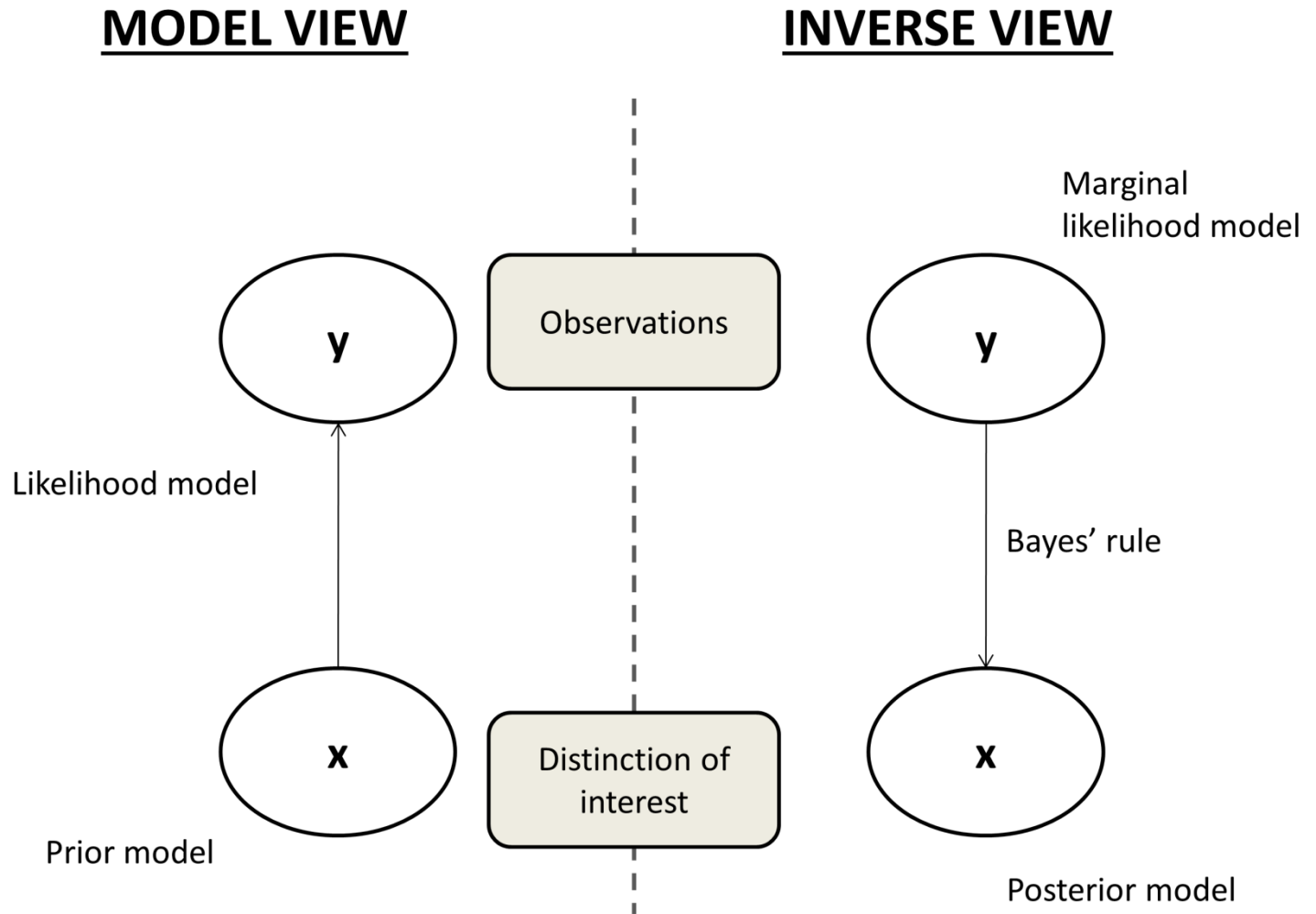
$$p(y = 0 | x = 0) = p(y = 1 | x = 1) = 0.95$$



Should the pirate pay to do a detector experiment?

Does the VOI of this experiment exceed the price of the test?

# Bayes rule - Detector experiment



# Bayes rule - Detector experiment

Likelihood:

$$p(y = 0 | x = 0) = p(y = 1 | x = 1) = 0.95$$



Marginal likelihood:

$$\begin{aligned} p(y = 1) &= p(y = 1 | x = 0) p(x = 0) + p(y = 1 | x = 1) p(x = 1) \\ &= 0.05 \cdot 0.99 + 0.95 \cdot 0.01 = 0.06 \end{aligned}$$

Posterior:

$$p(x = 1 | y = 1) = \frac{p(y = 1 | x = 1) p(x = 1)}{p(y = 1)} = \frac{0.95 \cdot 0.01}{0.06} \approx 0.16 = 16 / 100.$$

$$p(x = 1 | y = 0) = \frac{p(y = 0 | x = 1) p(x = 1)}{p(y = 0)} = \frac{0.05 \cdot 0.01}{0.94} \approx 0.0005 = 5 / 10000.$$



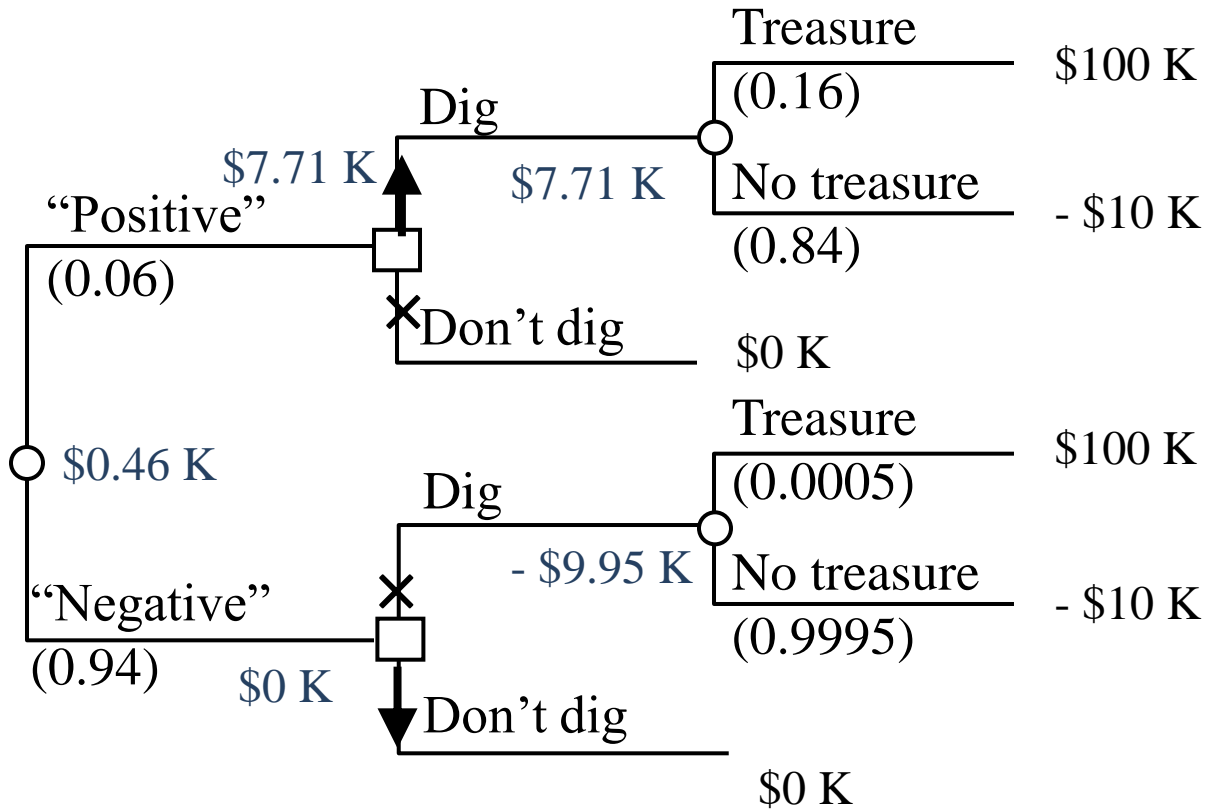
# VOI – Pirate considers detector test

$$\begin{aligned}PoV(y) &= \sum_y \max_{a \in A} \{E(v(x, a) | y)\} p(y) \\&= \left(0.06 \cdot \max\{0, (100 \cdot 0.16) + (-10 \cdot 0.84)\}\right) \\&\quad + \left(0.94 \cdot \max\{0, (100 \cdot 0.0005) + (-10 \cdot 0.9995)\}\right) \\&= \left(0.06 \cdot \max\{0, 7.71\}\right) + \left(0.94 \cdot \max\{0, -9.95\}\right) = \$0.46K.\end{aligned}$$

$$VoI(y) = PoV(y) - PV = 0.46 - 0 = \$0.46K$$

Conclusion: Purchase detector testing if its price is less than \$460.

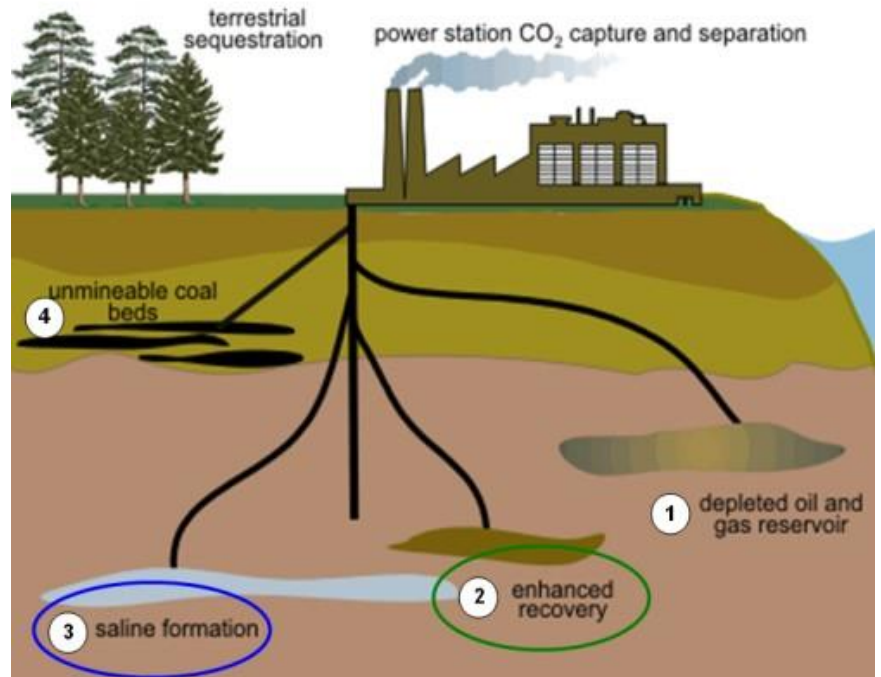
# PoV - imperfect information



# Exercise: CO<sub>2</sub> sequestration

CO<sub>2</sub> is sequestered to reduce carbon emission in the atmosphere and defer global warming.

Geological sequestration involves pumping CO<sub>2</sub> in subsurface layers, where it will remain, unless it leaks to the surface.



# VOI for CO2 sequestration

The decision maker can proceed with CO2 injection or suspend sequestration. The latter incurs a tax of 80 monetary units. The former only has a cost of injection equal to 30 monetary units, but the injected CO2 may leak ( $x=1$ ). If leakage occurs, there will be a fine of 60 monetary units (i.e. a cost of 90 in total). Decision maker is risk neutral.

$$p(x=1) = 0.3$$

$$p(x=0) = 0.7$$

Data: Geophysical experiment, with binary outcome, indicating whether the formation is leaking or not.

$$p(y=0 | x=0) = 0.95$$

$$p(y=1 | x=1) = 0.9$$

## Exercise:

1. Draw the decision tree without information.
2. Draw the decision tree with perfect information (clairvoyance).
3. Compute the VOI of perfect information.
4. Draw the decision tree with the geophysical experiment.
5. Compute conditional probabilities, expected values and the VOI of geophysical data.

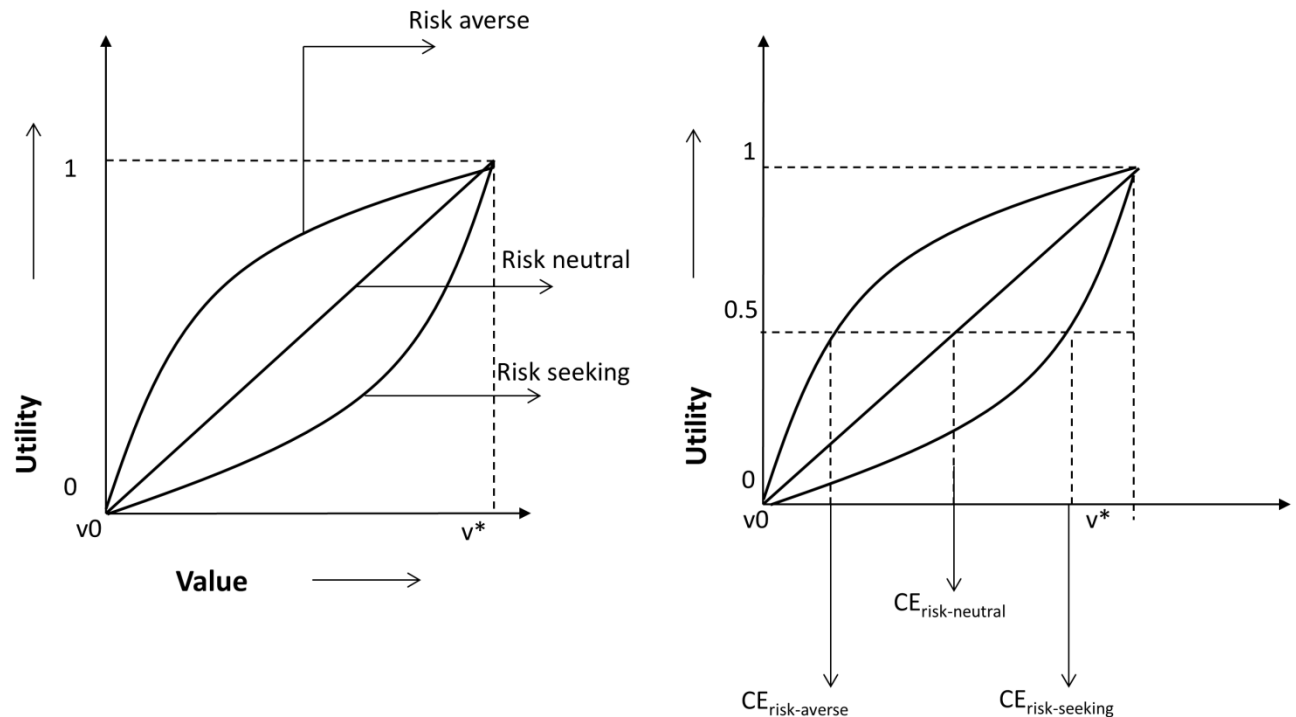
# Value of information (VOI)

## - More general formulation

- VOI analysis is used to compare the additional value of making informed decisions with the price of the information.
- If the VOI exceeds the price, the decision maker should purchase the data.

$$\text{VOI} = \text{Posterior value} - \text{Prior value}$$

# Risk and utility functions

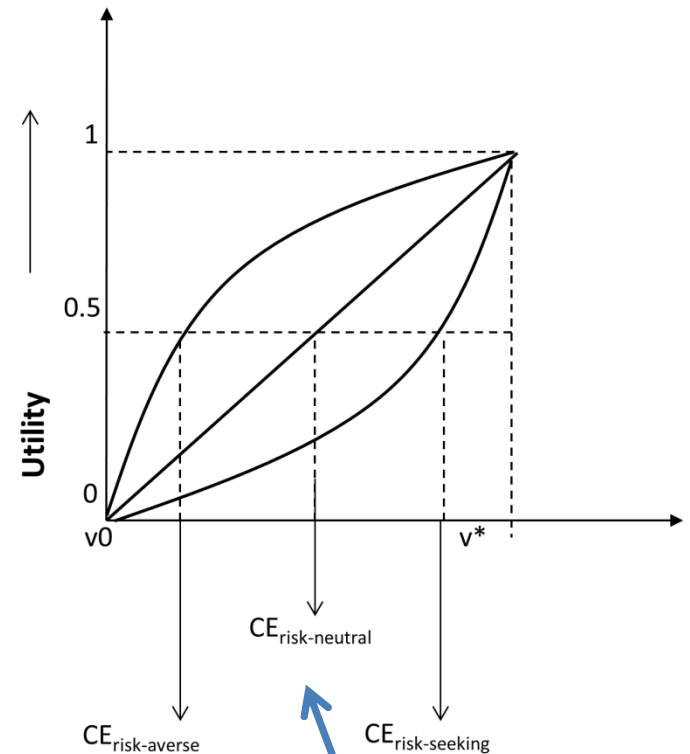
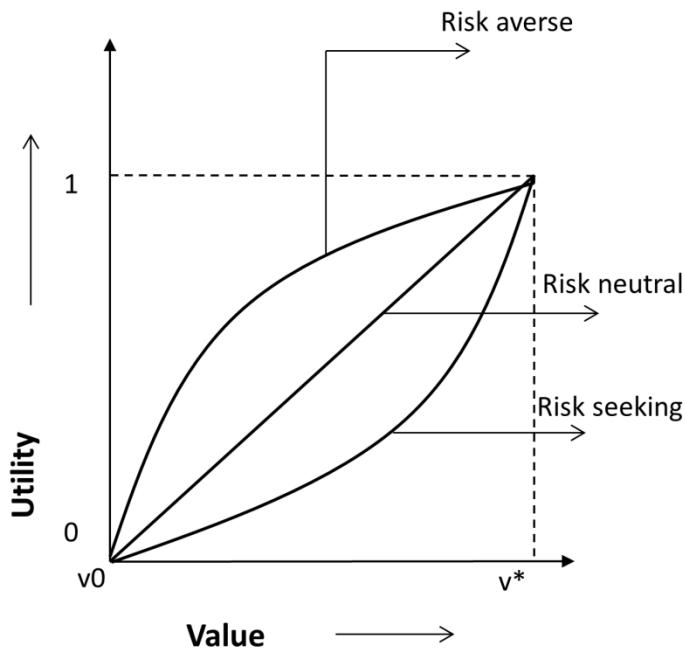


Exponential and linear utility have constant risk aversion coefficient:

$$\gamma = - \frac{u''(v)}{u'(v)}$$

# Certain equivalents (CE)

Utilities are mathematical. The certain equivalent is a measure of how much a situation is worth to the decision maker. (It is measured in value).



$$CE = u^{-1} \left( \max \left\{ E(u(v_{dig})), E(u(v_{don't dig})) \right\} \right)$$

What is the value of indifference? How much would the owner of a lottery be willing to sell it for?

# VOI - Clairvoyance

Price  $P$  of experiment makes the equality.

$$\sum_x \max_{a \in A} \{v(x, a) - P\} p(x) = \max_{a \in A} \{E(v(x, a))\}$$

$$\rightarrow P = VOI = \sum_x \max_{a \in A} \{v(x, a)\} p(x) - \max_{a \in A} \{E(v(x, a))\}$$

VOI=Posterior value – Prior value

Assuming risk neutral decision maker!



# Value of information- Imperfect

Price of indifference.

$$\sum_y \max_{a \in A} \{E(v(x, a) - P | y)\} p(y) = \max_{a \in A} \{E(v(x, a))\}$$

$$\sum_y \max_{a \in A} \{E(v(x, a) - P | y)\} p(y) = \max_{a \in A} \{E(v(x, a))\}$$

$$\rightarrow P = VOI = \sum_y \max_{a \in A} \{E(v(x, a) | y)\} p(y) - \max_{a \in A} \{E(v(x, a))\}$$

VOI=Posterior value – Prior value

Assuming risk neutral decision maker!

# Properties of VOI

a) VOI is always positive

- Data allow better, informed decisions.

$$\max \left\{ 0, \sum_i v_i \right\} \leq \sum_i \max \{ 0, v_i \}$$

b) If value is in monetary units, VOI is in monetary units.

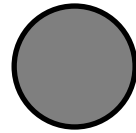
c) Data should be purchased if  $VOI > \text{Price of experiment } P$ .

d) VOI of clairvoyance is an upper bound for any imperfect information gathering scheme.

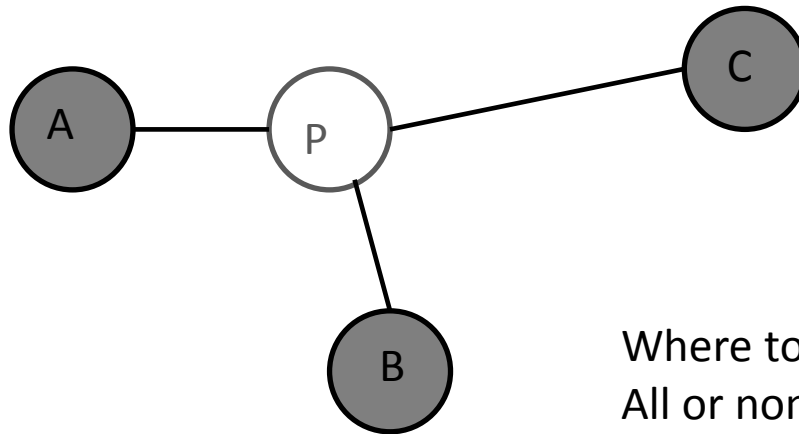
e) When we compare different experiments, we purchase the one with largest VOI compared with the price:

$$\arg \max \{ VOI_1 - P_1, VOI_2 - P_2 \}$$

What if several projects or treasures?



# What if several projects or treasures?



Where to invest?

All or none? Free to choose as many as profitable? One at a time, then choose again?

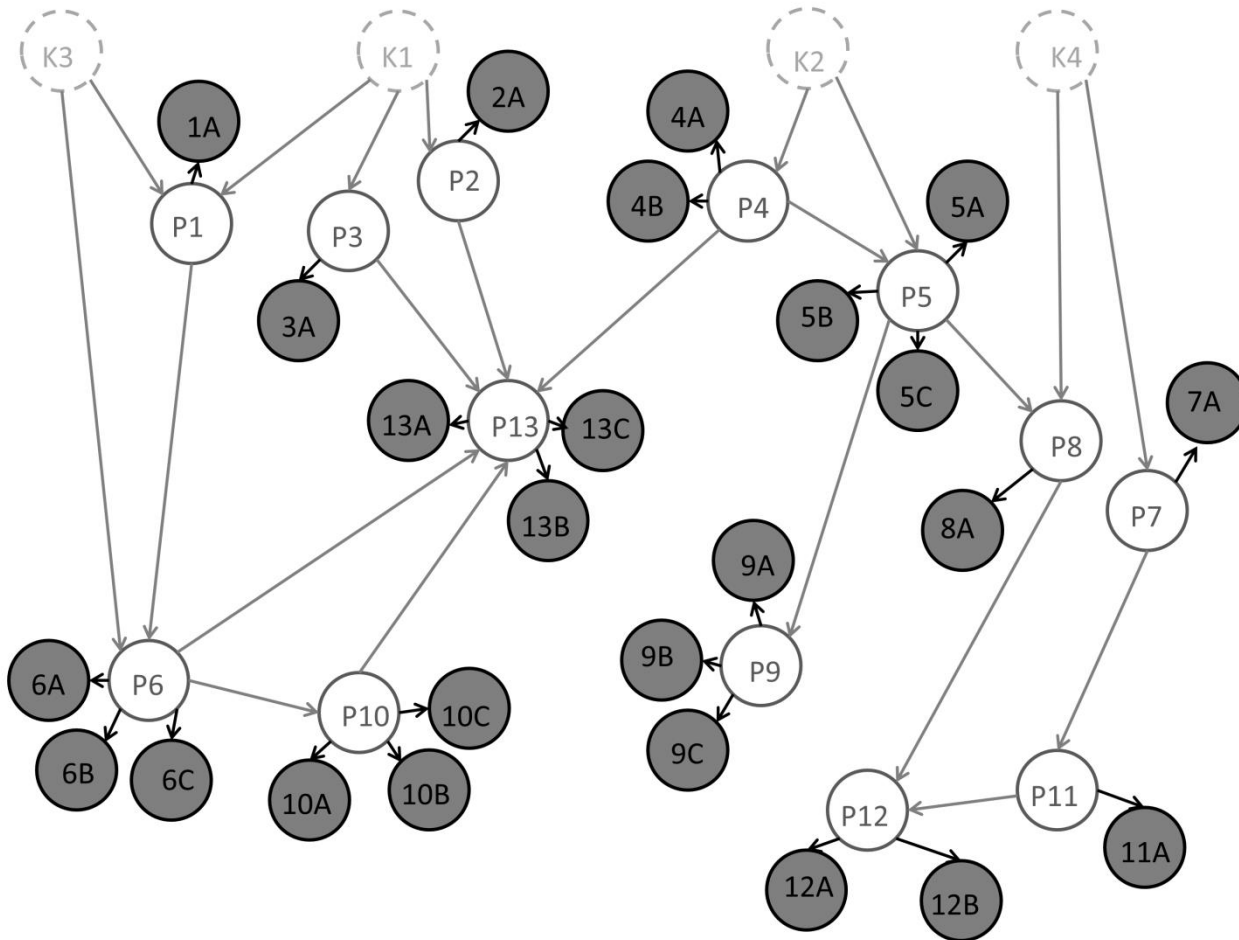
Where should one collect data? All or none? One only? Or two? One first, then maybe another?

# VOI and Earth sciences

- **Alternatives are spatial**, often with high flexibility in selection of sites, control rates, intervention, excavation opportunities, harvesting, etc.
- **Uncertainties are spatial**, with multi-variable interactions . Often both discrete and continuous.
- **Value function is spatial**, typically involving coupled features, say through differential equations. It can be defined by «physics» as well as economic attributes.
- **Data are spatial**. There are plenty opportunities for partial, total testing and a variety of tests (surveys, monitoring sensors, electromagnetic data, , etc.)

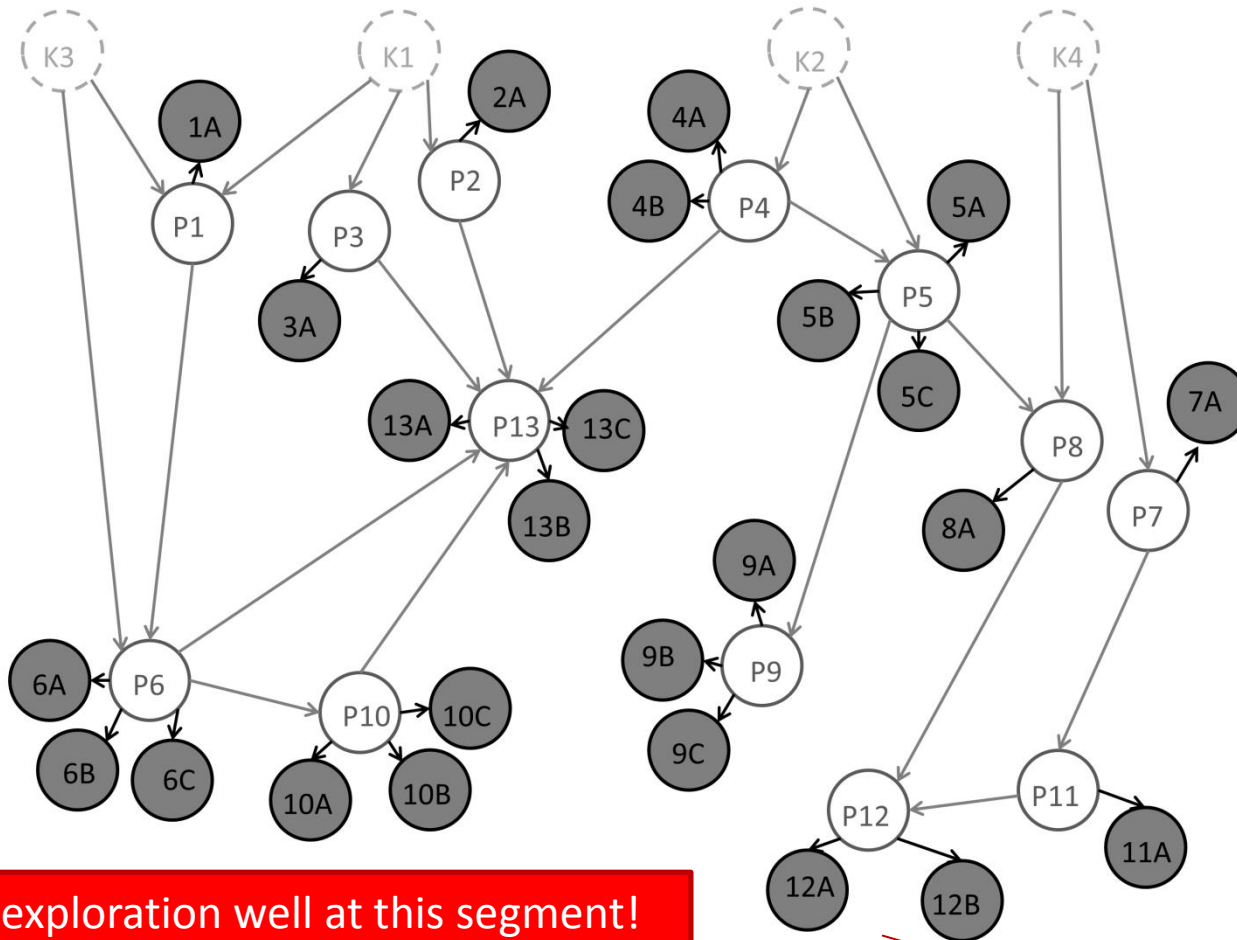
# Dependence matters?

Gray nodes are petroleum reservoir segments where the company aims to develop profitable amounts of oil and gas.



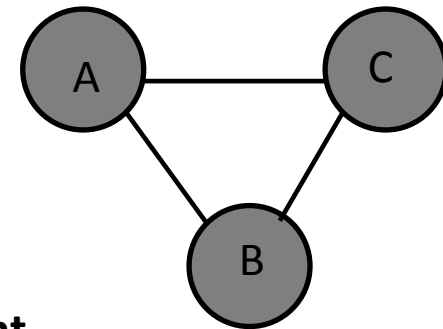
# Dependence matters – value matters?

Gray nodes are petroleum reservoir segments where the company aims to develop profitable amounts of oil and gas.



Drill the exploration well at this segment!  
The value of information is largest.

# Joint modeling of multiple variables



Independence is rare!

To study if dependence matter, we need to model the **joint** properties of uncertainties.

- What is the probability that variable A is 1 and, at the same time, variable B is 1 ?
- What is the probability that variable C is 0, and both A and B are 1 ?

If information is available about one variable, it affects our knowledge about other variables.



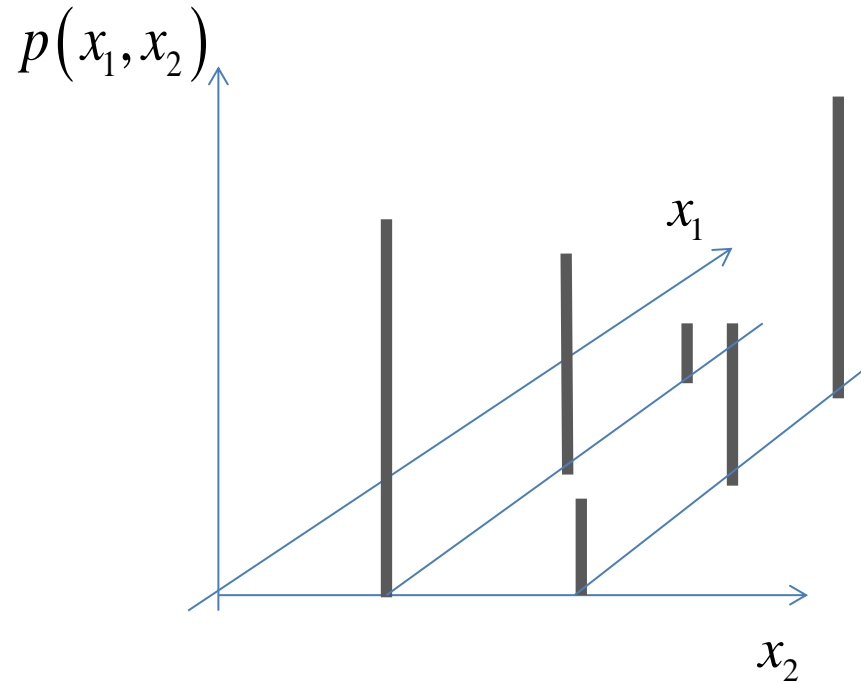
# Joint pdf

$$p(\mathbf{x}) = p(x_1, \dots, x_n)$$

Discrete sample  
space:

$$p(\mathbf{x}) \geq 0, \quad \mathbf{x} \in \Omega,$$
$$\sum_{x_1 \in \Omega_1} \dots \sum_{x_n \in \Omega_n} p(\mathbf{x}) = 1.$$

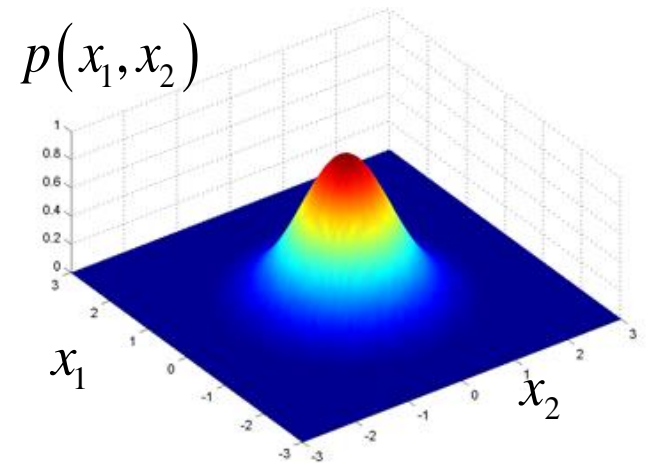
Probability mass function (pmf)



Continuous  
sample space:

$$p(\mathbf{x}) \geq 0, \quad \mathbf{x} \in \Omega,$$
$$\int_{x_1 \in \Omega_1} \dots \int_{x_n \in \Omega_n} p(\mathbf{x}) dx_1 \dots dx_n = 1.$$

Probability density function (pdf)



# Marginal and conditional probability

$$\mathbf{x} = (\mathbf{x}_{\mathbb{K}}, \mathbf{x}_{\mathbb{L}})$$

$$p(\mathbf{x}_{\mathbb{K}}) = \int p(\mathbf{x}) d\mathbf{x}_{\mathbb{L}}$$

$$p(\mathbf{x}_{\mathbb{K}} | \mathbf{x}_{\mathbb{L}}) = \frac{p(\mathbf{x})}{p(\mathbf{x}_{\mathbb{L}})} = \frac{p(\mathbf{x})}{\int p(\mathbf{x}) d\mathbf{x}_{\mathbb{K}}}$$

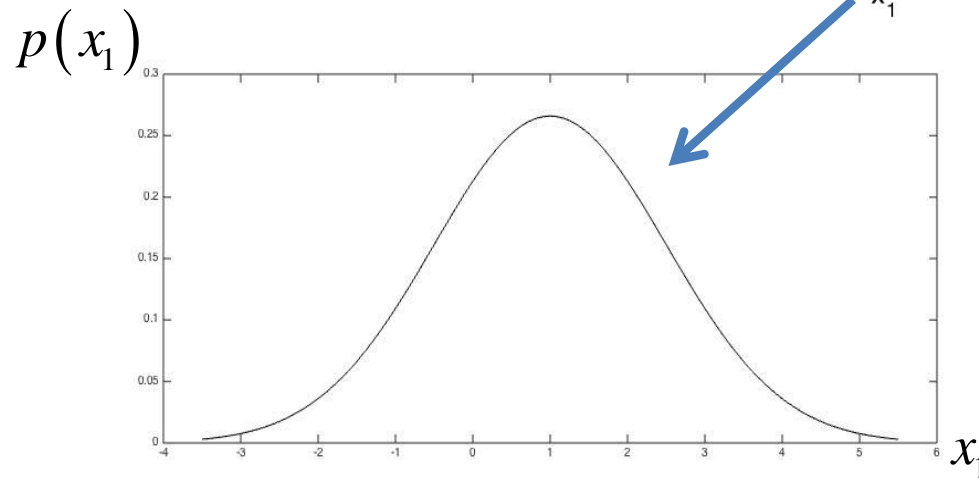
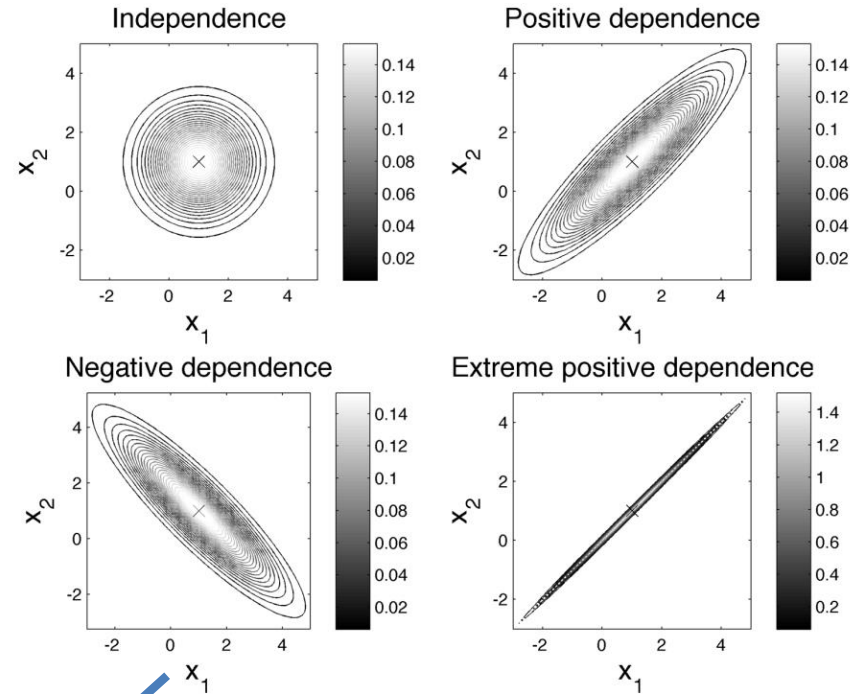
Marginalization in joint pdf.

Conditioning in joint pdf.

# Marginalization

$$\mathbf{x} = (\mathbf{x}_{\mathbb{K}}, \mathbf{x}_{\mathbb{L}})$$

$$p(\mathbf{x}_{\mathbb{K}}) = \int p(\mathbf{x}) d\mathbf{x}_{\mathbb{L}}$$

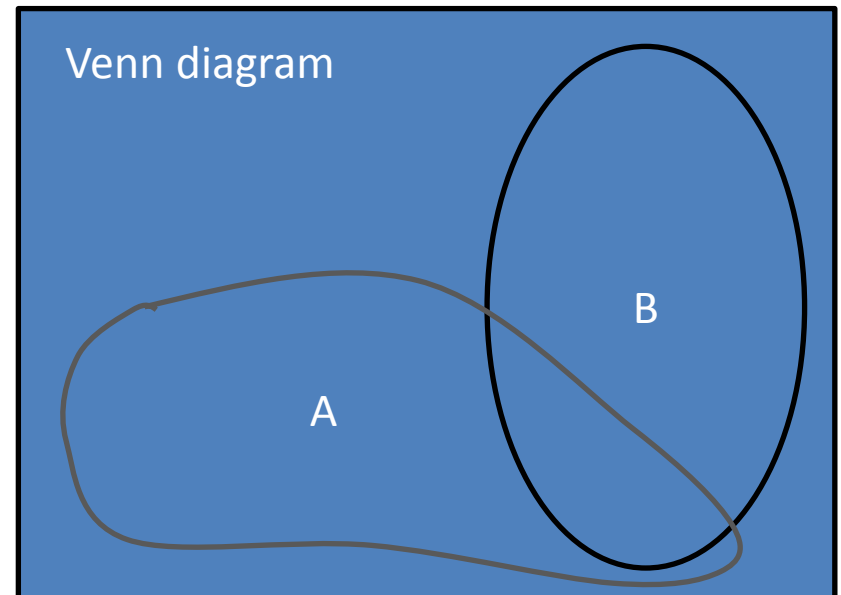


# Conditional probability

$$p(\mathbf{x}_{\mathbb{K}} | \mathbf{x}_{\mathbb{L}}) = \frac{p(\mathbf{x})}{p(\mathbf{x}_{\mathbb{L}})} = \frac{p(\mathbf{x})}{\int p(\mathbf{x}) d\mathbf{x}_{\mathbb{K}}}$$

$$p(A | B) = \frac{\text{Area}(A \cap B)}{\text{Area}(B)}$$

$$B = (A \cap B) \cup (A^c \cap B)$$



# Modeling by conditional probability

The **joint** pdf can be difficult to model directly.

Instead we can build the joint pdf from **conditional** distributions.

$$p(\mathbf{x}) = p(\mathbf{x}_{\mathbb{K}} | \mathbf{x}_{\mathbb{L}}) p(\mathbf{x}_{\mathbb{L}})$$

$$p(\mathbf{x}) = p(x_1) p(x_2 | x_1) \dots p(x_n | x_{n-1}, \dots, x_1)$$



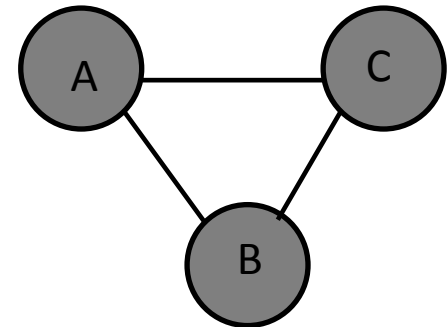
Holds for any ordering of variables.

# Modeling by conditional probability

Modeling can be done by conditional statements, rather than joint assessment:

- What is likely to happen for variable A when variable B is 1?
- What is the probability of variable C being 1 when variables A and B are both 0?

Such statements might be easier to specify,  
and can more easily be derived from physical principles.



# Modeling by conditional probability

$$p(\mathbf{x}) = p(x_1) p(x_2 | x_1) \dots p(x_n | x_{n-1}, \dots, x_1)$$

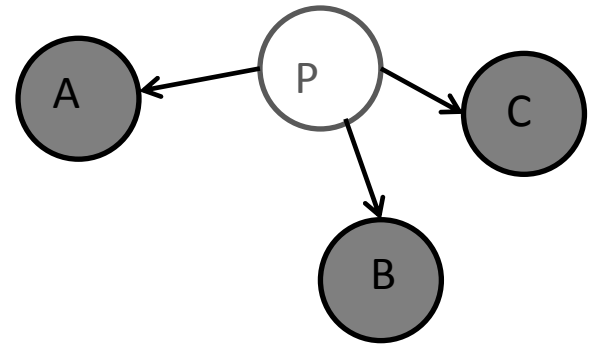


Holds for any ordering of variables. Some conditioning variables can often be skipped.  
Conditional independence in modeling.  
This simplifies modeling and interpretation! And computing!

# Modeling by conditional probability

Conditional independence:

$$p(x_A, x_B, x_C | x_P) = \prod_{i \in \{A, B, C\}} p(x_i | x_P)$$



- What is the chance of success at B, when there is success at parent P?
- What is the chance of success at B, when there is failure at parent P?

$$p(x_B = 1 | x_P = 1) = 0.9$$

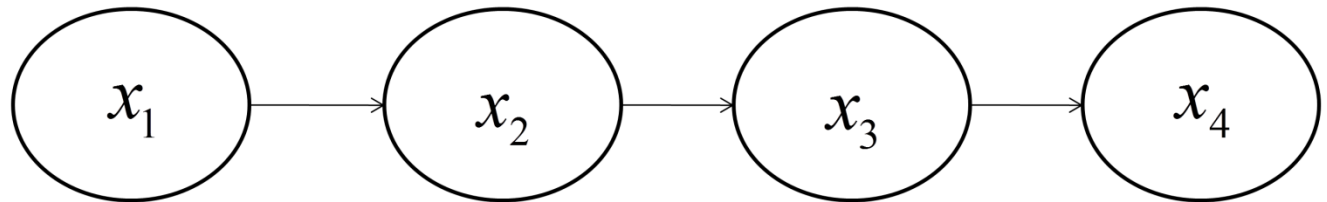
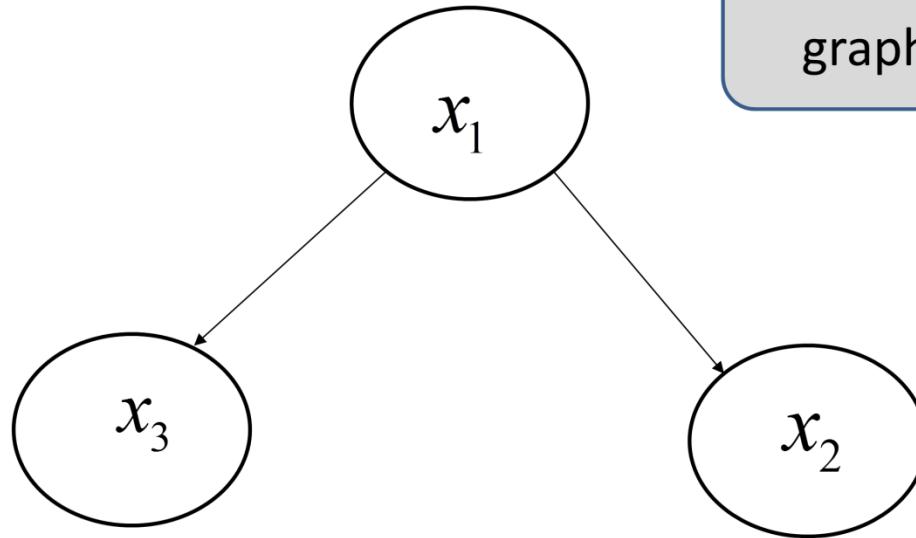
$$p(x_B = 1 | x_P = 0) = 0$$

Must set up models for all nodes, using marginals for root nodes, and conditionals for all nodes with edges.



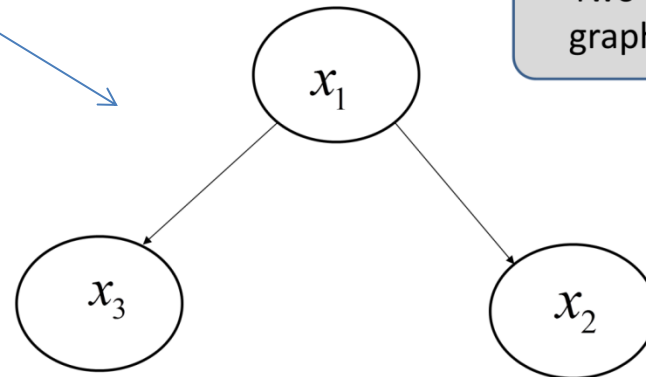
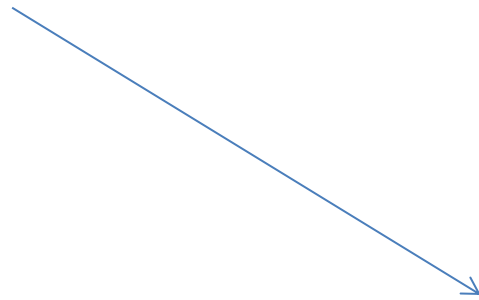
# Bayesian networks and Markov chains

Two examples of graphical models

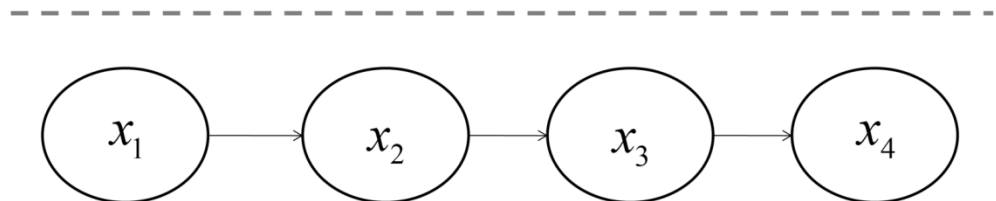


# Bayesian networks and Markov chains

$$p(\mathbf{x}) = p(x_1, x_2, x_3) = p(x_1)p(x_2|x_1)p(x_3|x_1)$$

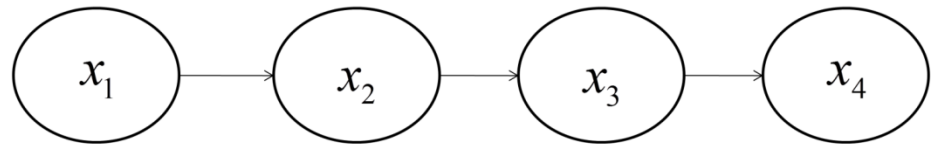
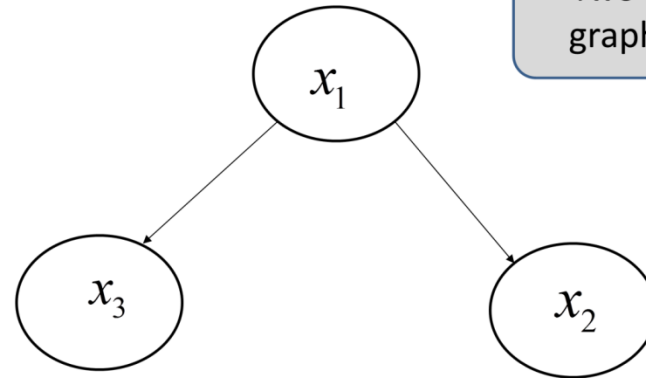


Two examples of graphical models



# Bayesian networks and Markov chains

Two examples of graphical models



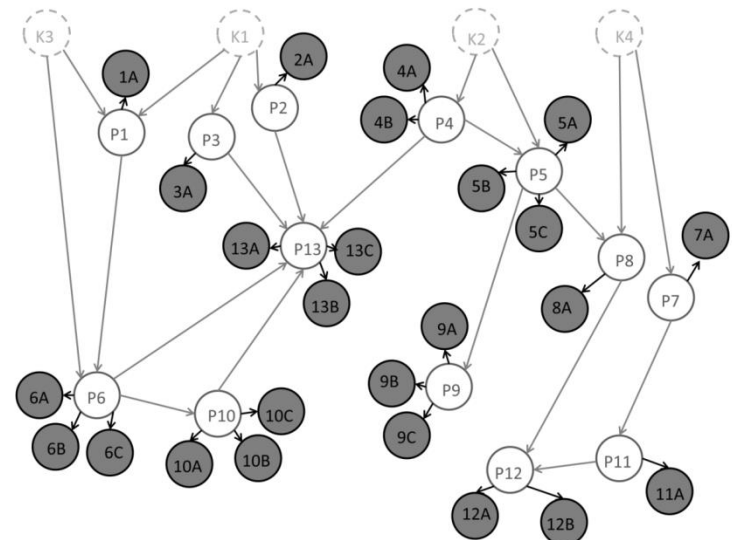
$$p(\mathbf{x}) = p(x_1, x_2, x_3, x_4) = p(x_1) p(x_2 | x_1) p(x_3 | x_2) p(x_4 | x_3)$$

# Larger networks / graphs - computation

Algorithms have been developed for efficient marginalization and conditioning.

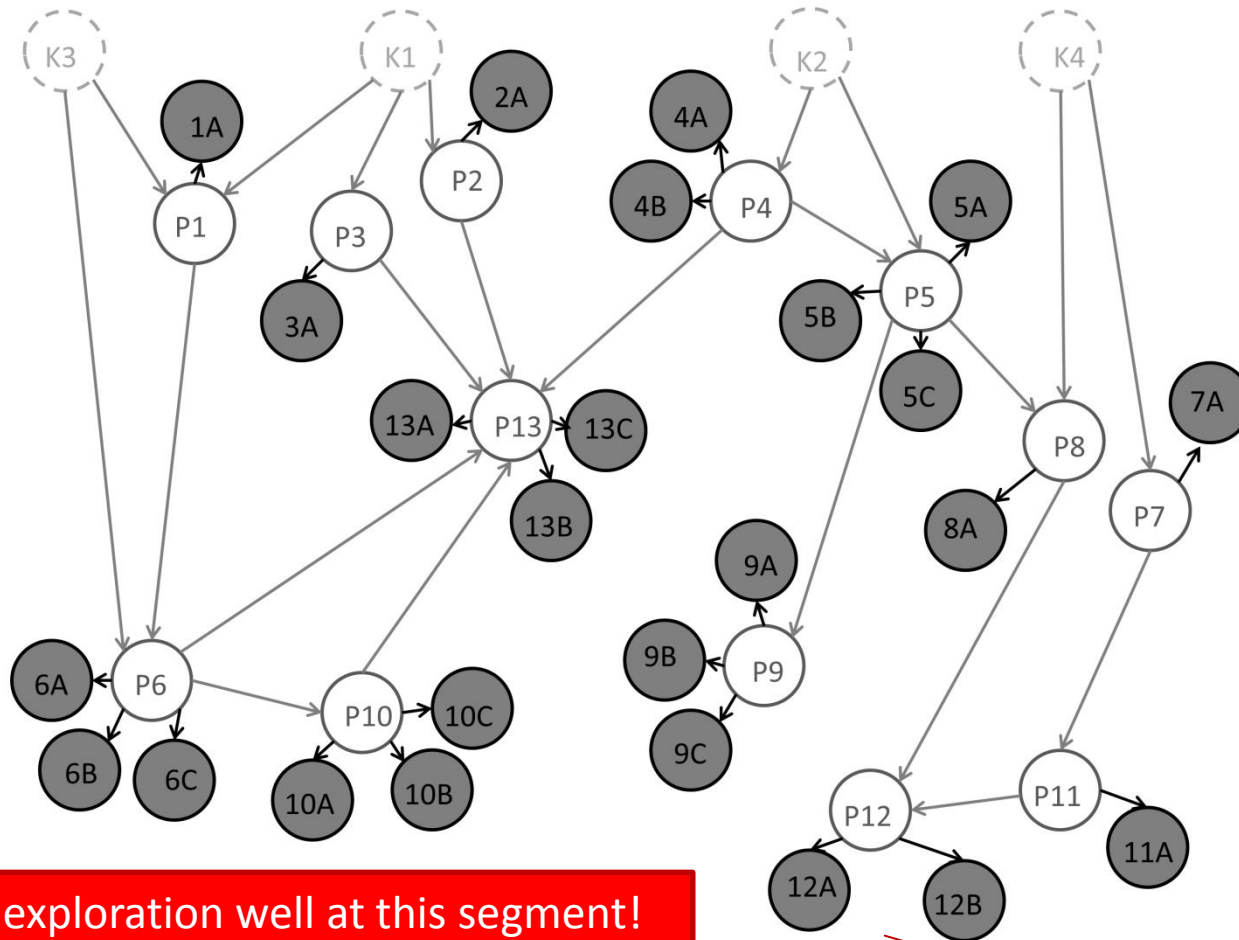
This enables fast evidence propagation and VOI calculations.

- Bayesian network models (Junction tree algorithm).
- (hidden)) Markov models (Forward-backward algorithms)



# Example of VOI analysis

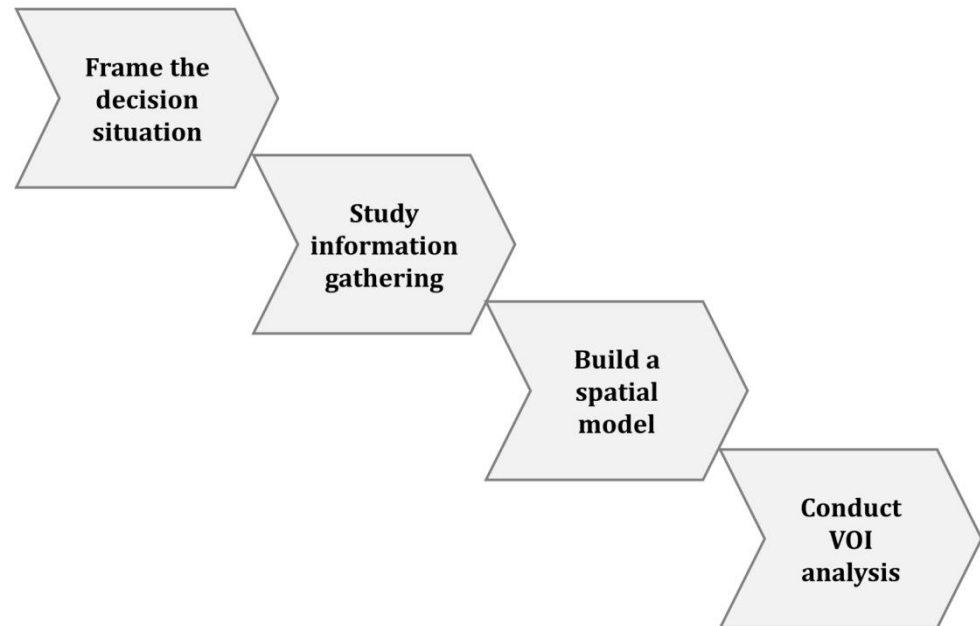
Gray nodes are petroleum reservoir segments where the company aims to develop profitable amounts of oil and gas.



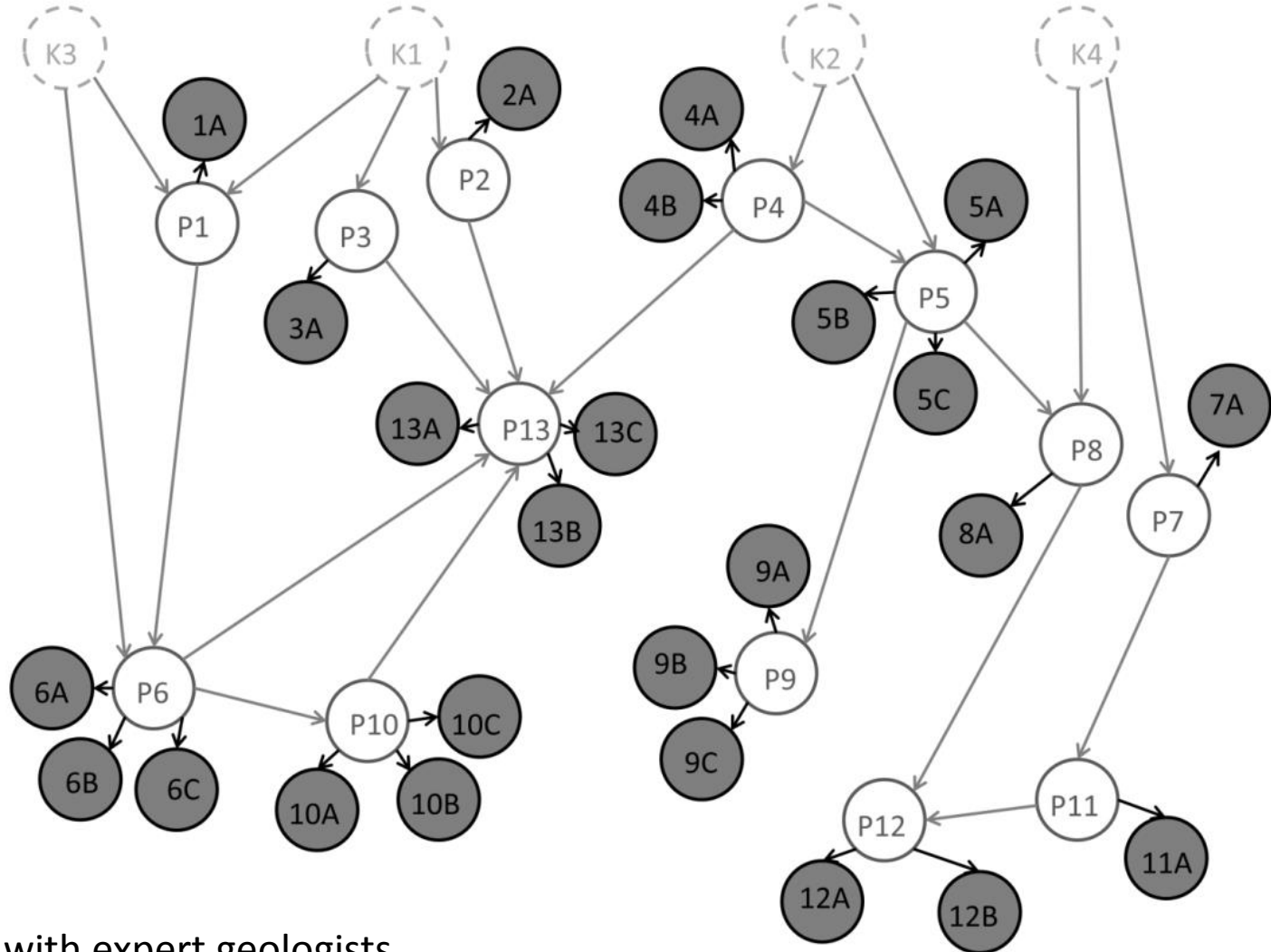
Drill the exploration well at this segment!  
The value of information is largest.

# VOI workflow

- Develop prospects separately. Shared costs for segments within one prospect.
- Gather information by exploration drilling. One well.
- Model is a Bayesian network model elicited from expert geologists in this area.
- VOI analysis done by exact computations for Bayesian networks. Conducted for all possible single exploration wells.



# Bayesian network , reservoir segments



Model elicited with expert geologists.

Source migration of gas (and oil) from kitchens (K).

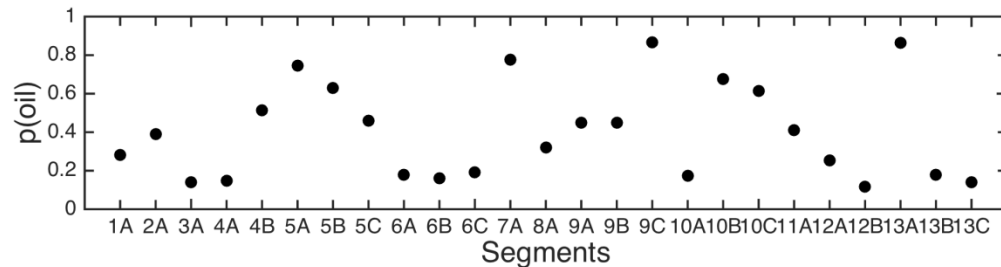
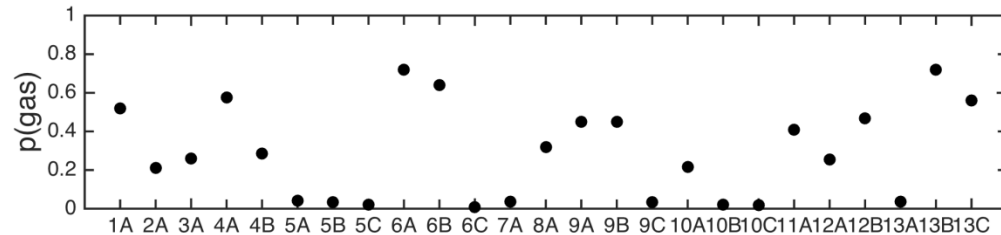
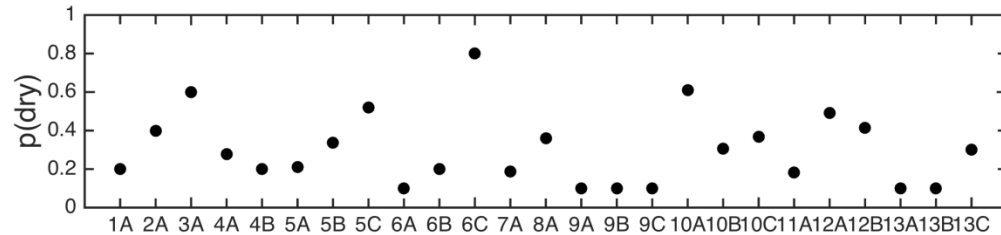
i) Local failure probability of migration

and ii) 
$$p(x_i = \text{dry} \mid \mathbf{x}_{Pa(i)} = \mathbf{dry}) = 1$$

# Prior marginal probabilities

Three possible  
classes at all  
nodes:

- Dry
- Gas
- Oil





# Prior values

Development fixed cost.  
Infrastructure at prospect r.

$$PV = \sum_{r=1}^{13} \max \left\{ 0, \sum_{i \in \text{Pr}} IV(x_i) - DFC \right\}$$

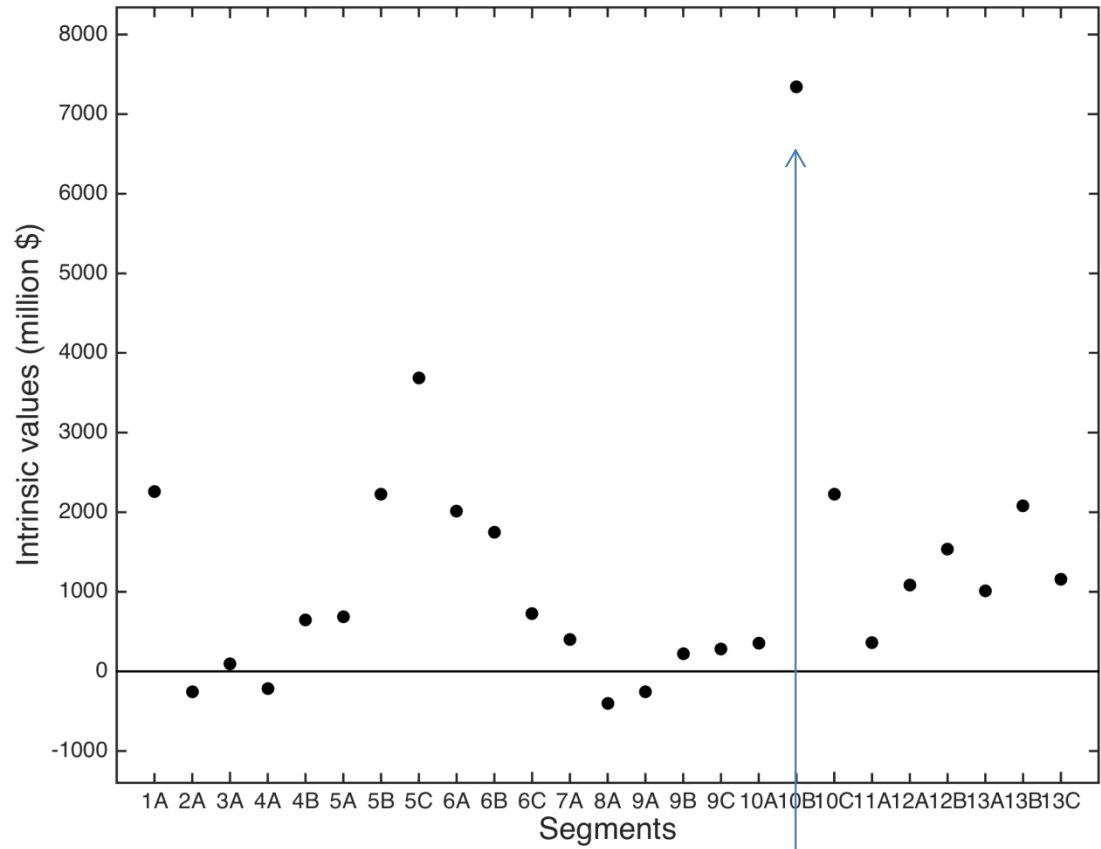
$$IV(x_i) = \sum_{k=1}^3 \left( \text{Rev}_{i,k} p(x_i = k) - \text{Cost}_{i,k} p(x_i = k) \right) - \text{Cost}_{i,0}$$

Revenues of oil/gas,  
0 otherwise.

Cost if dry,  
0 otherwise.

Cost of drilling  
segment i.

# Values




Most lucrative. But might not be most informative.

# Posterior values and VOI

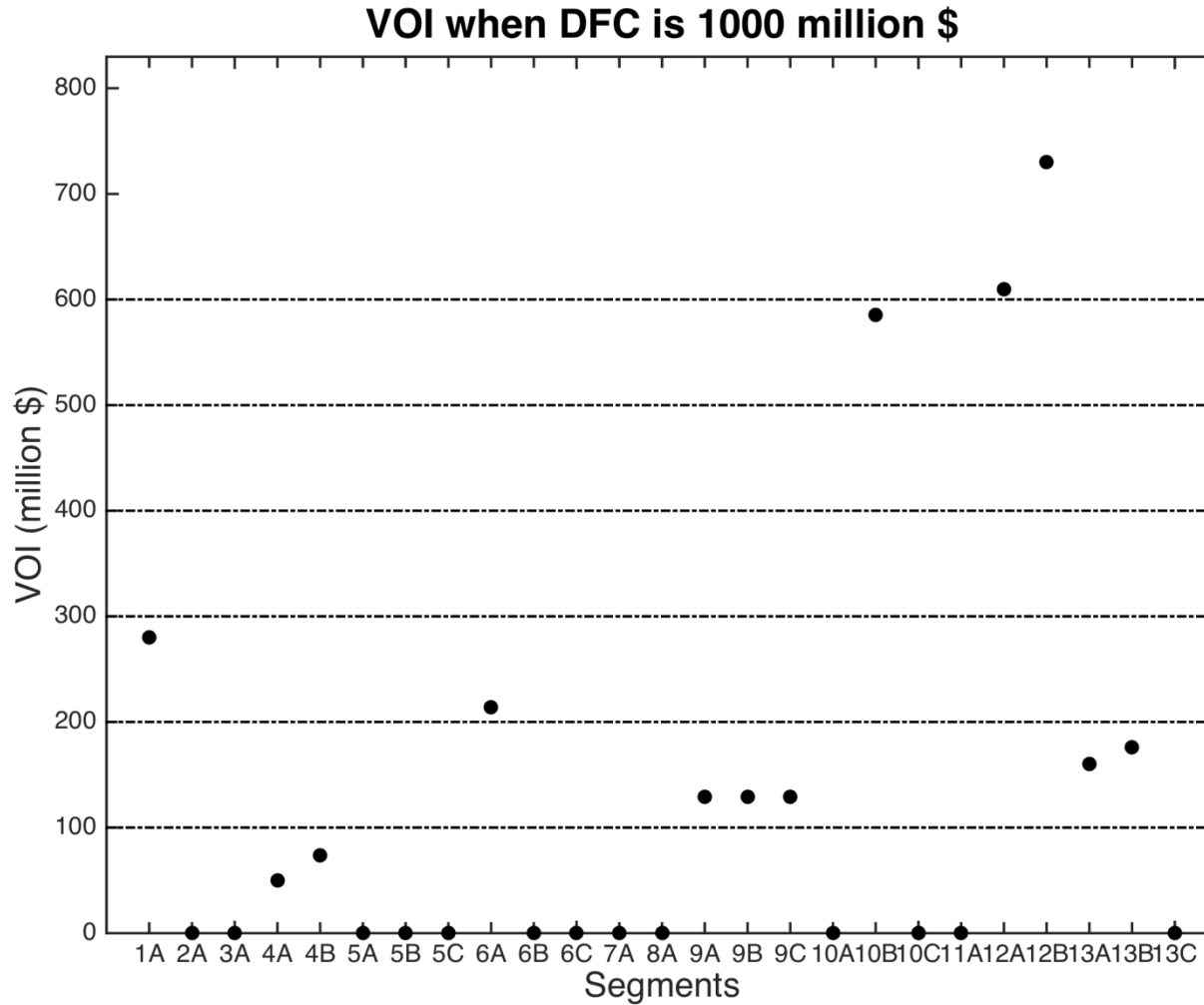
$$PoV(x_{\mathbb{K}}) = \sum_{l=1}^3 \sum_{r=1}^{13} \max \left\{ 0, \sum_{i \in Pr} IV(x_i | x_{\mathbb{K}} = l) - DFC \right\} p(x_{\mathbb{K}} = l)$$

$$VOI(x_{\mathbb{K}}) = PoV(x_{\mathbb{K}}) - PV$$

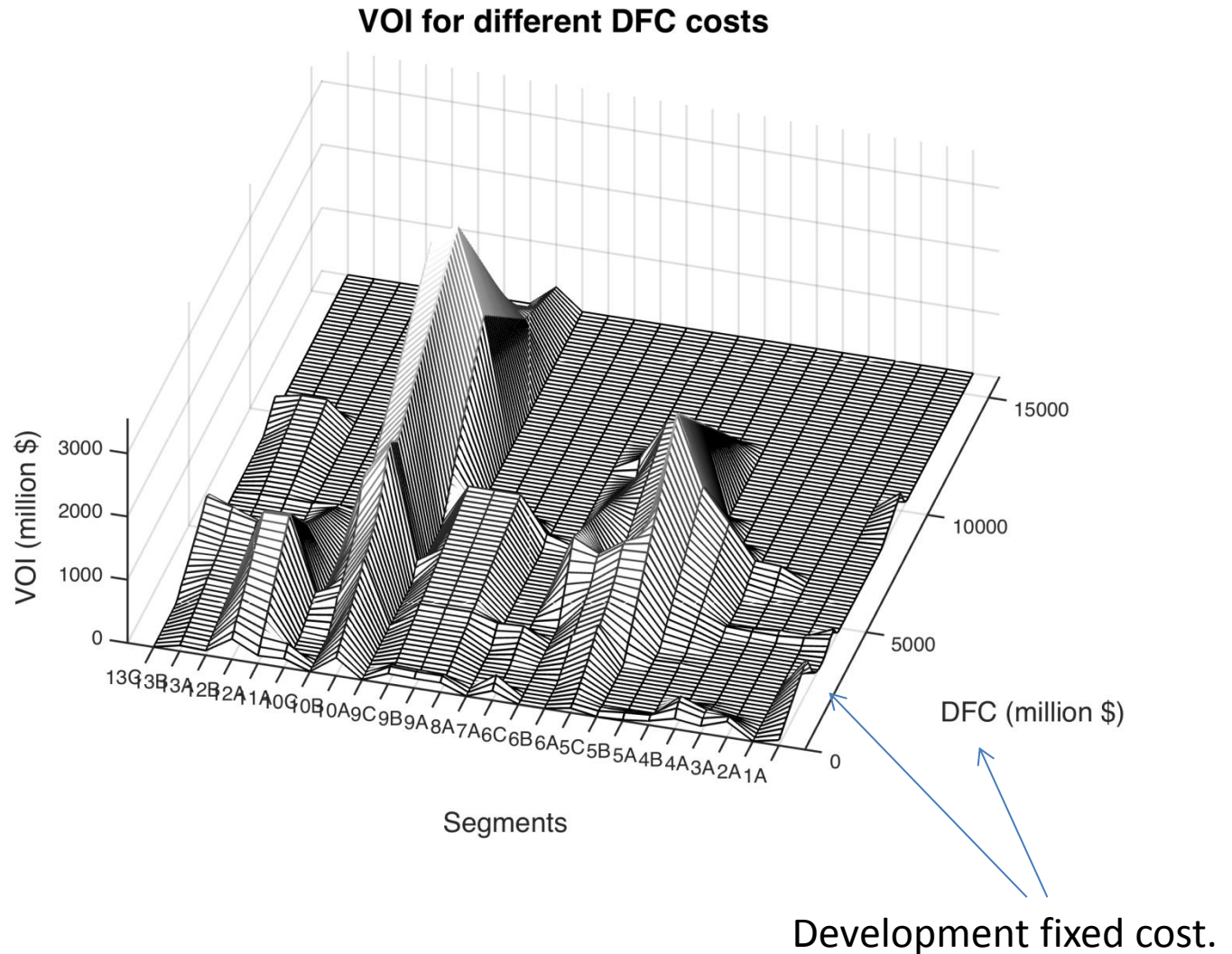
Data acquired at single well.



# VOI single wells

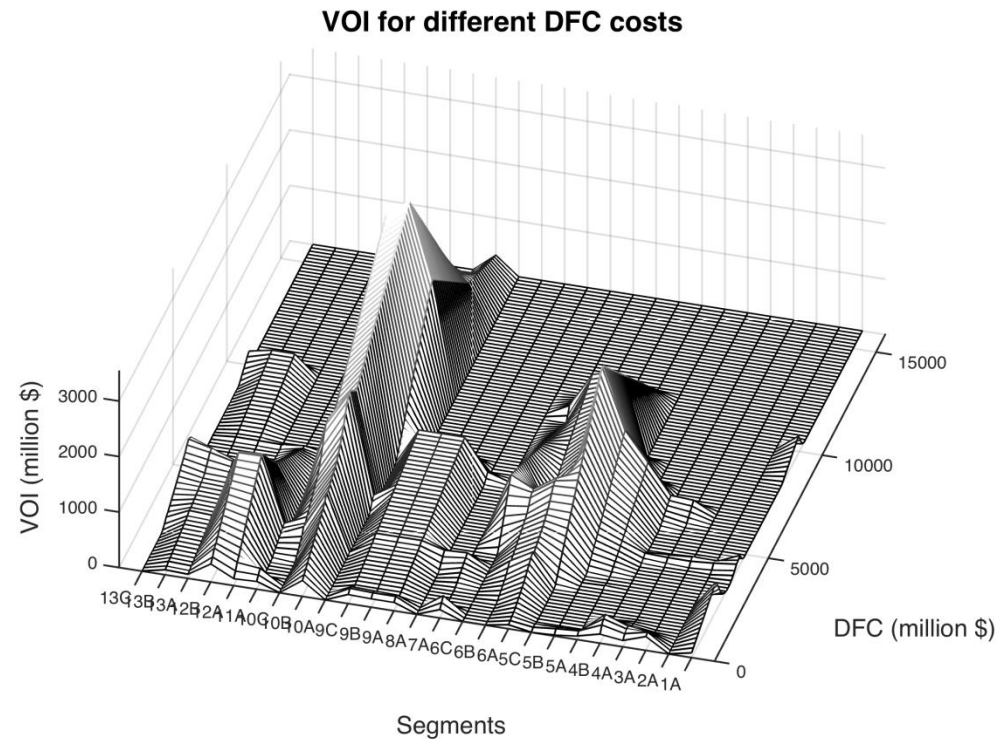


# VOI for different costs



# VOI for different costs

- For each segment VOI starts at 0 (for small costs), grows to larger values, and decreases to 0 (for large costs).
- VOI is smooth for segments belonging to the same prospect. Correlation and shared costs.
- VOI can be multimodal as a function of cost, because the information influences neighboring segments, at which we are indifferent of at other costs.



# Markov chains

Markov chains are special graphs, defined by initial probabilities and transition matrices.

$$p(\mathbf{x}) = p(x_1, x_2, \dots, x_n) = p(x_1) p(x_2 | x_1) \dots p(x_n | x_{n-1})$$

$$p(x_1 = k), \quad k = 1, \dots, d$$

$$p(x_{i+1} = l | x_i = k) = P(k, l), \quad k, l = 1, \dots, d$$

$d = 2$

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

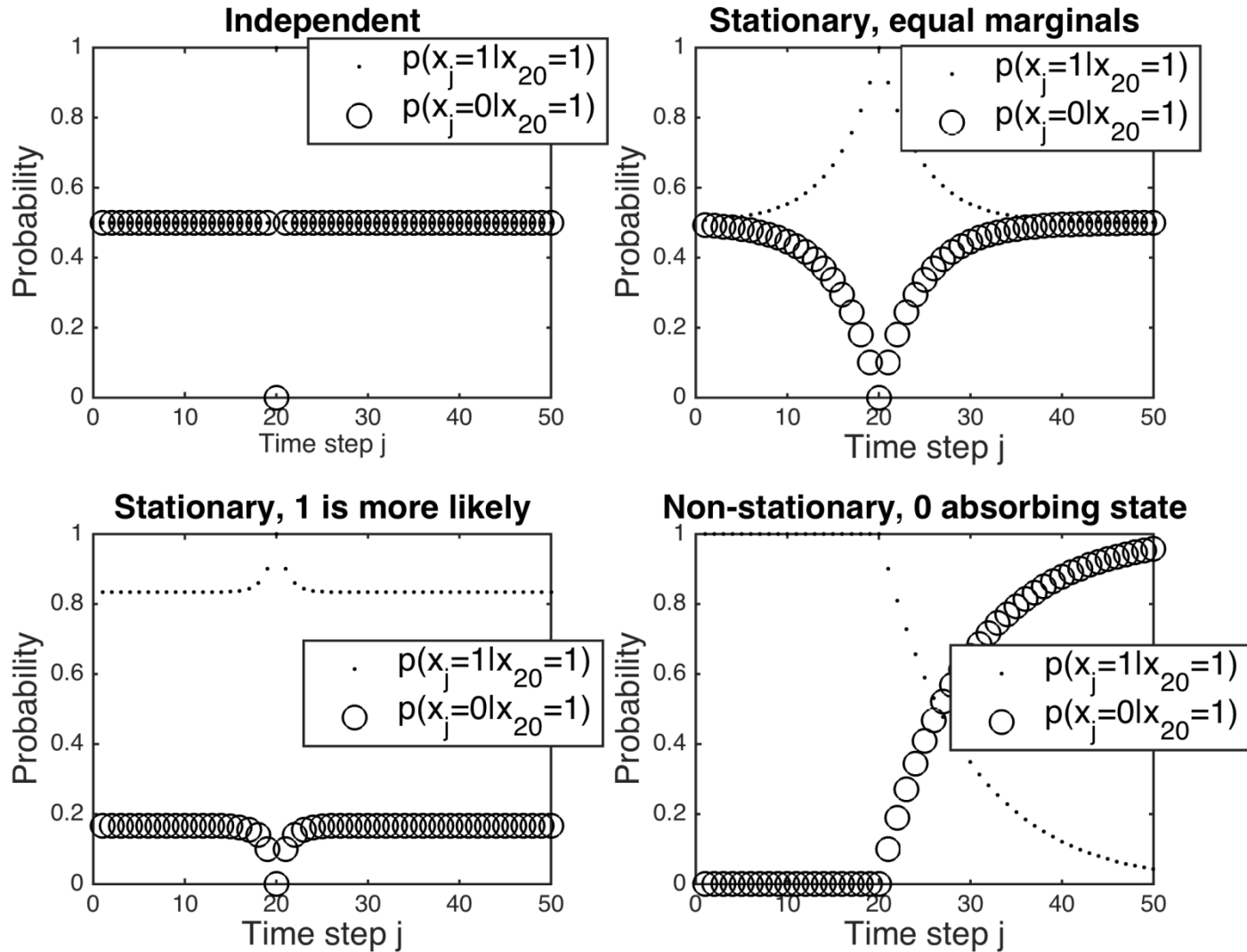
$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0 & 1 \end{bmatrix}$$

Independence

Absorbing

# Markov chains (given perfect information)





# Avalanche decisions and sensors

Suppose that parts along a road are at risk of **avalanche**.

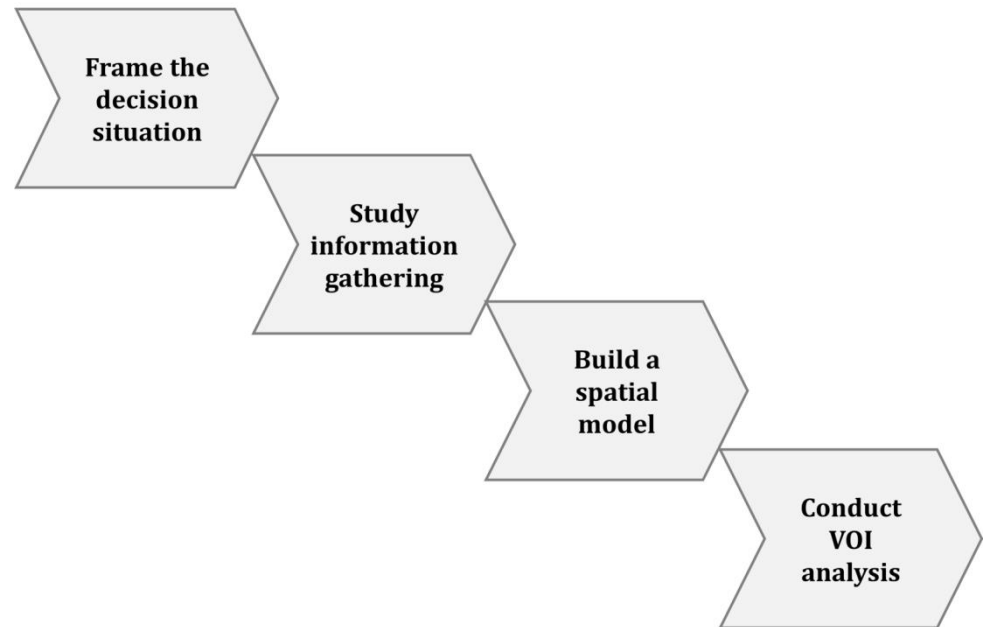
- One can remove risk by clearing roads, at a cost.
- Otherwise, the repair cost depends on the **unknown risk class: 1) low, 2) high**.

Data, sensor at a particular location, can help classify the risk class and hence improve the decisions made regarding cleaning / wait and see.



# VOI workflow

- Clear entire road up front (fixed cost), or wait and see (uncertain cost at each location).
- Gather information by sensor at one location, indicating risk class at that location.
- Model is a Markov chain with increasing probability of high risk for later indices (altitude).
- VOI analysis done by Markov chain calculations. Conducted for all possible sensor locations.



# Avalanche decisions - risk analysis

n=50 identified locations along railroad track, at increasing altitude and risk of **avalanche**.  
One can remove risk entirely by cost 100 000.

If it is not removed, the repair cost, at each location, depends on the unknown risk class:

$$C_j, \quad j \in \{1, 2\},$$

$$C_1 = 0, C_2 = 5000,$$

Decision maker must choose whether to

- i) **clean tracks** up front, with fixed price.
- ii) **wait and see**, with the uncertain price at each location.

The decision is based on the minimization of expected costs.

Prior value:

$$PV = \max \left\{ -100000, -5000 \sum_{i=1}^{50} p(x_i = 2) \right\}$$

Clean up front

Expected value when  
wait and see.

# Learning risk of avalanche

- Plan to install a sensor at one location, getting perfect information at that location.

$$j \in \{1, \dots, 50\}$$

- Compute the posterior value, with sensor location at one location.  
Compute the VOI.
- What is the optimal sensor location, if the goal is to improve risk decisions?

# Results – VOI

$$PV = \max \left\{ -100000, -5000 \sum_{i=1}^{50} p(x_i = 2) \right\}$$

$$PoV(x_j) = \sum_{k=1}^2 \max \left\{ -100000, -5000 \sum_{i=1}^{50} p(x_i = 2 | x_j = k) \right\} p(x_j = k)$$

$$VOI(x_j) = PoV(x_j) - PV$$

Best location near  $j=30$ .  
The VOI is about 13000

