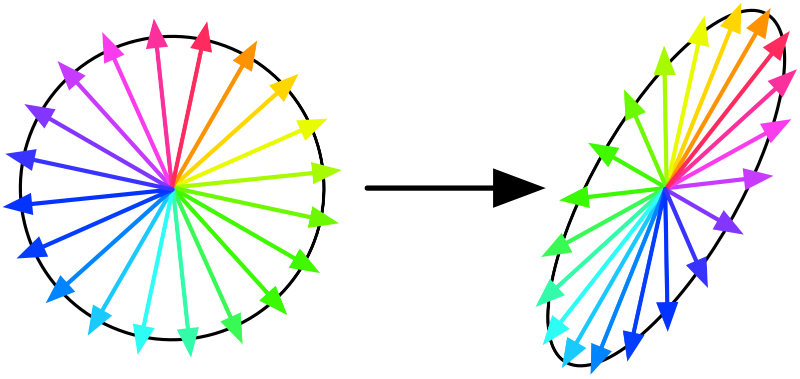


Tensor field visualization – My view on the field

January 2016

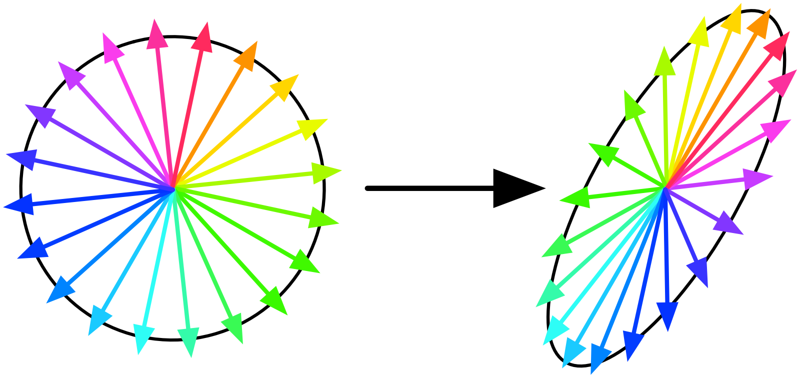
Geilo winter School – Scientific Visualization

Ingrid Hotz – Linköping University

LINKÖPINGS UNIVERSITET

From the general perspective …

… to digging in the sand



Tensor field visualization – My view on the field

January 2016

Geilo winter School – Scientific Visualization

Ingrid Hotz – Linköping University

LINKÖPINGS UNIVERSITET

Tensors – Why Do We Care?

- Tensors are everywhere (Simulations, physical theories)
- Can hardly be seen anywhere – nobody cares?

Tensors – Why Do We Care?

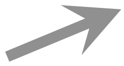
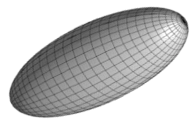
If you're sitting at a cocktail party with a bunch of engineers, physicists, and mathematicians, and you want to start a heated debate,

Just ask out loud: "What is a tensor?"

- One person will say that, for all practical purposes, a tensor is just a **fancy word for a matrix**.
- Then someone else will pipe up indignantly and insist that a tensor is a **linear transformation from vectors to vectors**.
- Yet another person will say that a tensor is an ordered **set of numbers that transform in a particular way upon a change of basis**.
- Other folks (like us) will start babbling about **"dyads" and "dyadics."**

[R. Brannon, *Functional and Structured Tensor Analysis for Engineers,* UNM Book Draft, 2003]

Tensors – Why Do We Care?

• Vector visualization	• Tensor visualization
• Intuitive notion of a vector 	• Intuitive notion? 
• Prevalent Applications	• Many different applications
• Some driving questions	• Only a few shared questions
• **Long visualization history**, well accepted	• **No visualization history**, hardly known

Tensors – Why Do We Care?

Data ??

Head **Computer**

No common language across applications
But terminology mostly related to **specific tensor invariants**
→ Basis of our visualization concept

Eigenvalues
Eigenvectors

Questions

Fuzzy **Crisp**

Often scientists have no glue how to expect from the tensor data

adapted from talk by Miriah Meyer given at Dagstuhl 2011??

Tensors – Why Do We Care?

Tensor visualization is field that is still in its infancy
(maybe except for Diffusion Tensor Imaging
however this does not really happen in the visualization community)

Overview

I. Tensors
   As mathematical objects
   As physical descriptors
II. Some basic visualization methods
III. Tensor Invariants for feature definition
IV. The story of a collaboration

I. Tensors as mathematical objects

I Tensors as mathematical objects

V *n*-dim vector space

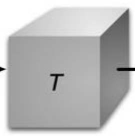
Second-order tensor *T*
is a bilinear function

$$T : V \otimes V \rightarrow \mathbb{R}$$
$$T(v,w) = w^T \cdot \mathbf{T} \cdot v, \quad \forall v,w \in V, \; \mathbf{T} \in \mathbb{R}^{n \times n}$$

Second-order tensor *T*
as lin. operator

$$T : V \rightarrow V$$
$$T(v) = \mathbf{T} \cdot v, \quad \forall v \in V$$

$v_1, v_2 \in V^2$ →

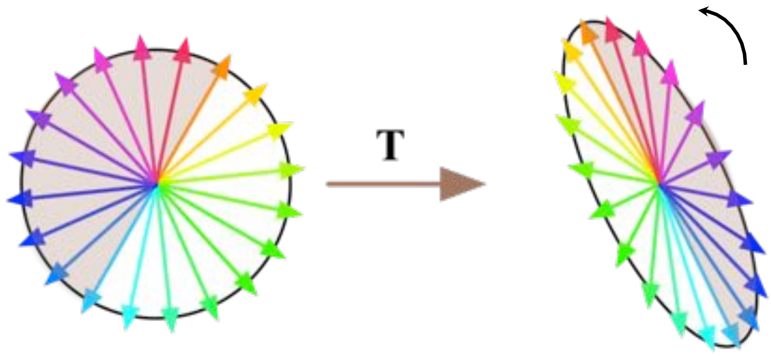


→ $T(v_1, v_2) \in \mathbb{R}$

$$\mathbf{T} = \begin{pmatrix} t_{11} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots \\ t_{n1} & \cdots & t_{nn} \end{pmatrix}$$

This is not a tensor but a matrix
Often a tensor is mixed up with its representation

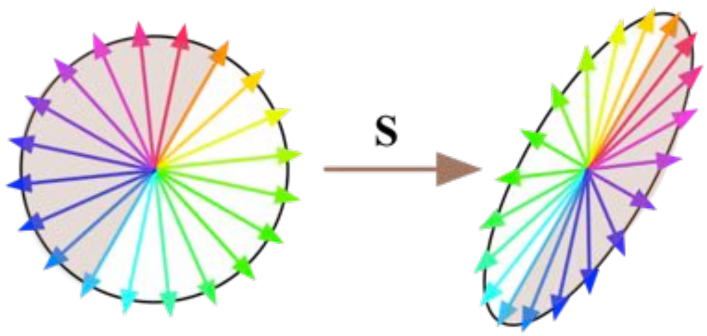
I Tensors as mathematical objects



The tensor is uniquely determined by its action on all unit vectors

Rotation, deformation, reflection

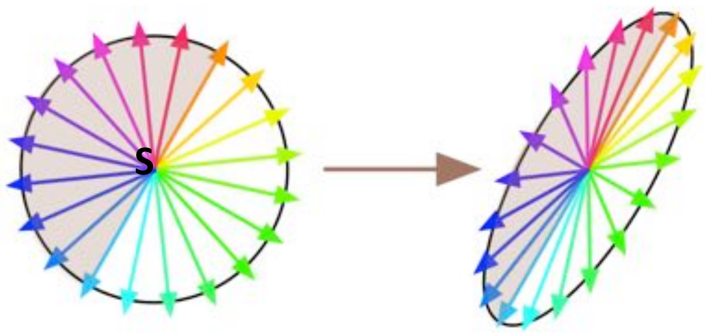
I Tensors as mathematical objects



Symmetric tensor

Deformation, reflection – no rotation

I Tensors as mathematical objects



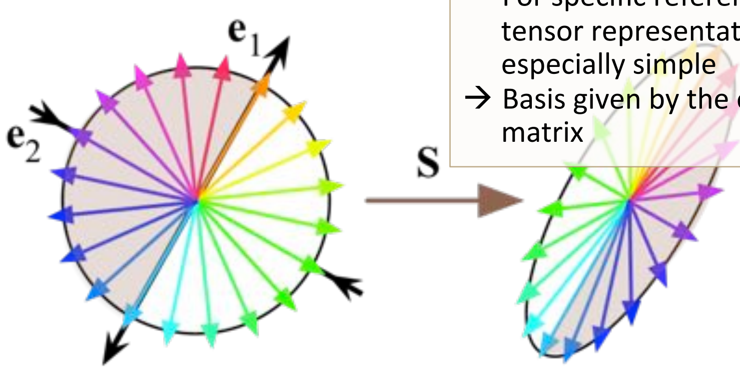
Positive definite, symmetric tensor

Deformation – no rotation, no reflection

I Tensors as mathematical objects

- Tensors can be analyzed using any convenient reference frame
- For specific reference frames, the tensor representation becomes especially simple

→ Basis given by the eigenvectors of the matrix



Symmetric tensor

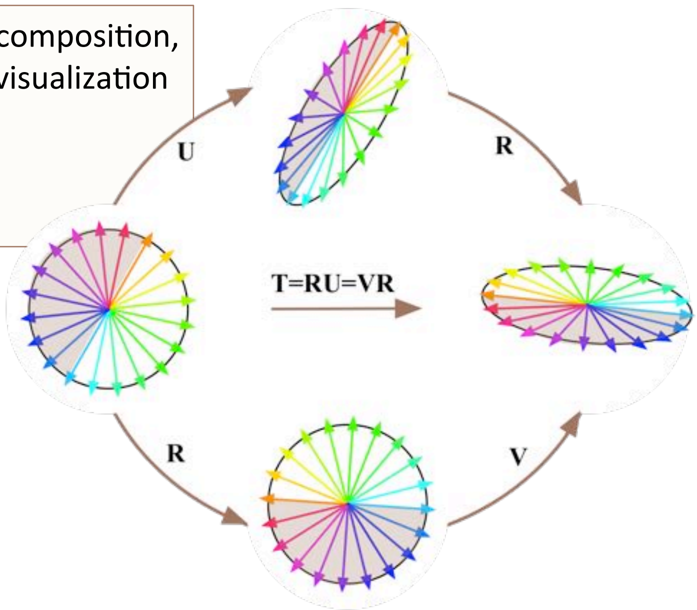
Eigenvectors and eigenvalues

$$Te_i = \lambda_i e_i$$

I Tensors as mathematical objects

There are different tensor Decomposition, which are often the basis for visualization methods

- Polar decomposition – stretch-rotation



Concatenation of mappings

Tensor field visualization – My view on the field

I Tensors as mathematical objects

There are different tensor Decomposition, which are often the basis for visualization methods

- Symmetric and anti-symmetric part

Sum of mappings

$$\mathbf{S} = \frac{1}{2}\left(\mathbf{T} + \mathbf{T}^T\right),$$

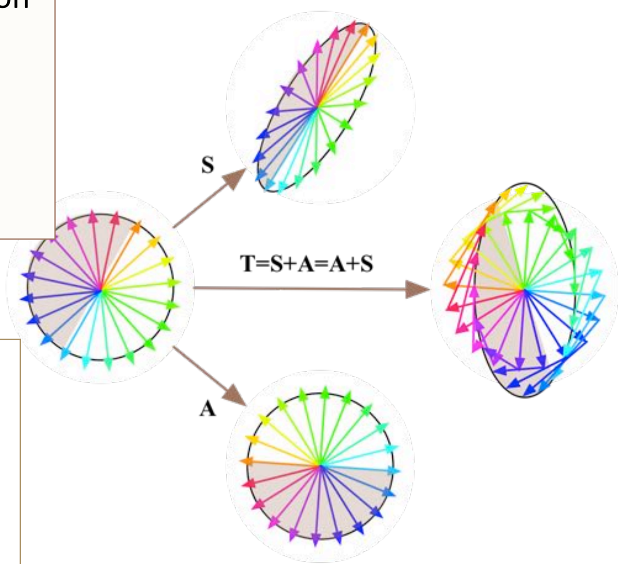
$$s_{ij} = \frac{1}{2}\left(t_{ij} + t_{ji}\right).$$

Symmetric part

$$\mathbf{A} = \frac{1}{2}\left(\mathbf{T} - \mathbf{T}^T\right),$$

$$a_{ij} = \frac{1}{2}\left(t_{ij} - t_{ji}\right),$$

Anti-symmetric part

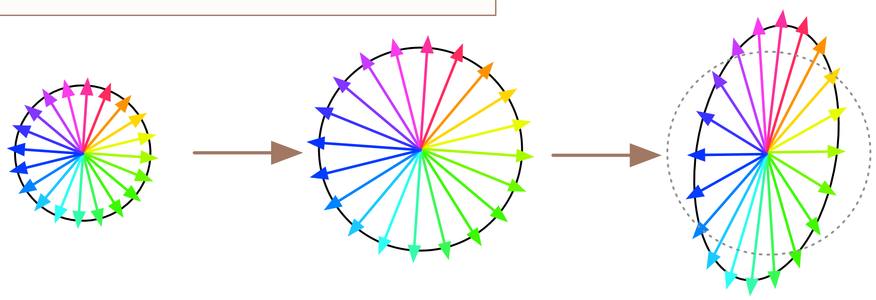


I Tensors as mathematical objects

There are different tensor Decomposition, which are often the basis for visualization methods

- Isotropic scaling, anisotropic deformation (deviator)

- Many different measures for anisotropy available



I Tensors as mathematical objects

Tensor Invariants

Entities that do not depend on the representation

Properties inherent to the tensor

Examples

- Eigenvalues
- Determinant
- Trance
- All functions that only depend on the eigenvalues

I Tensor as physical descriptors

I Tensor as physical descriptors

Tensors are everywhere in Physics and Engineering

- Not because the world is linear
- But because linear is simple

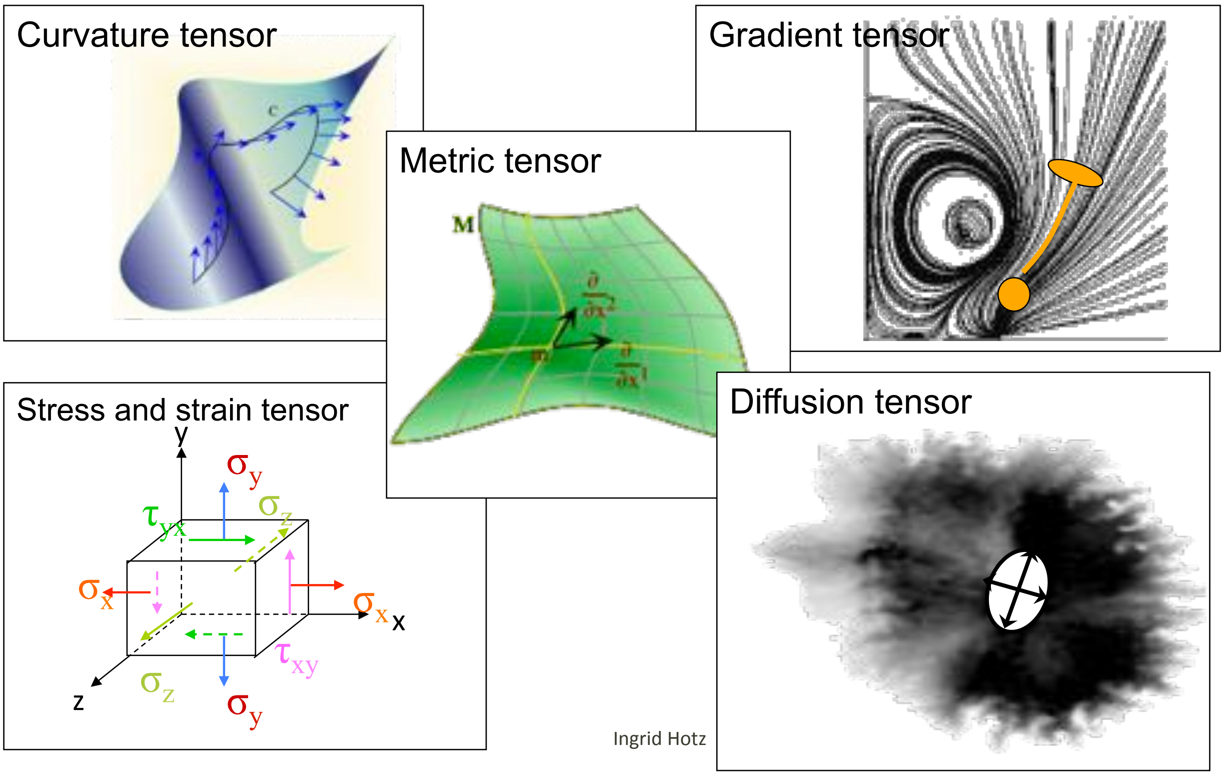
First order descriptors of the dependence of two vector fields v, w

→ first term of the Taylor expansion

$$v(w) = \mathbf{T} \cdot w + \text{higher order terms}$$

They provide a more or less good approximation of the reality

I Tensor as physical descriptors

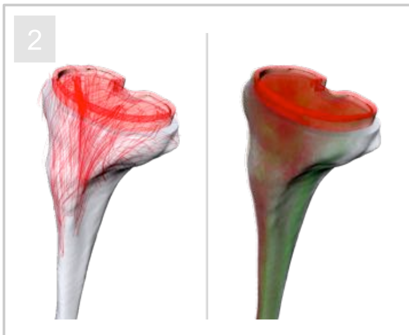


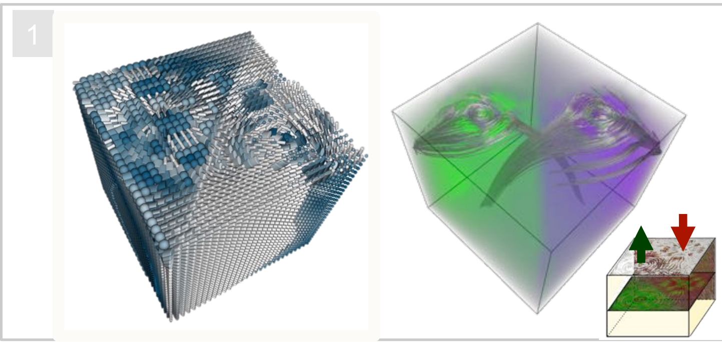
I Tensor as physical descriptors

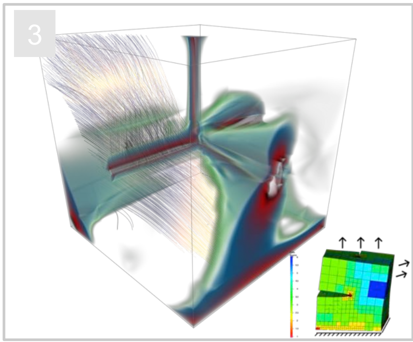
Stress Tensor

Geo- and Material Sciences

1. Solid block, with two applied forces
2. Implant design, stress simulation in human bone
3. Notched block with external forces







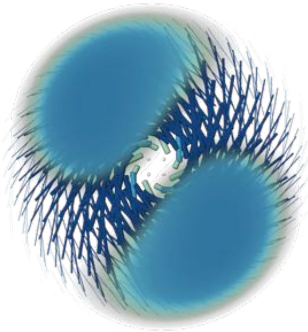
Images: Kratz, ZIB

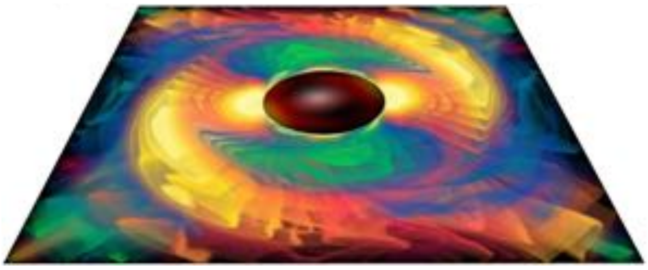
I Tensor as physical descriptors

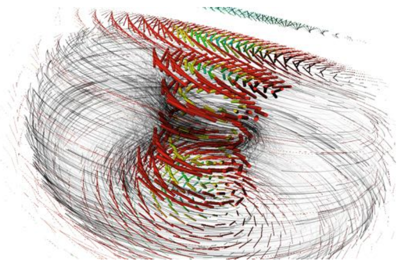
Metric, Curvature, Stress

General Relativity

- Simulation of gravitational field of a rotating black hole, respectively neutron star







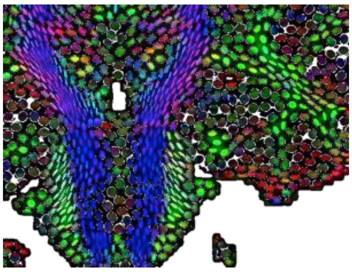
Images: Benger, Kratz, ZIB

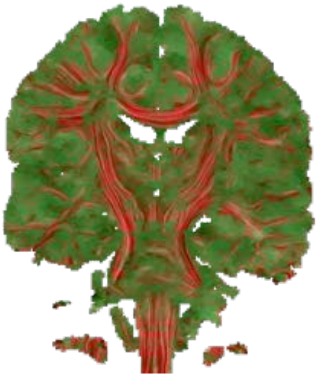
Tensor field visualization – My view on the field

I Tensor as physical descriptors

Diffusion Tensor – Medicine

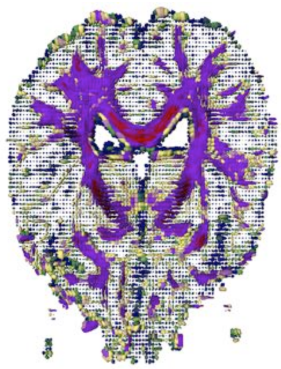
- Imaging method: based on magnetic resonance tomography (MRT) measuring the diffusion of water molecules in tissues
- Application: Reconstruction of neural fibers in human brain (tractography)





Textured slice

3D Fiber tracking



Glyph representations

Images: Kratz, Breßler, Hotz, ZIB

I Tensor as physical descriptors

Structure Tensor – Image Analysis

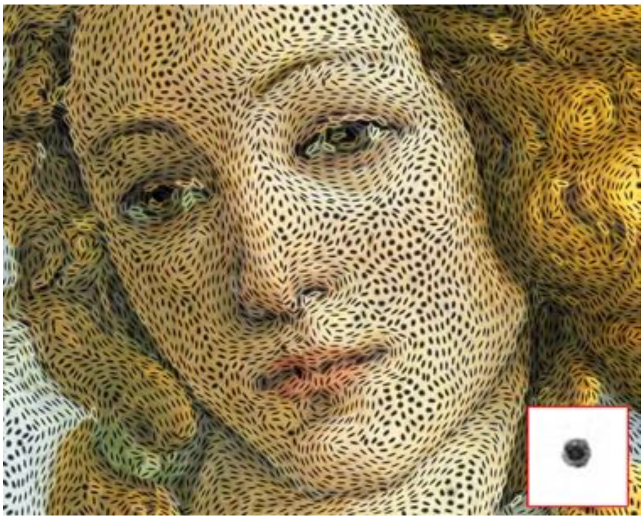


Image: Kratz, ZIB

I Tensor as physical descriptors

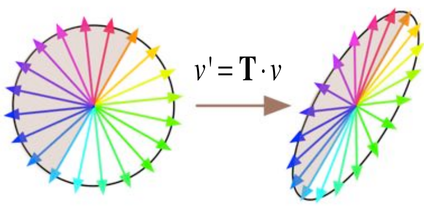
Different Characters of Tensors - Examples

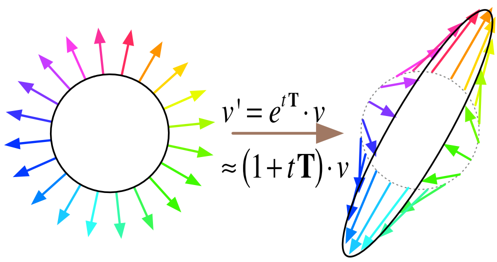
Describes a deformation

- Positive definite
- E.g. deformation tensor
- Determinant *det*$\mathbf{T}$ → volume change

Describes a generator of a deformation

- Indefinite
- E.g. stress tensor (forces/ area)
- *Trace*$\mathbf{T}$ → volume change





I Tensor as physical descriptors

Tensor Invariants

- Play a fundamental role in the understanding of application specific tensors
- Every application has its own invariants that are especially important. Thy often come with a domain specific language

→ Relevant invariants should guide the choice of the visualization

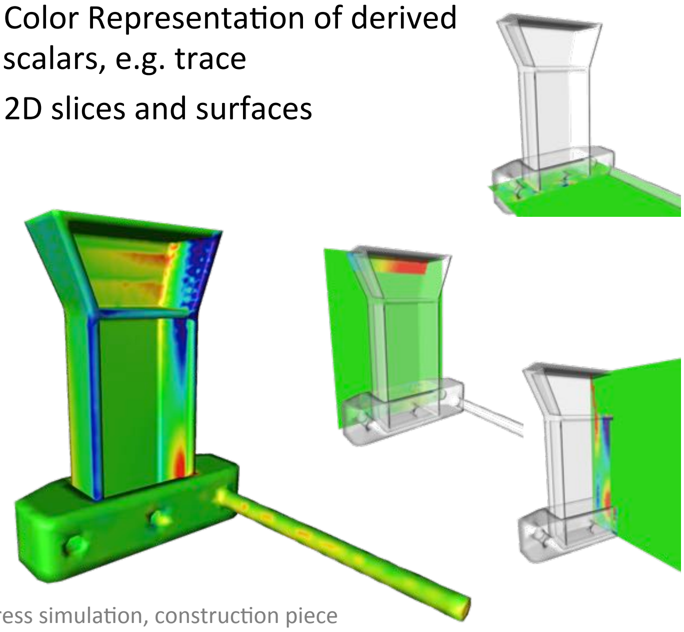
II Some basic visualization methods

What has visualization currently to offer

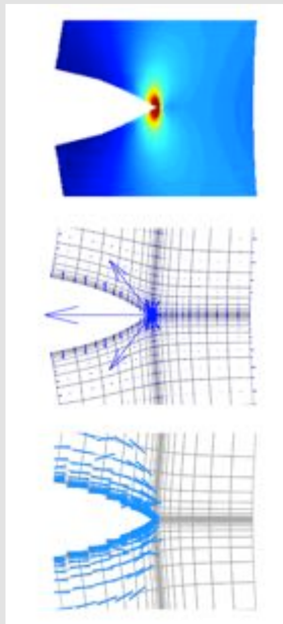
II Some basic visualization methods

Common practice

- Color Representation of derived scalars, e.g. trace
- 2D slices and surfaces



Stress simulation, construction piece
M. Stommel, Uni Saarbrücken

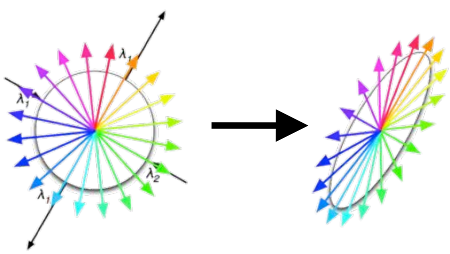


B. Hirschberger (2004), Uni Kaiserslautern

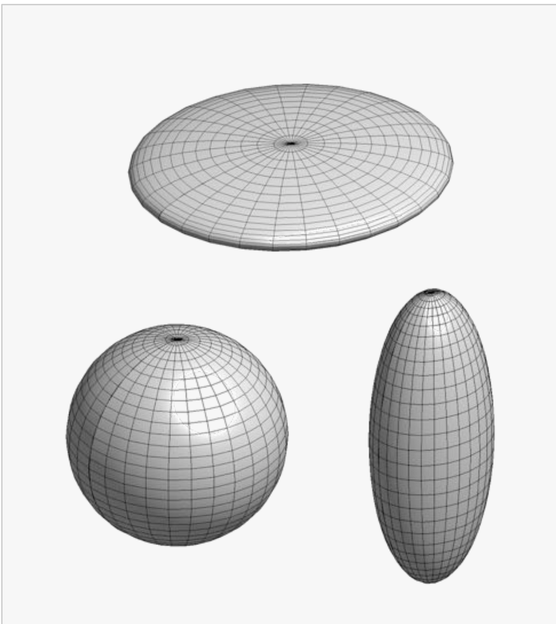
II Some basic visualization methods

Local Methods - Glyphs

- Geometric objects representing **tensor characteristics**
- Here: Ellipses
- Most frequently used visualization



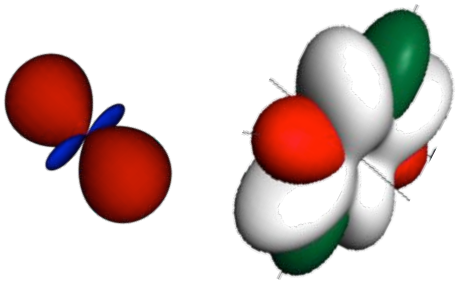
The typical glyph: Ellipsoid



II Some basic visualization methods

Local Methods - Glyphs

Many different glyph types in use



Example from Material Sciences

When to use?

- Evaluation of data quality
- Visualization overview
- Probing

Glyph design

- What should be represented?
- How should it be represented?

Glyph placement

- How many glyphs?
- Where to place them?

[Schultz2010, Kratz2014]

II Some basic visualization methods

Local Methods - Glyphs

How should it be represented – general design guidelines

- Preservation of symmetry
  e.g. eigenvectors have no orientation, isotropic tensors have not distinguished direction
- Continuity
  similar tensors should have similar glyph representations
- Disambiguate
  tensors with different values should be reflected by different glyphs

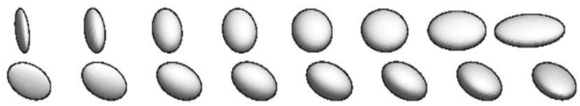
  [Schultz and Kindlmann 2010]
- Use glyphs that have been used in the community befor

What should be represented – application specific guidelines

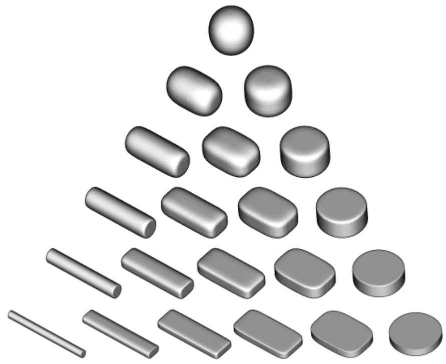
- Use application specific invariants for the design

II Some basic visualization methods

Local Methods - Glyphs



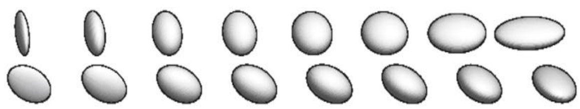
Resolving Perceptional Issues
Superquadrics



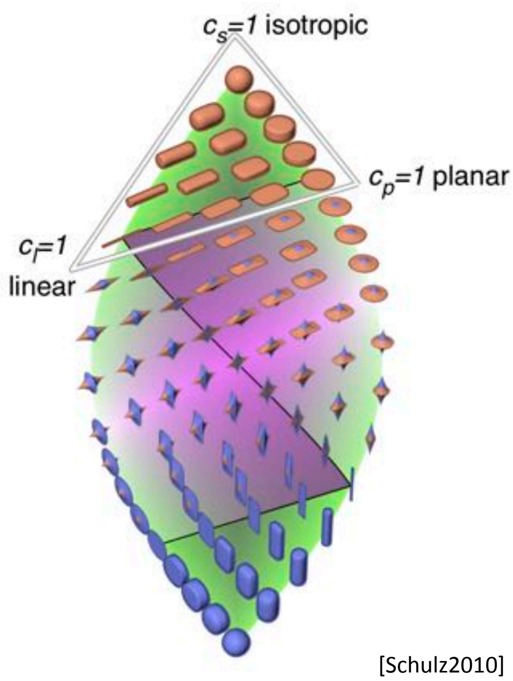
[Kindlmann2004]

II Some basic visualization methods

Local Methods - Glyphs



Resolving Perceptional Issues
Superquadrics



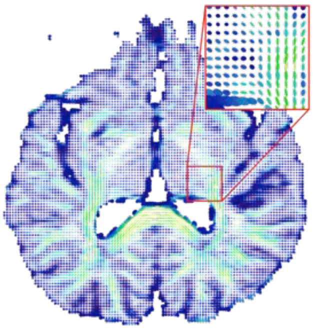
[Schulz2010]

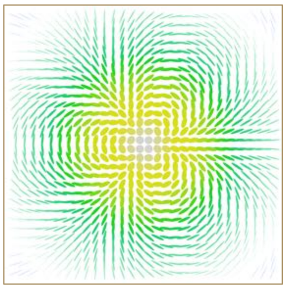
II Some basic visualization methods

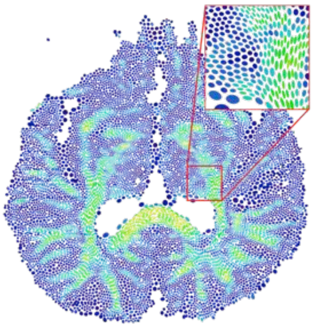
Local Methods - Glyphs

Glyph placement

- How many glyphs?
- Where to place them?







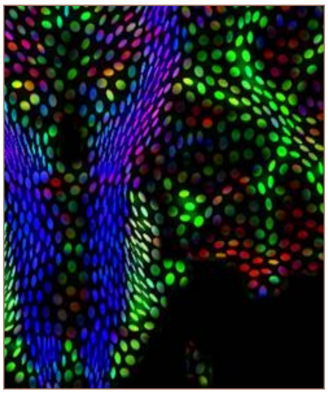
Anisotropic sampling

Images: Kratz, Bressler ZIB, Amira

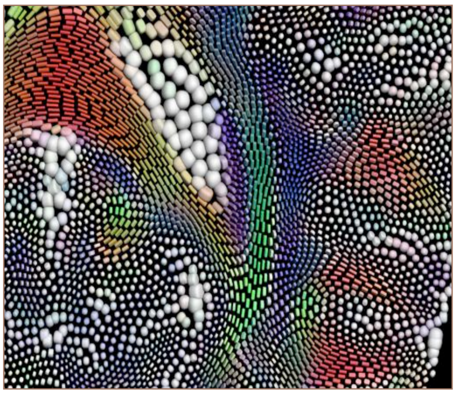
II Some basic visualization methods

Local Methods - Glyphs

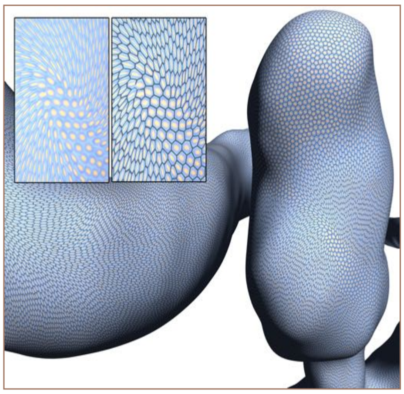
Particle based and geometric methods
on planes, in 3d, on surfaces



[Feng2008]



[Kindlmann2006]

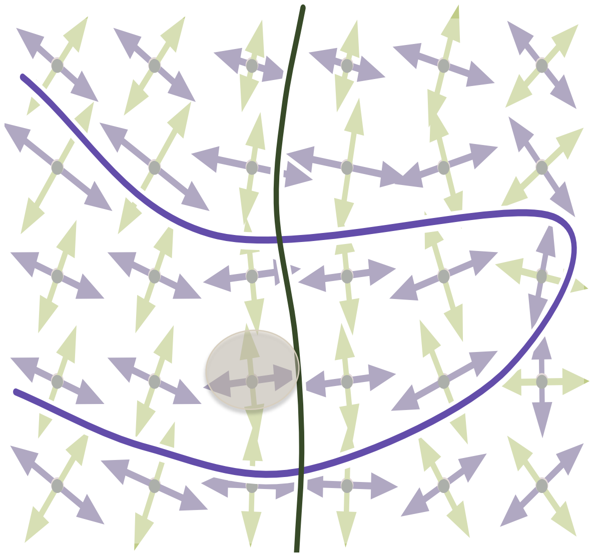


[Kratz2013]

II Some basic visualization methods

Tensor lines

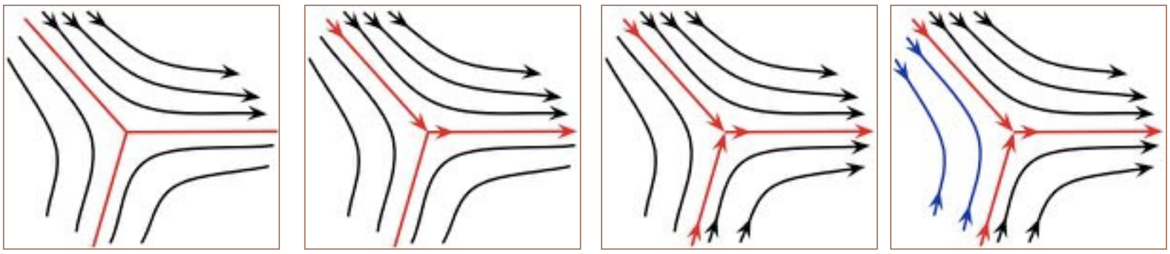
- Integral lines similar to streamlines
- Follow one eigenvector field



II Some basic visualization methods

Tensor lines

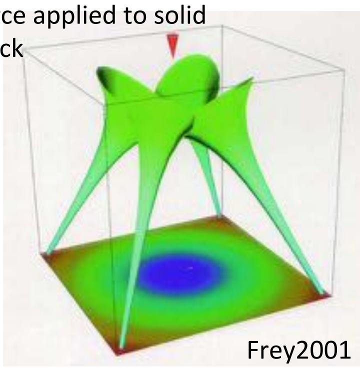
Direction field is not orientable.
It is not a vector field!!

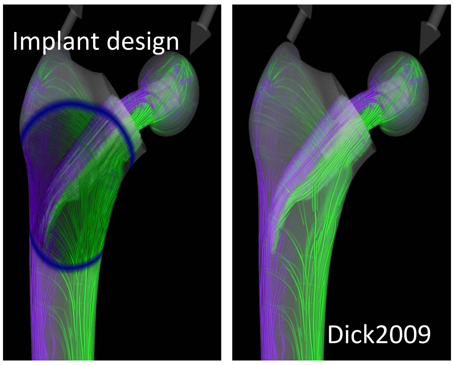


II Some basic visualization methods

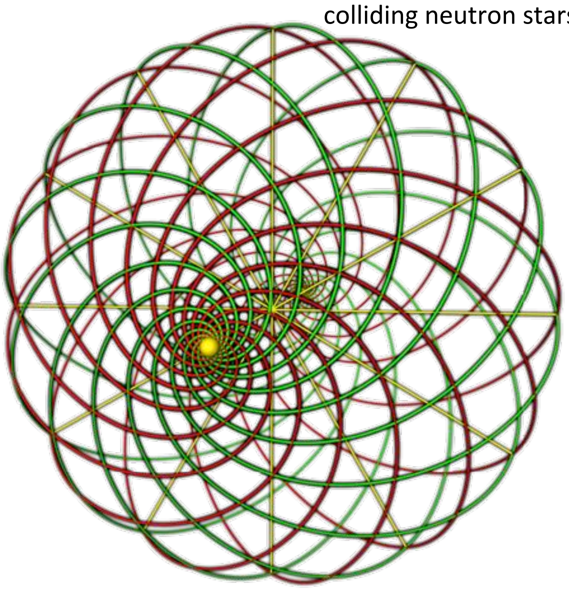
Tensor lines

Force applied to solid block





Gravitational field of two colliding neutron stars



Images: Zobel, ZIB

II Some basic visualization methods

Tensor lines – Fiber tracking

Tensors from diffusion MRI

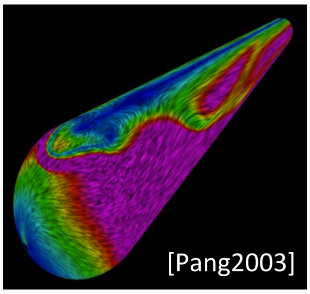
Tracking of **neural fibers**

- Major tensor lines can be used to approximate fibers
- Line tracing only for regions of high anisotropy

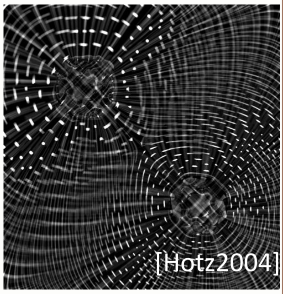
II Some basic visualization methods

Textures

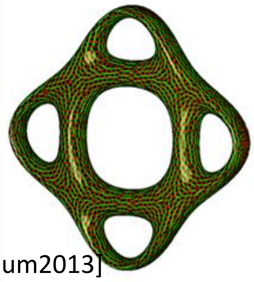
HyperLIC -2d kernel



Fabric Texture

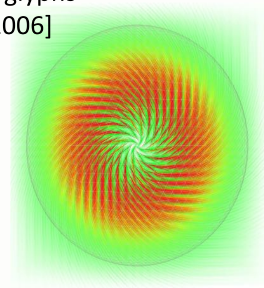


Fabric Texture

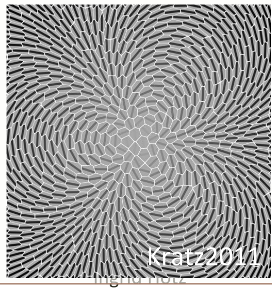


[Eichelbaum2013]

Splatting glyphs
[Benger2006]



Sampling based textures

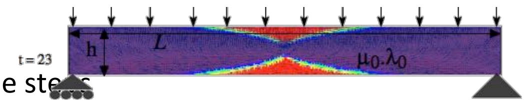


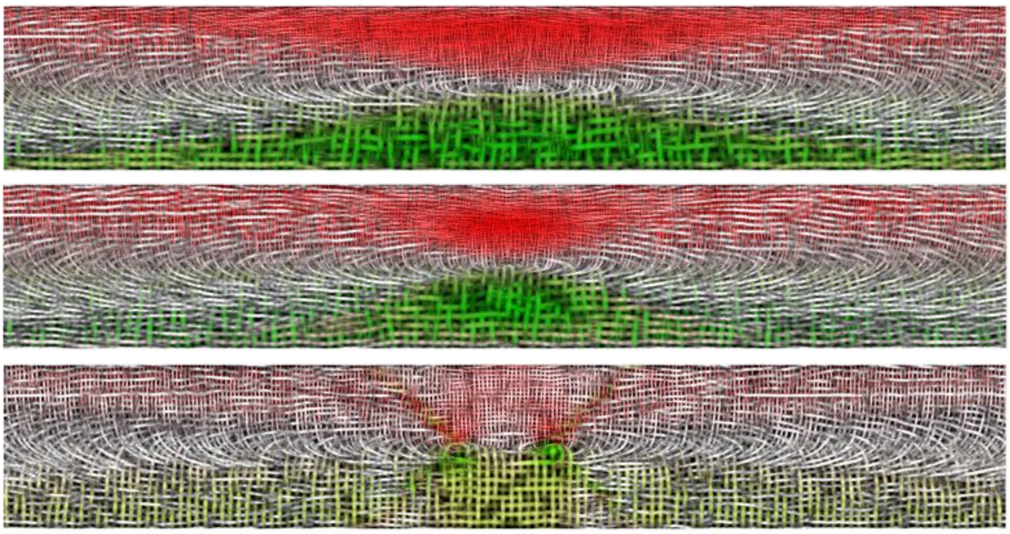
Topology based textures

II Some basic visualization methods

Textures – Principal directions and values

- Beam bend under its own load.
- Simulation respecting evolving damage, 3 time steps
- Data: Louise Kellog University of California.





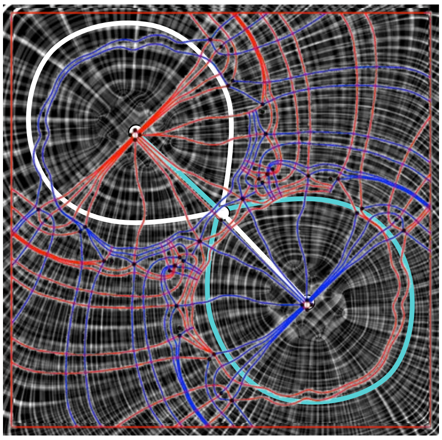
Data: Louise Kellogg, Department of Geology, University of California, Images: Louis Feng UCD.

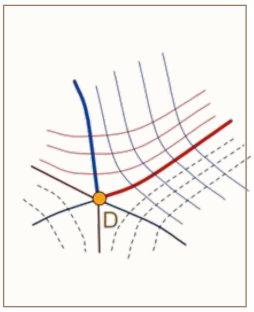
II Some basic visualization methods

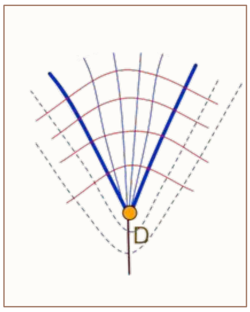
Topology

Topological structure

- Segmentation of domain in areas of uniform directional behavior
- Similar to vector field topology – but different basic structures





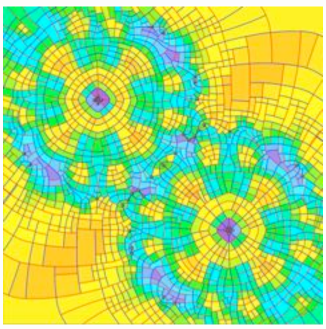


Tensor field visualization – My view on the field

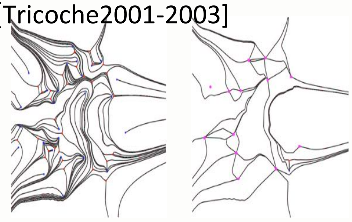
II Some basic visualization methods

Topology

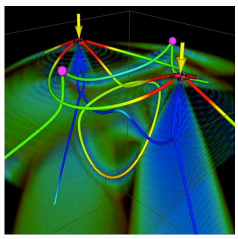
- Topology based Segmentation
- Topology based Textures [Auer2013]



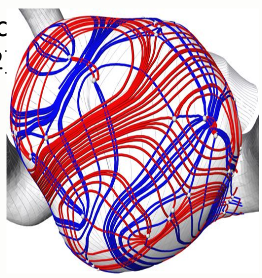
Computation, Simplification, Tracking [Tricoche2001-2003]



3D topology, critical lines [Zheng2005]



On Surfac [Auer2012]



Tensor field design [Zhang2007]

II Some basic visualization methods

What has been done so far?

Some technical stuff

- Glyphs from all perspectives
- **Technical generalization of vector field visualization methods**
- Textures

Much work on tensor processing

- Interpolation / reconstruction
- Morphology
- Topology

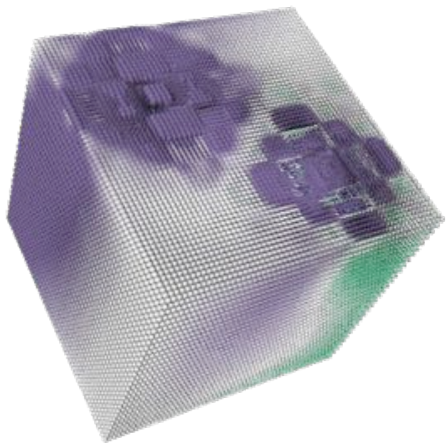
What is still missing

- Applications – link form visualization techniques to physical interpretations
- Notion of features
- Questions

III Tensor Invariants for feature definition

- **Exploarative framework**
  supporting multiple applications
  with different questions

- **Structuring the data**
  - Manage complexity of the data
  - Highlight trends
  - Point at critical/atypical behavior



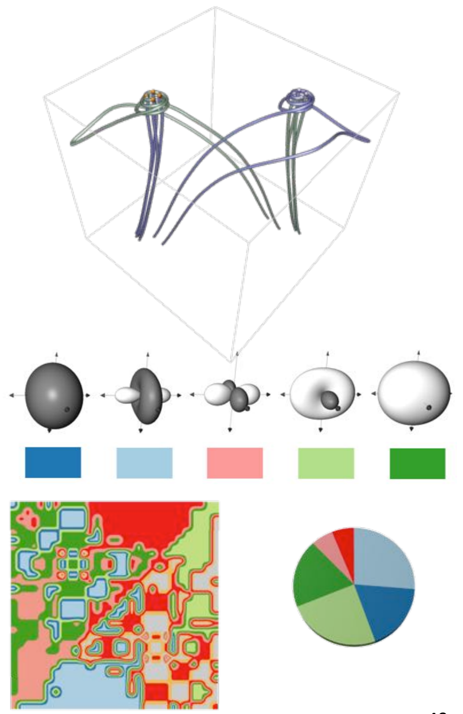
III Tensor Invariants for feature definition

- **Explorative framework**
  supporting multiple applications with different questions

→ **Domain specific feature spaces**

- **Structuring the data**
  - Manage complexity of the data
  - Highlight trends
  - Point at critical/atypical behavior

→ **Data atlas using a thumbnail like representation**



III Tensor Invariants for feature definition

Tensor is uniquely defined by

- 3 eigenvalues → point in '**shape-space**'
- 3 eigenvectors → point in '**direction-space**'

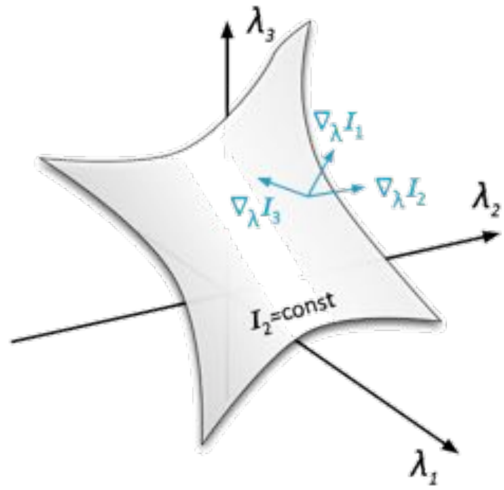
Tensor invariant

- Are the language of the applications

Use application specific invariants to parameterize the shape space

→ **shape descriptors** $I_i(\lambda_1, \lambda_2, \lambda_3)$

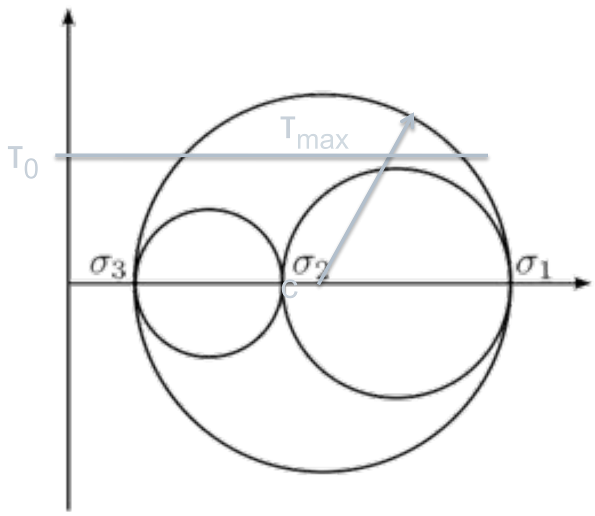
→ basis for the definition statistic views, glyph design, similarity measure



The Stress Tensor and Failure Models

Example for failure analysis in mechanical engineering

→ Mohr Circle

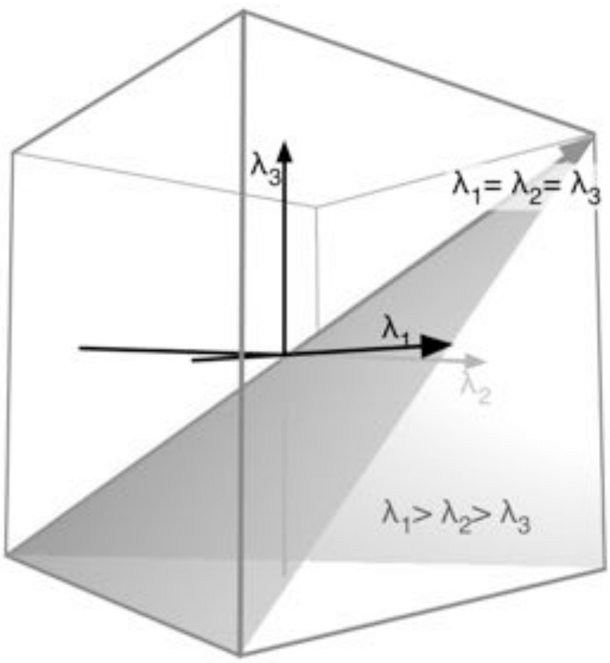


III Tensor Invariants for feature definition

Example

- Ordered shape space

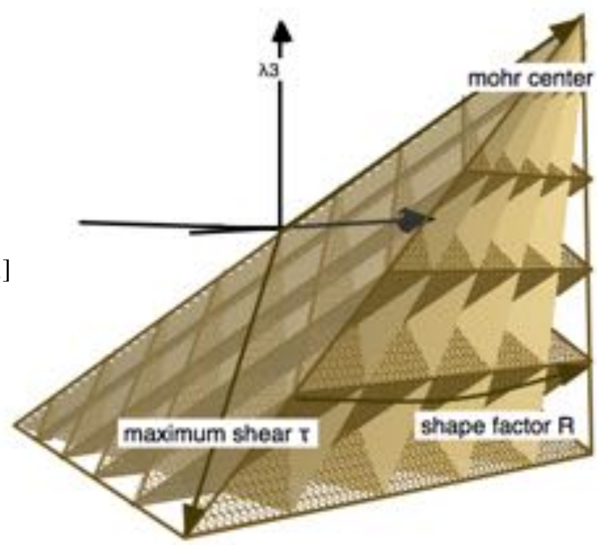
$$\lambda_1 \geq \lambda_2 \geq \lambda_3$$



III Tensor Invariants for feature definition

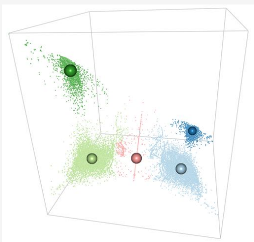
Example – Coulomb failure

- Parametrized by
  - Mohr center $c = \frac{\lambda_1 + \lambda_3}{2}$
  - Max Shear $\tau = \frac{\lambda_1 - \lambda_3}{2}$
  - Shape factor $R = \frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_3} \in [0,1]$

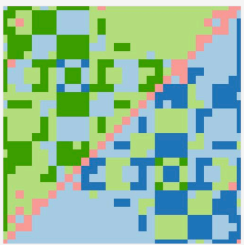


III Tensor Invariants for feature definition

Cluster in feature space

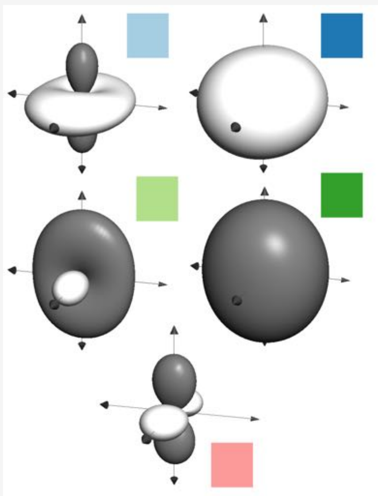


Spatial view: Top view of data



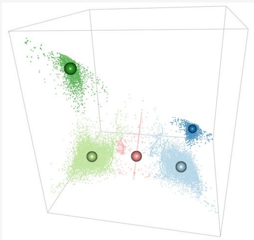
Cluster representatives

- Reynolds glyphs

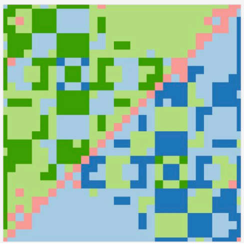


III Tensor Invariants for feature definition

Cluster in feature space

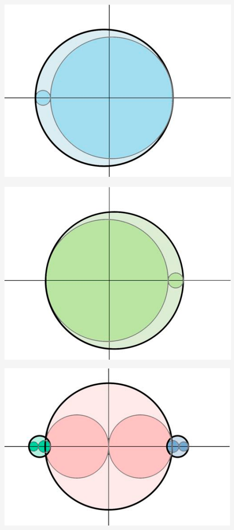


Spatial view: Top view of data



Cluster representatives

- Mohr circles

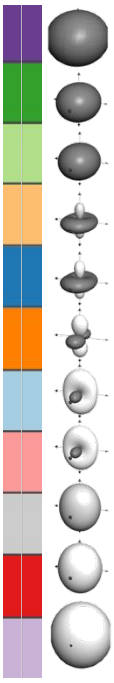


III Tensor Invariants for feature definition

Clustering

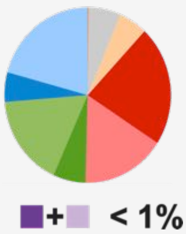
- Mean shift
- Full feature space
  + spatial coordinates

Top-view of the two-point load

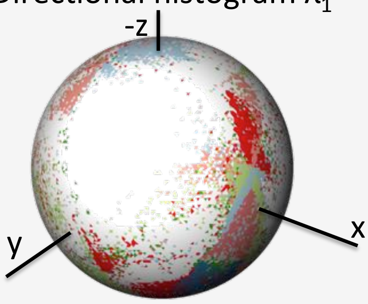


Cluster statistics

- Cluster size



- Directional histogram λ_1

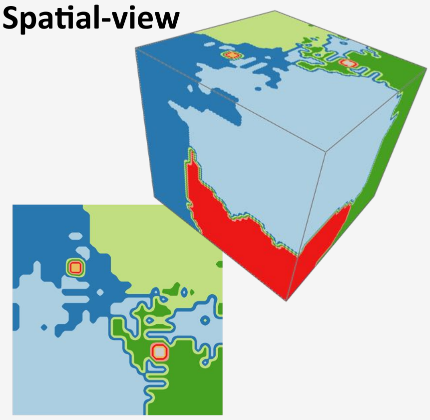


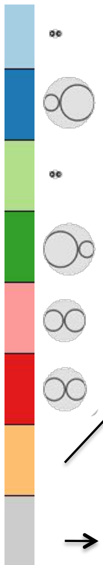
III Tensor Invariants for feature definition

Clustering
- Mean shift
- Feature space (shear, shape) + spatial coordinates

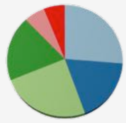
Two-point load

Spatial-view

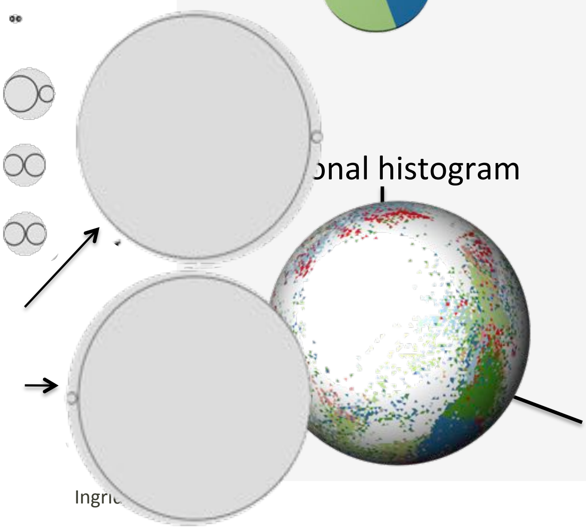




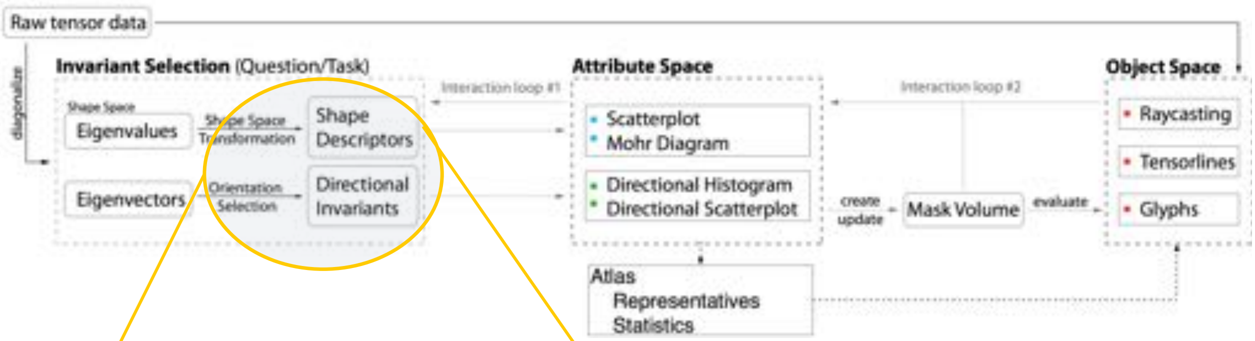
Cluster statistics
- Cluster size

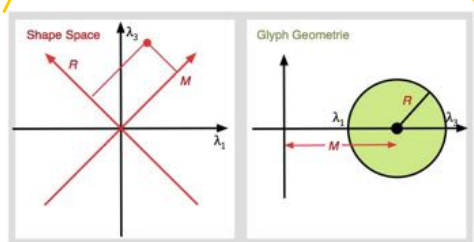


onal histogram



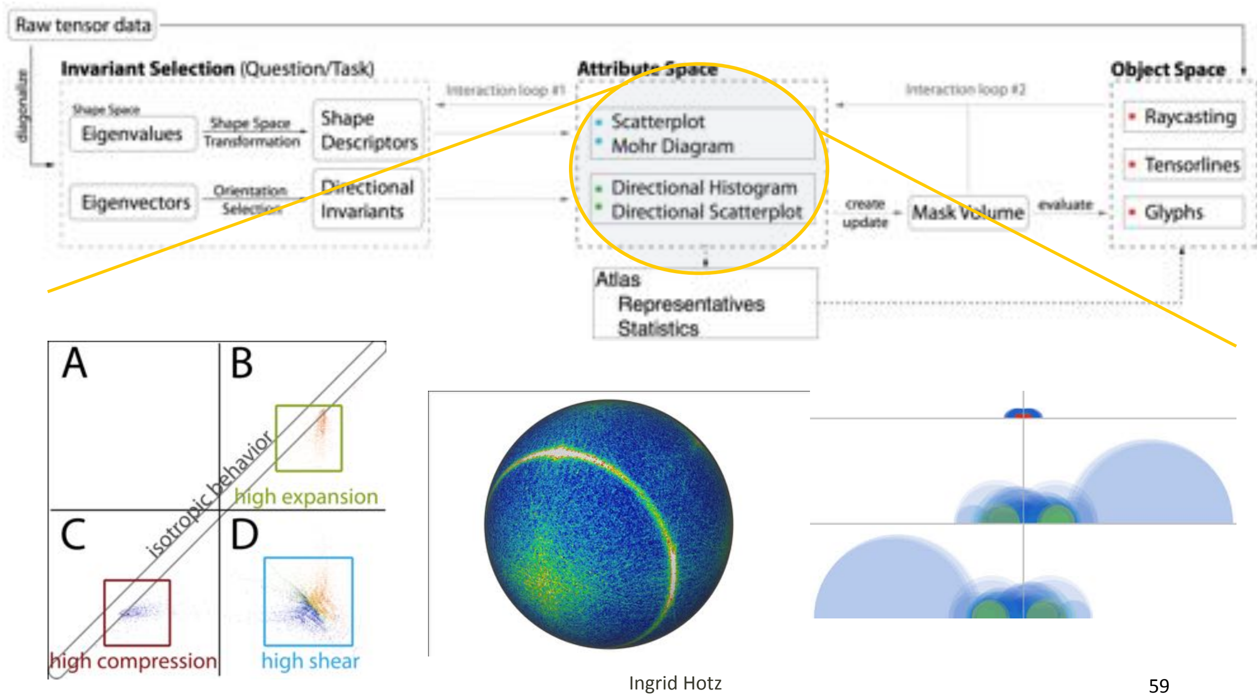
III Tensor Invariants for feature definition



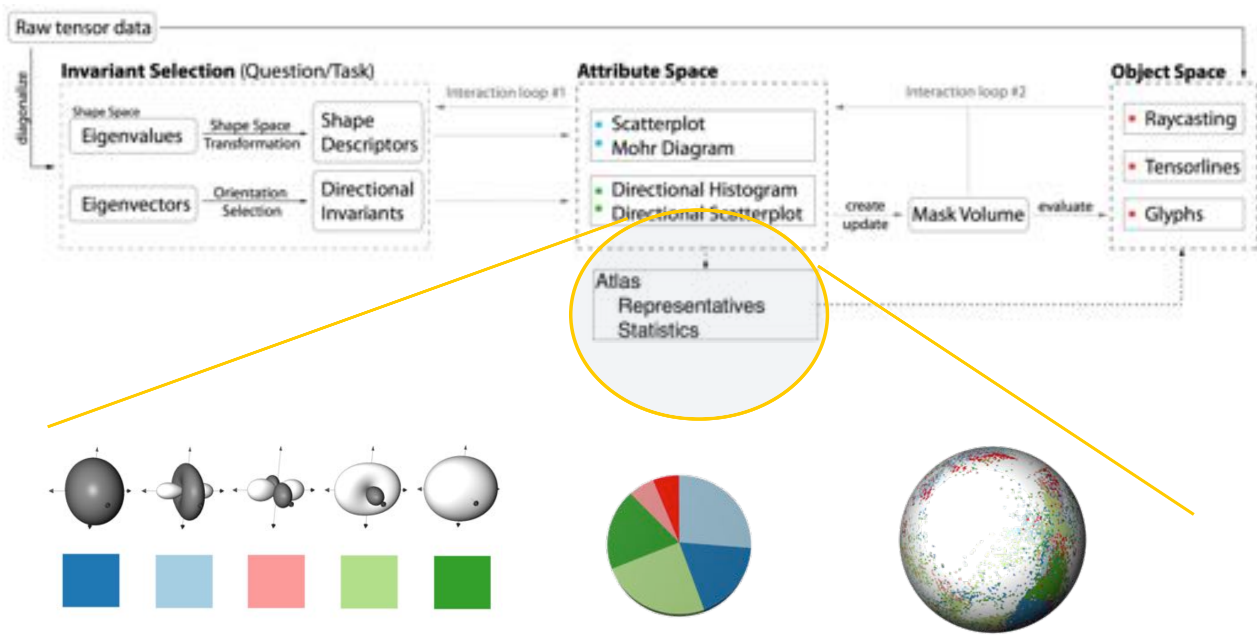


Feature Space Selection
- Change of basis results in another set of shape descriptors
- Determines
  - Exploration space
  - Metric, similarity measure
  - Glyph design

III Tensor Invariants for feature definition

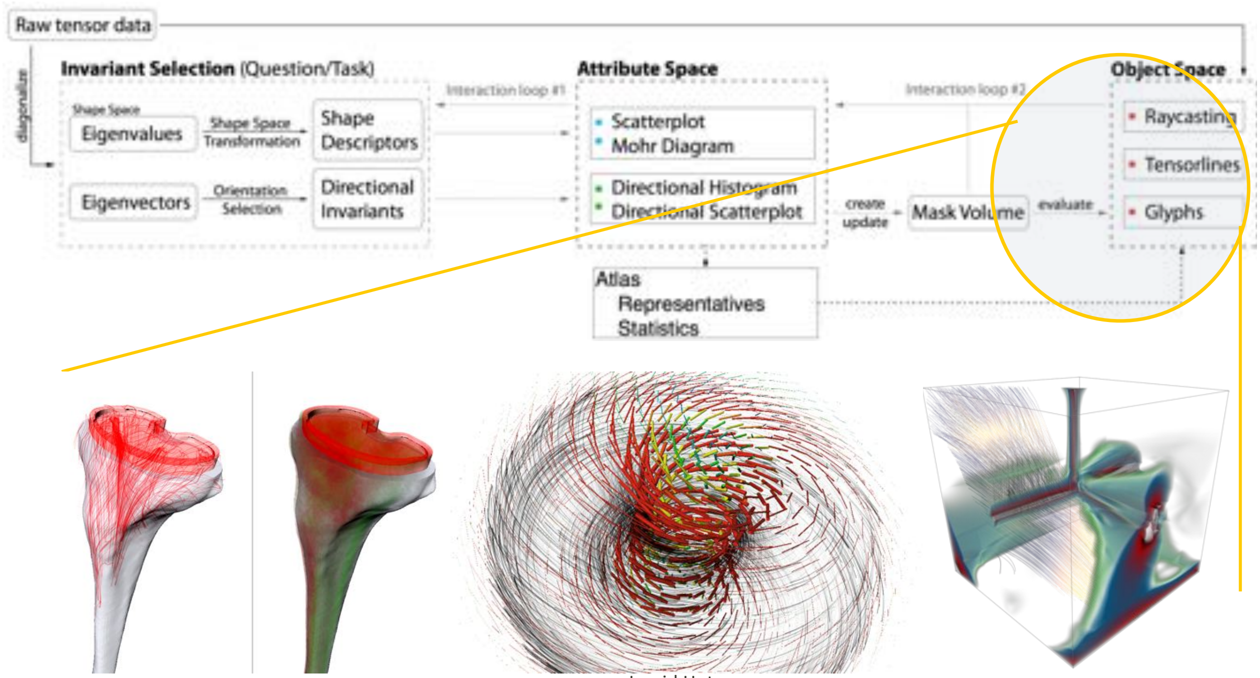


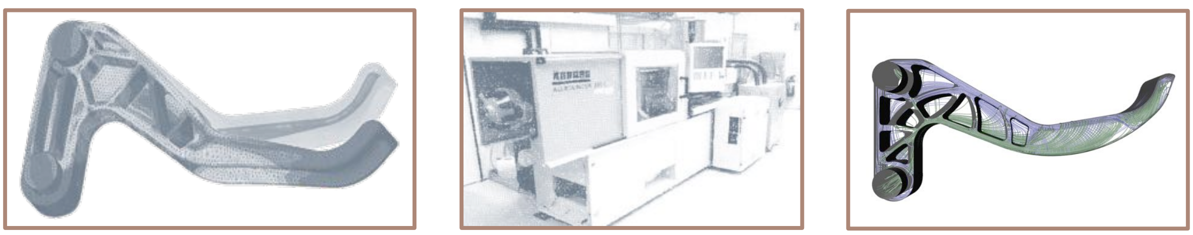
III Tensor Invariants for feature definition



Tensor field visualization – My view on the field

III Tensor Invariants for feature definition





Tensor Visualization Driven Mechanical Component Design

or

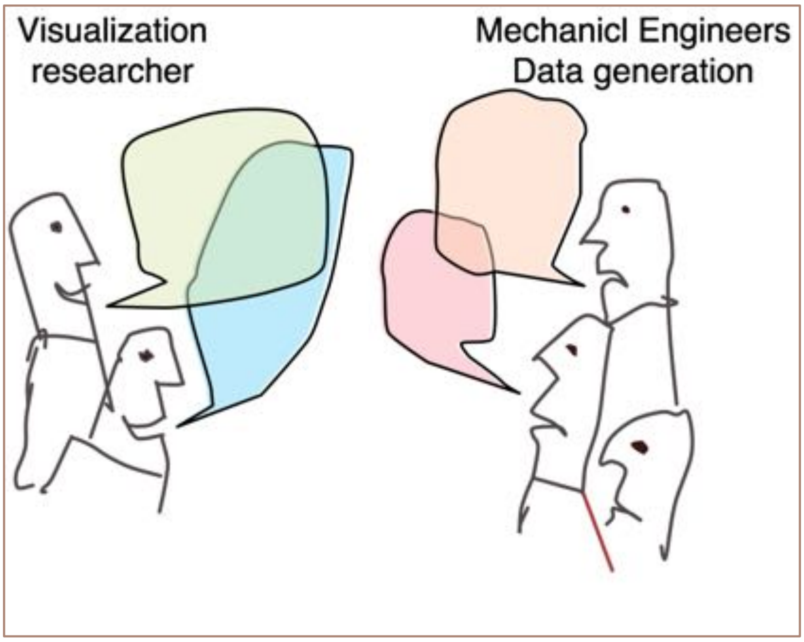
The story of a collaboration

Kratz, Schoeneich, Zobel, Burgeth, Scheuermann, Hotz, Stommel, Pacific Vis 2014

IV Story of a Collaboration

Starting point

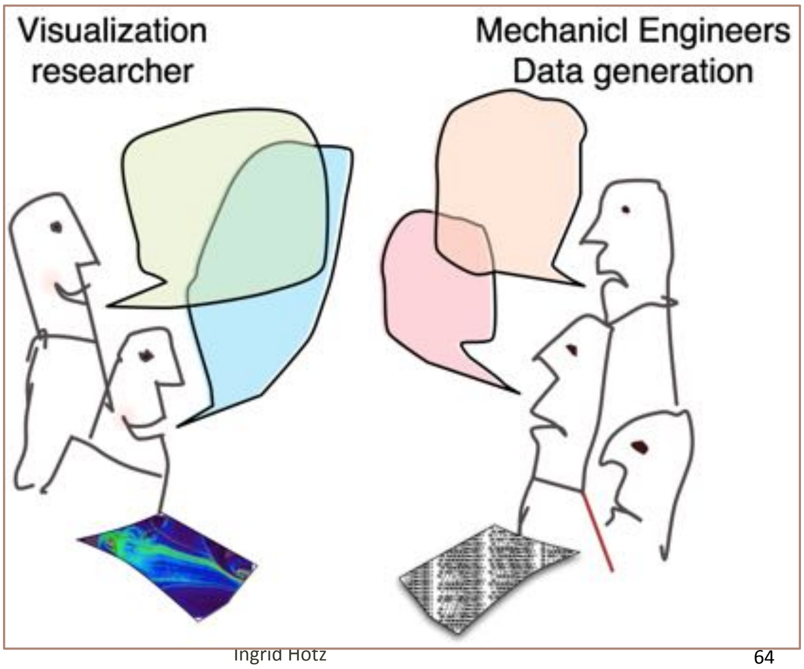
- Unspecific goals
- Different language



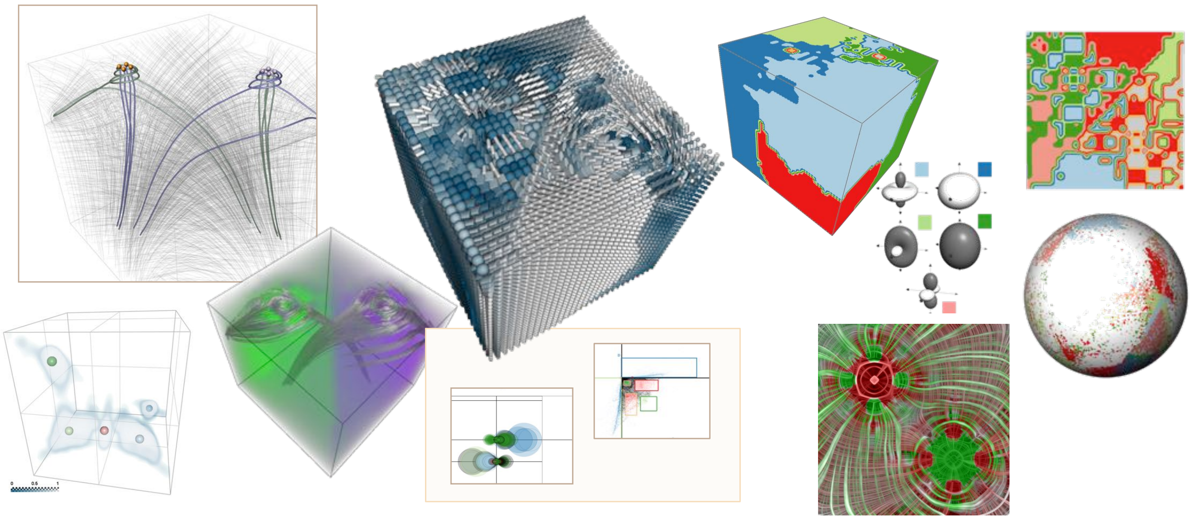
IV Story of a Collaboration

First Experiments

- Everybody shows what they have



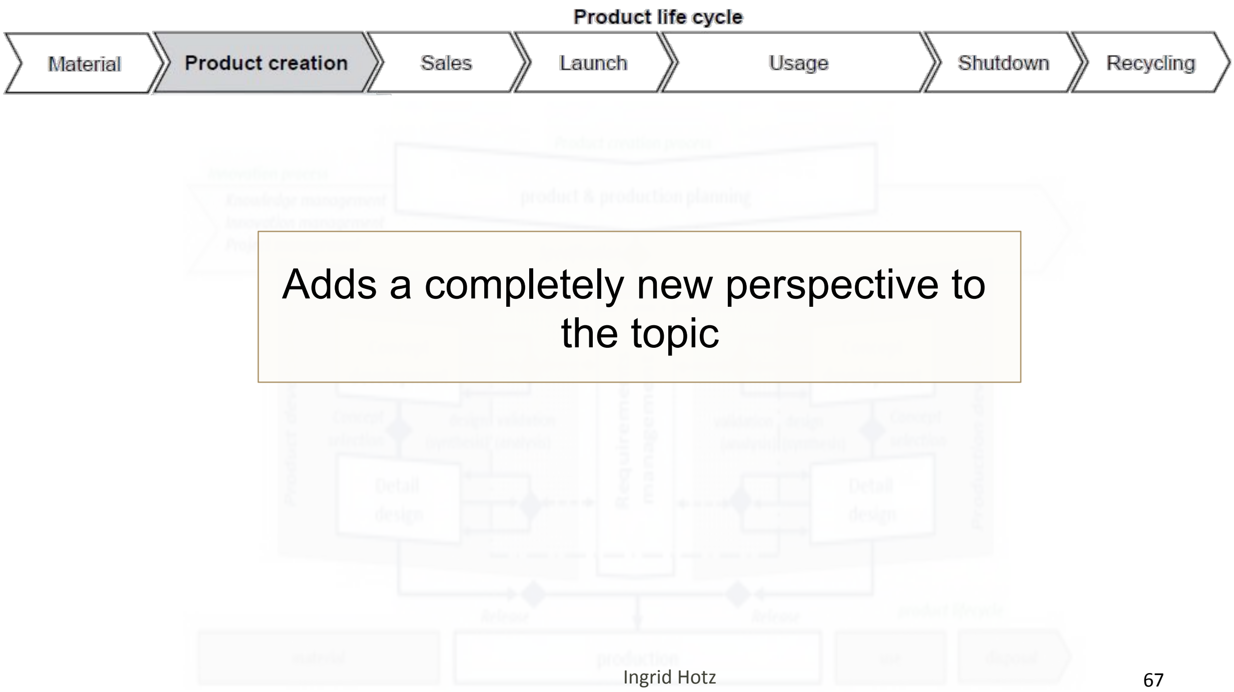
Visualization – Framework



Images: Kratz, ZIB, Amira

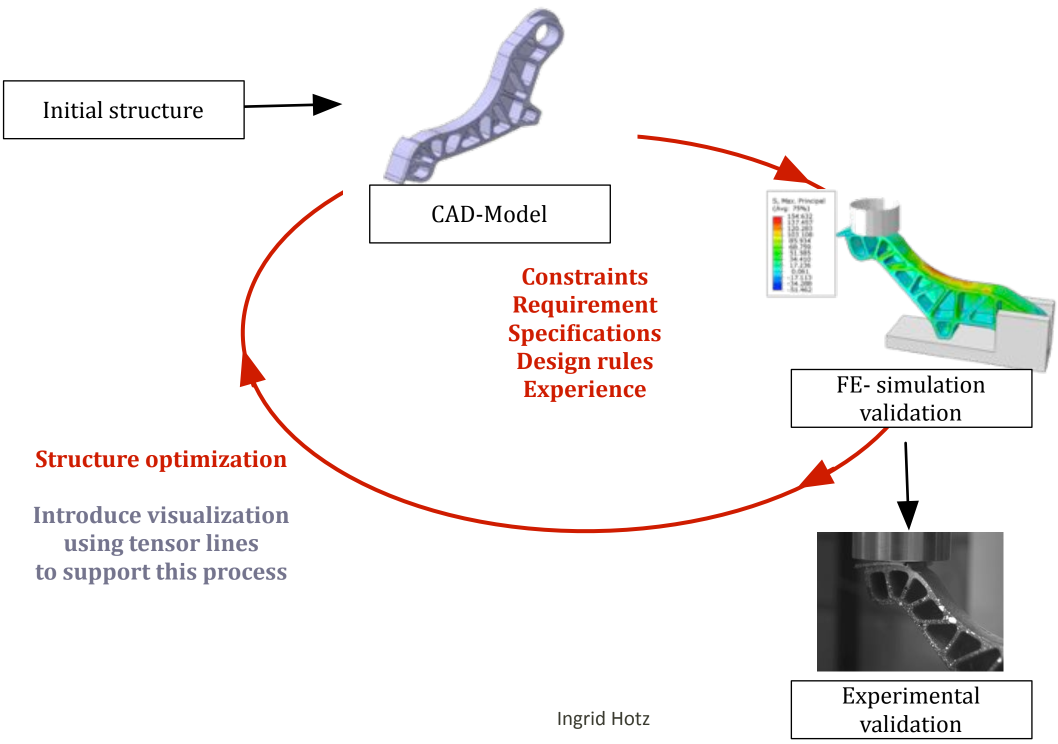
Engineering – Product Development Process

Tensors in Material Sciences
Engineering – Product Development Process



Adds a completely new perspective to the topic

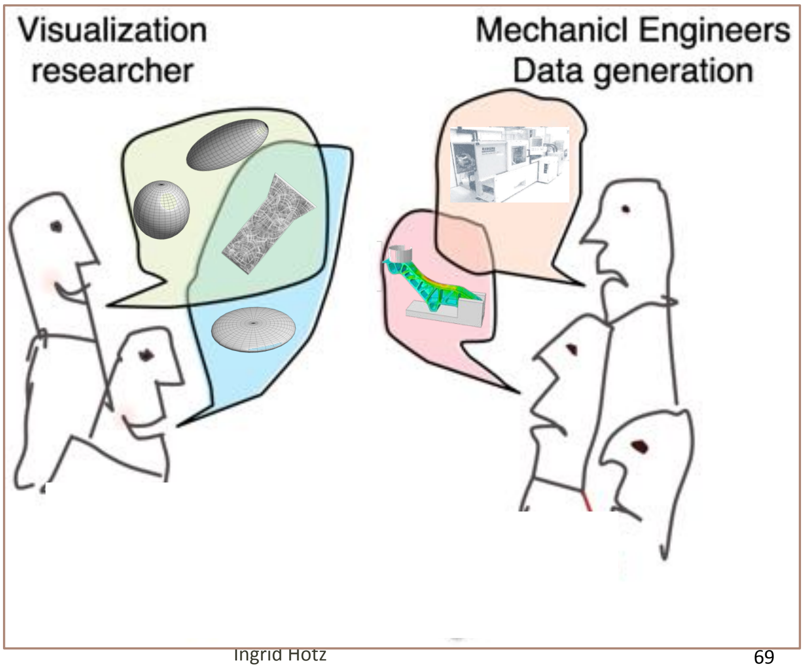
Engineering – Product Development Process



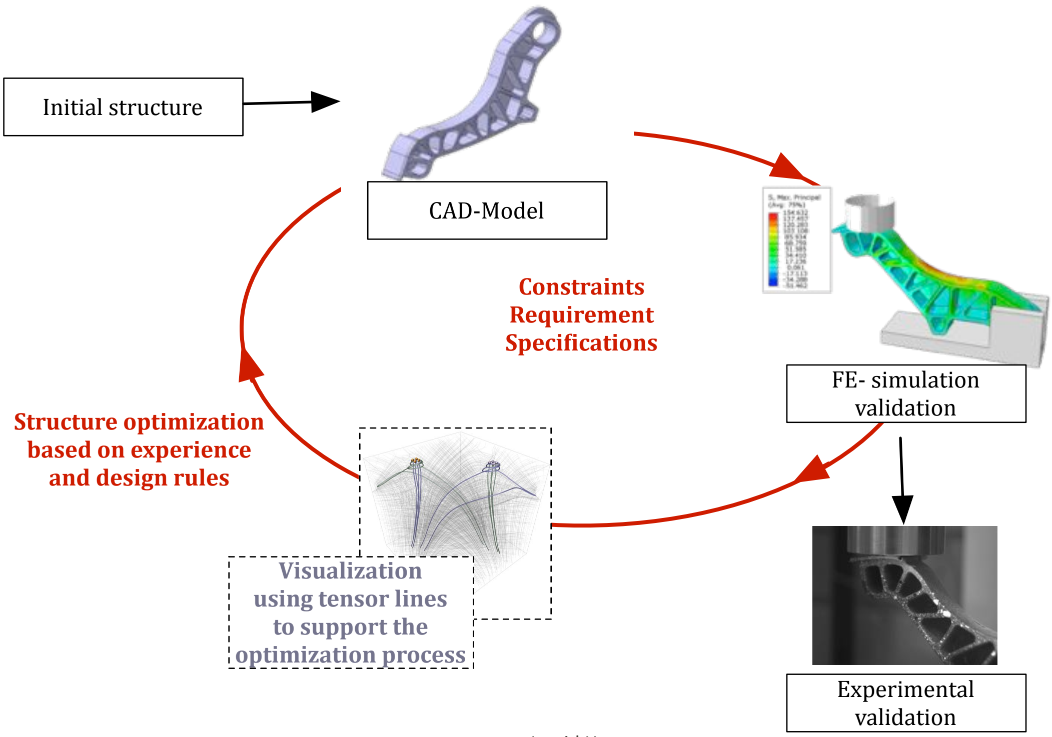
IV Story of a Collaboration

First Experiments

- Everybody shows what they have



IV Story of a Collaboration

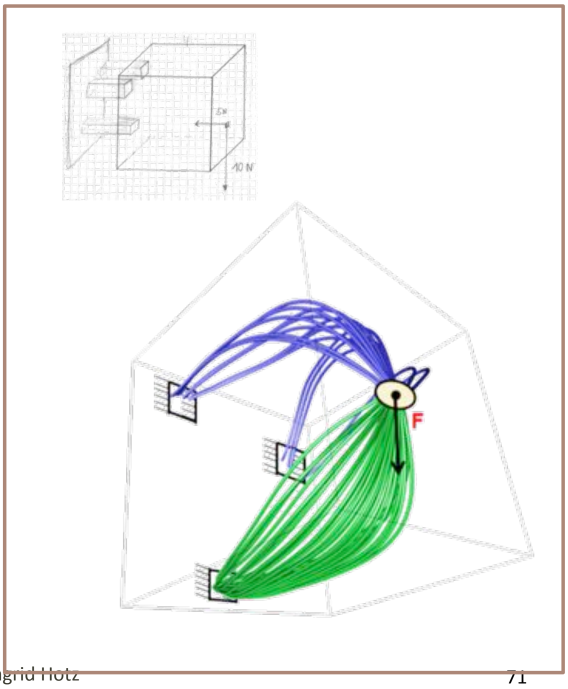


IV Story of a Collaboration

Hypothesis resulting from first experiments and discussions

- The **tensor lines** for a stress tensor field are related to the **major load paths** from the **operating loads to the fixation** points of a technical part

- Tensor lines can be used to support the **design of reinforcement structures**



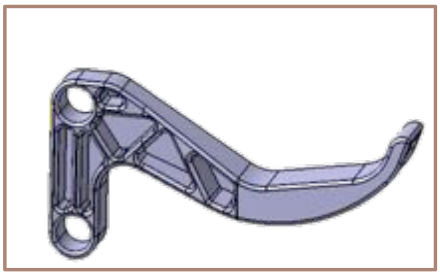
IV Story of a Collaboration

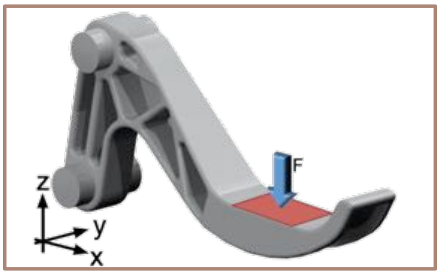
Design of a break lever

- Build reinforcement structure on basis of tensor line visualization
- **Combination of an automatic analysis with the expert's knowledge**
- Manageable but realistic case

Evaluation

- Comparison to reference structure – same volume
- Numerical validation
- Experimental validation



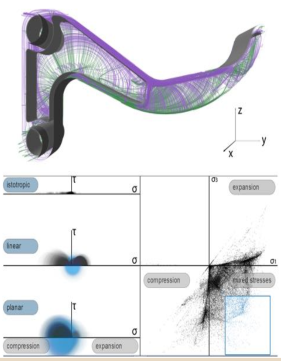


IV Story of a Collaboration

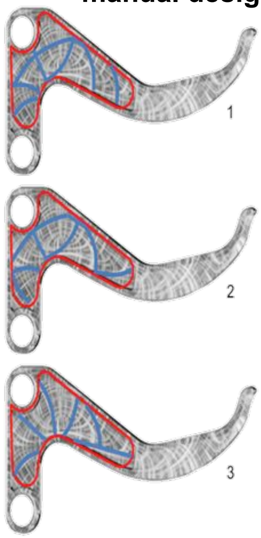
Design of the rib structure

- New geometries based on tensor visualization
- No additional steps in the workflow

Stress Tensor Visualization



Vis supported manual design



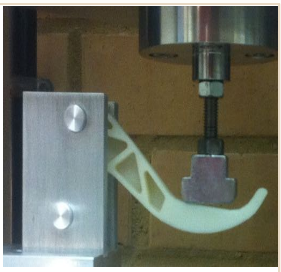
CAD - Model

IV Story of a Collaboration

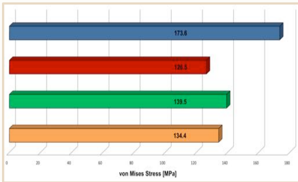
Numerical and experimental tests

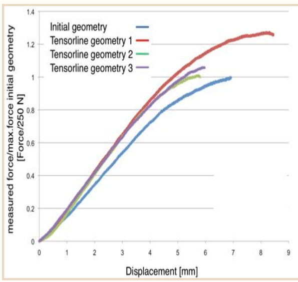
- New designs (red, green, yellow)
- Reference geometry (blue)

→ All new geometries performed better as the reference geometry



Rapid Prototyping and experimental validation: Test setup



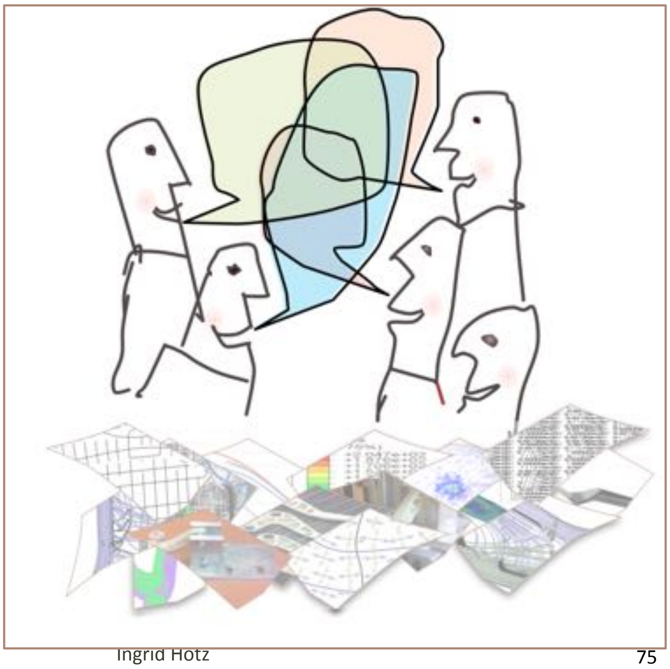


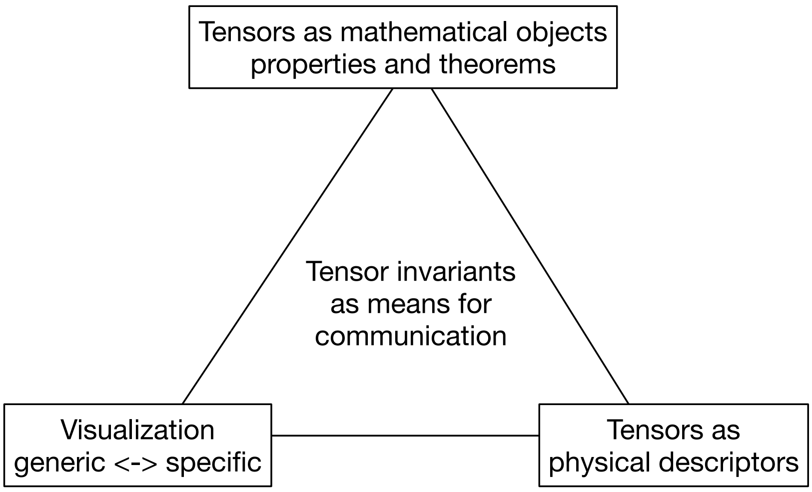
IV Story of a Successful Collaboration

Advances in both fields

Continuing Story

- Many exiting questions and ideas
- Much fun
- Proposal submitted





Relevance and meaning of mathematical properties and theorems is application dependent

- *Anisotropic Sampling of Planar and Two-Manifold Domains for Texture Generation and Glyph Distribution,* Kratz, Baum, Hotz, TVCG, 2013
- *Three-Dimensional Second-Order Tensor Fields: Exploratory Visualization and Anisotropic Sampling* (phdthesis), Andrea Kratz, 2013
- *Visualization and Analysis of Second-Order Tensors: Moving Beyond the Symmetric Positive-Definite Case*, Kratz, Auer, Stommel, Hotz, Computer Graphics Forum - State of the Art Reports, 2013
- *Tensor Invariants and Glyph Design*, Kratz, Auer, Hotz, Visualization and Processing of Tensors and Higher Order Descriptors for Multi-Valued Data (Dagstuhl'11), Springer, 2014

- **http://www.tensorvis.org/**