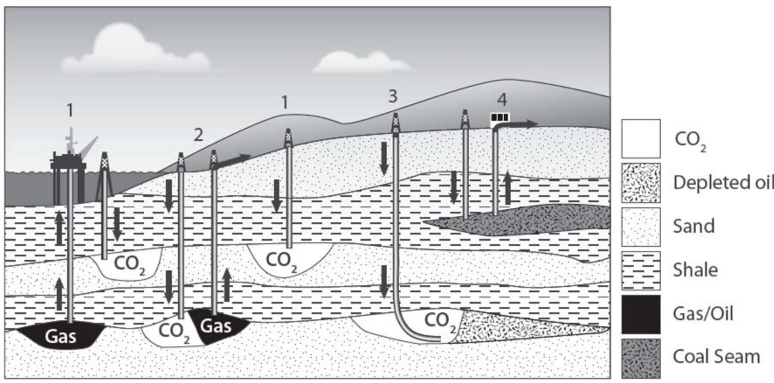
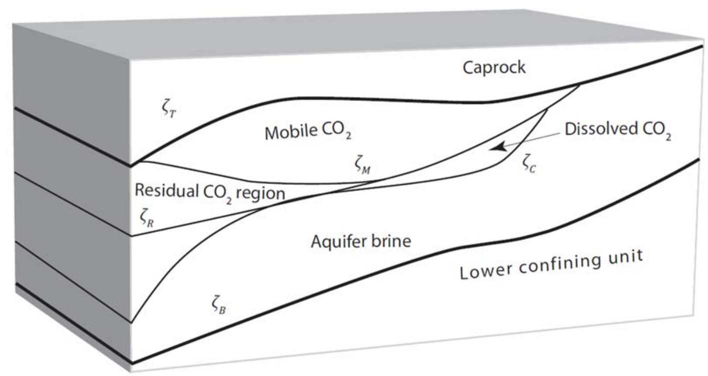
# Multiscale modeling:
# A modern look at old equations



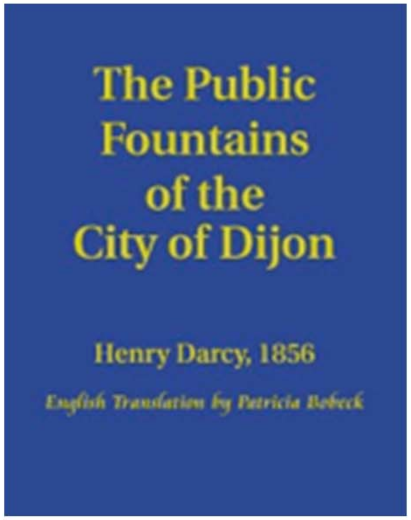




Geilo, January 25, 2011

# Topics

- Foundation 1: Multi-phase flow in porous media
- Foundation 2: Multiscale modeling
- Application: Understanding a generation of engineering models as a family of consistent multiscale models.
- Bonus: Novel, synergetic, models.

# Henry Darcy


The Public Fountains of the City of Dijon
Henry Darcy, 1856
English Translation by Patricia Bobeck

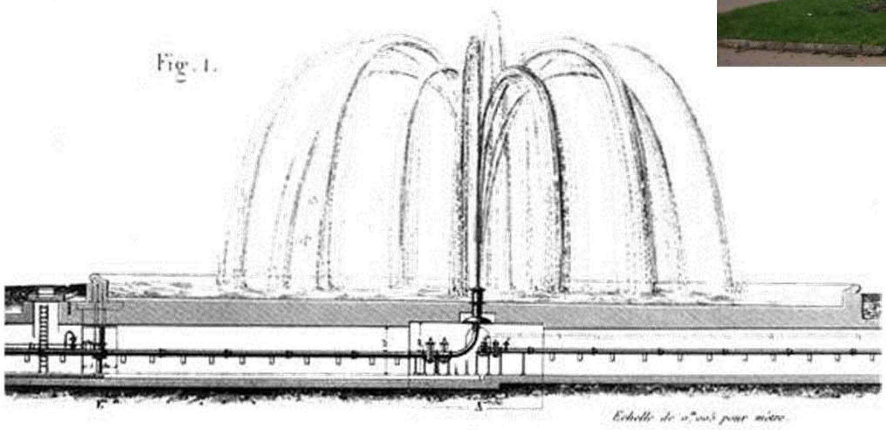
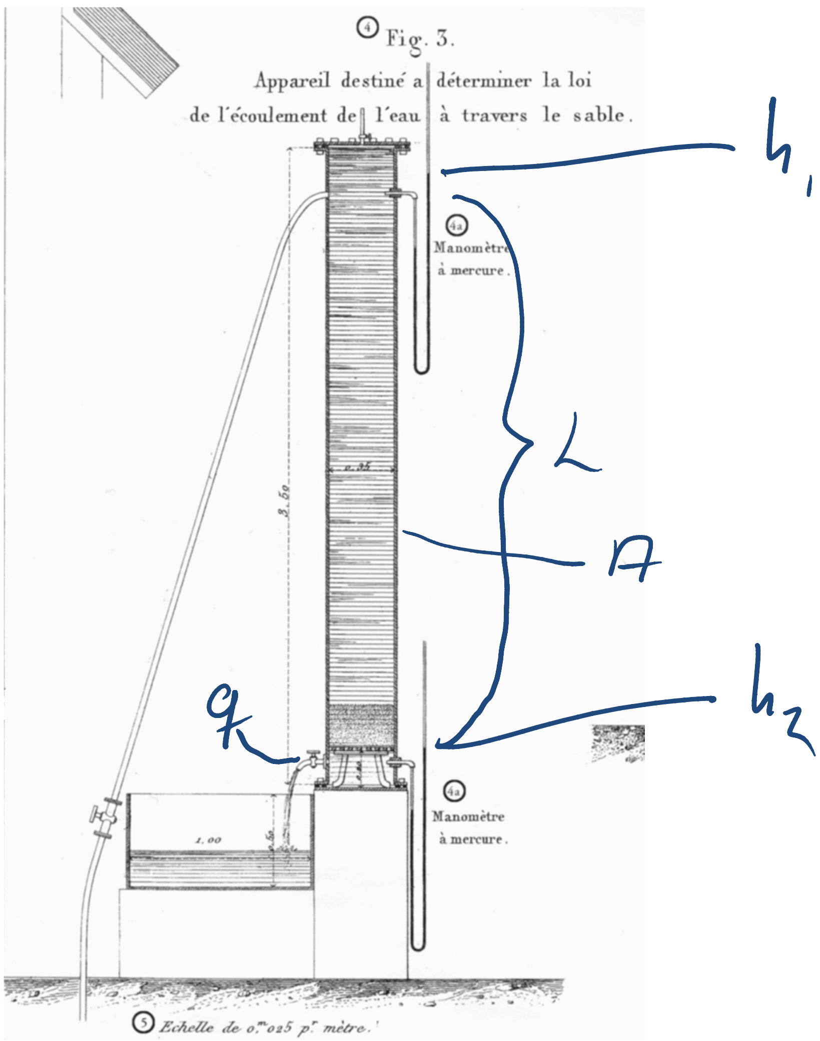


Fig. 1.

# Filtration of water




$$q \sim A \cdot \frac{h_1 - h_2}{L}$$

$$q = -K \nabla h$$

$$h = p - \rho g \cdot z$$

# Flow through porous rocks

- Proportional to (potential) energy gradient
- Inversely proportional to viscosity
- Rock dependent
- Dependent on fluid occupancy
- No momentum conservation!
- Interaction with other physical phenomena:
  - Transport, (geo)mechanics, (geo)chemistry, free-fluid flow, thermodynamics, radioactivity...

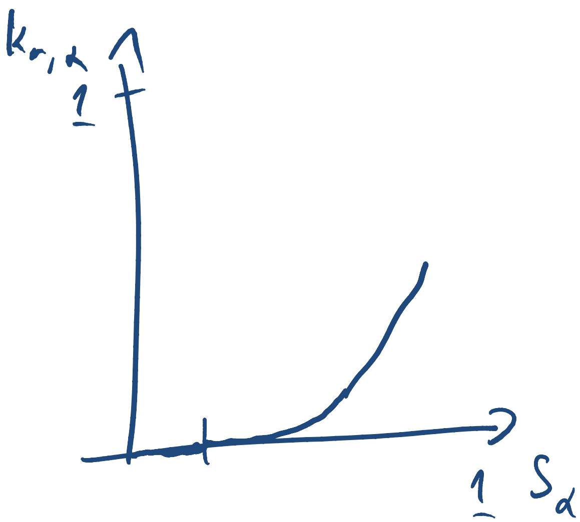
# Summary of «simple» flow equations

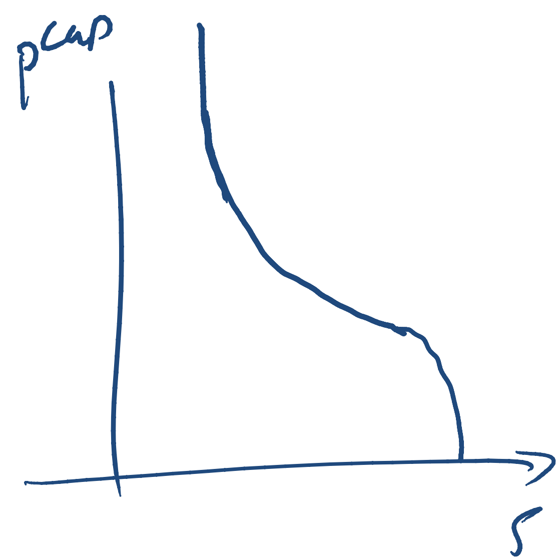
$$q_\alpha = -\frac{k\, k_{r,\alpha}(s)}{\mu_\alpha} \nabla (p_\alpha - \rho g z)$$

$$\sum_\alpha S_\alpha = 1$$

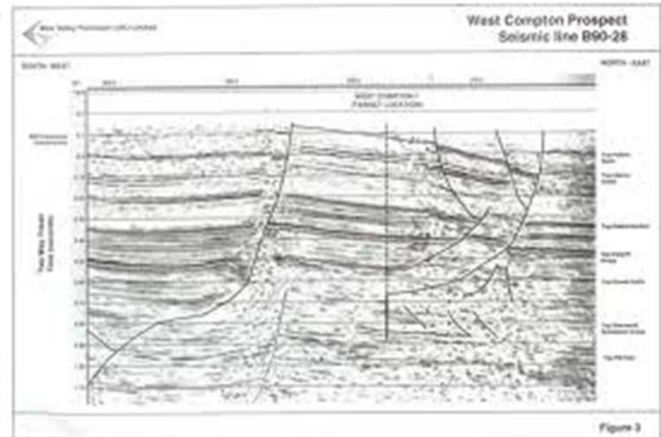
$$\nabla \cdot q = 0$$

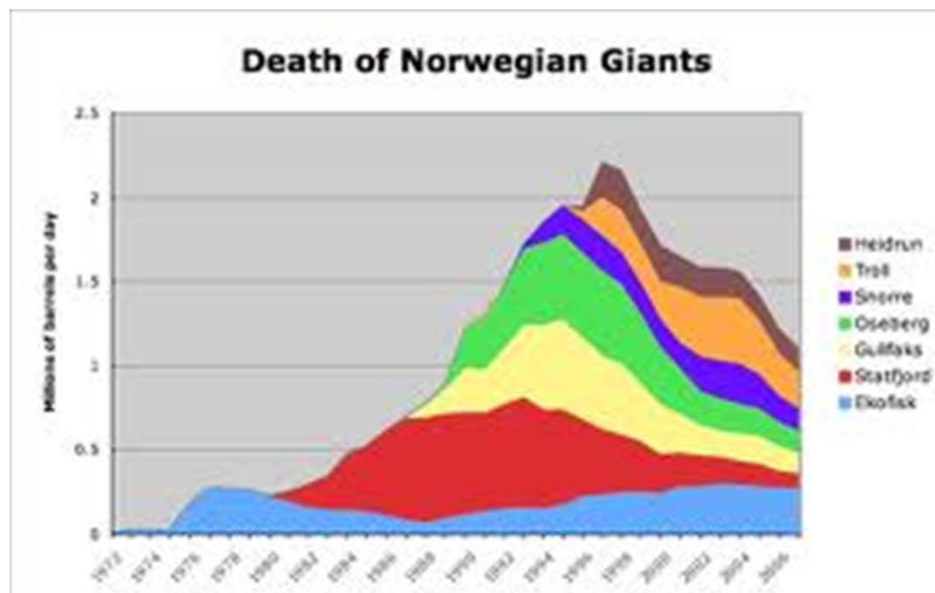
$$p^{cap} \doteq p_a - p_w = p^{cap}(s)$$



# Multiscale porous media

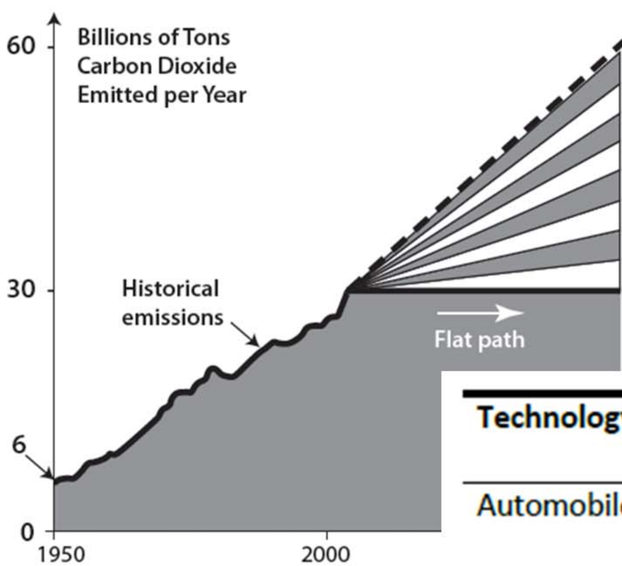


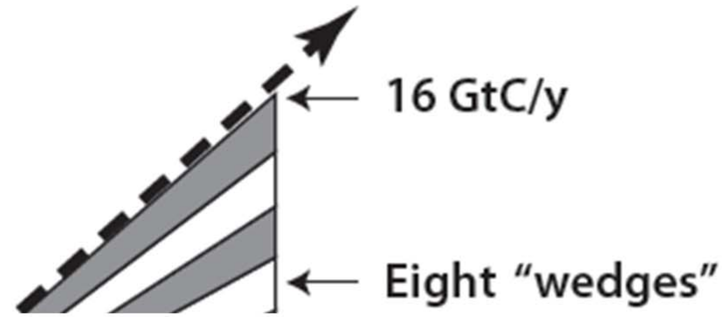



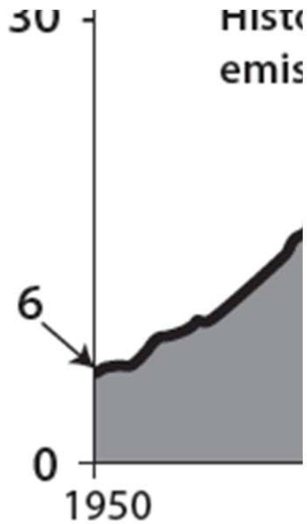
# Multiscale properties

- Parameters:
  - Heterogeneous at all length scales (no separation!)
  - Only «known» at coarse resolution
  - Highly uncertain in practice
- Solution:
  - Singularities near wells
  - Singularities near parameter discontinuities
  - Unstable displacement may give fractal displacement

# Fixing global CO2 emissions

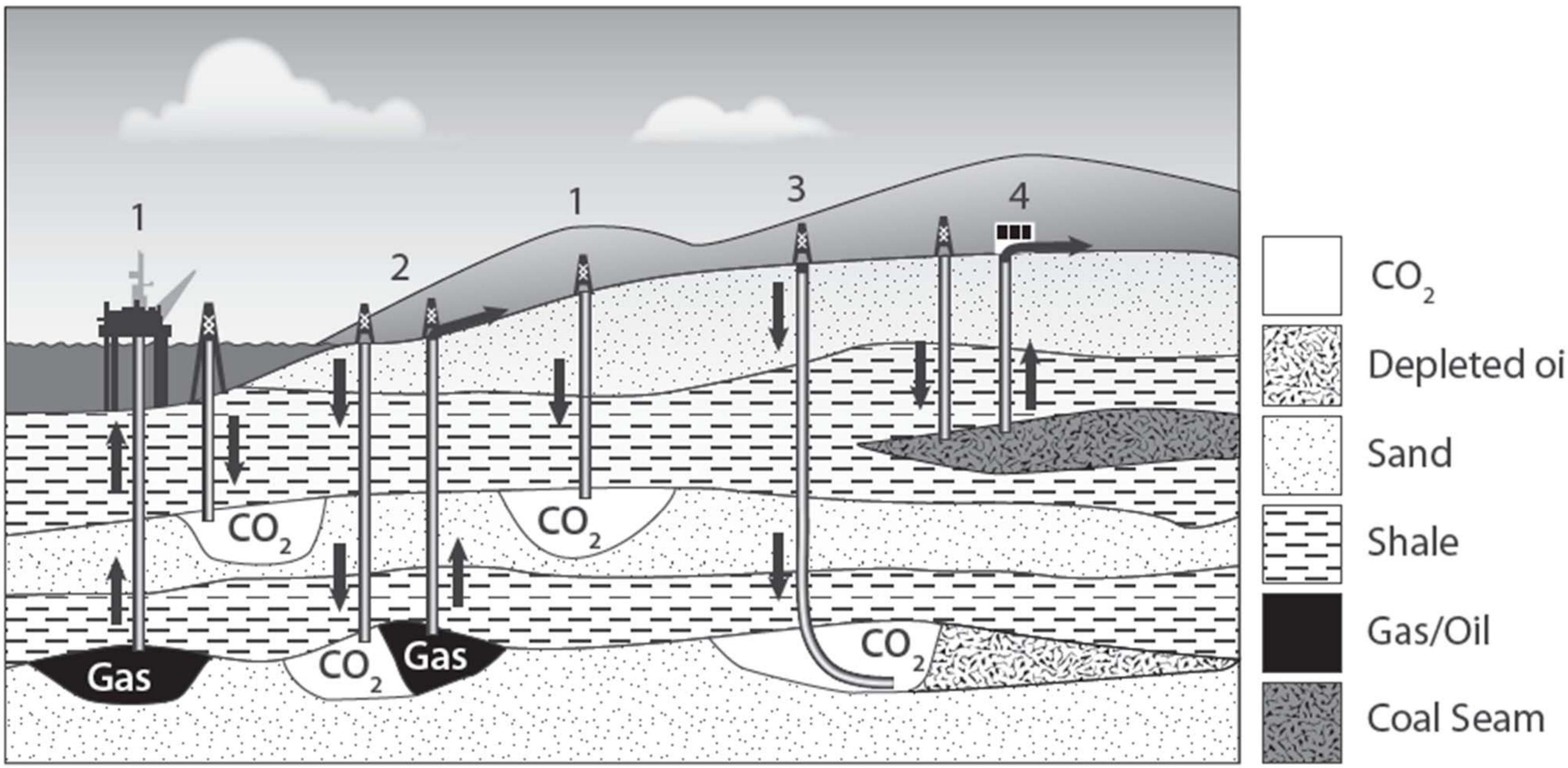


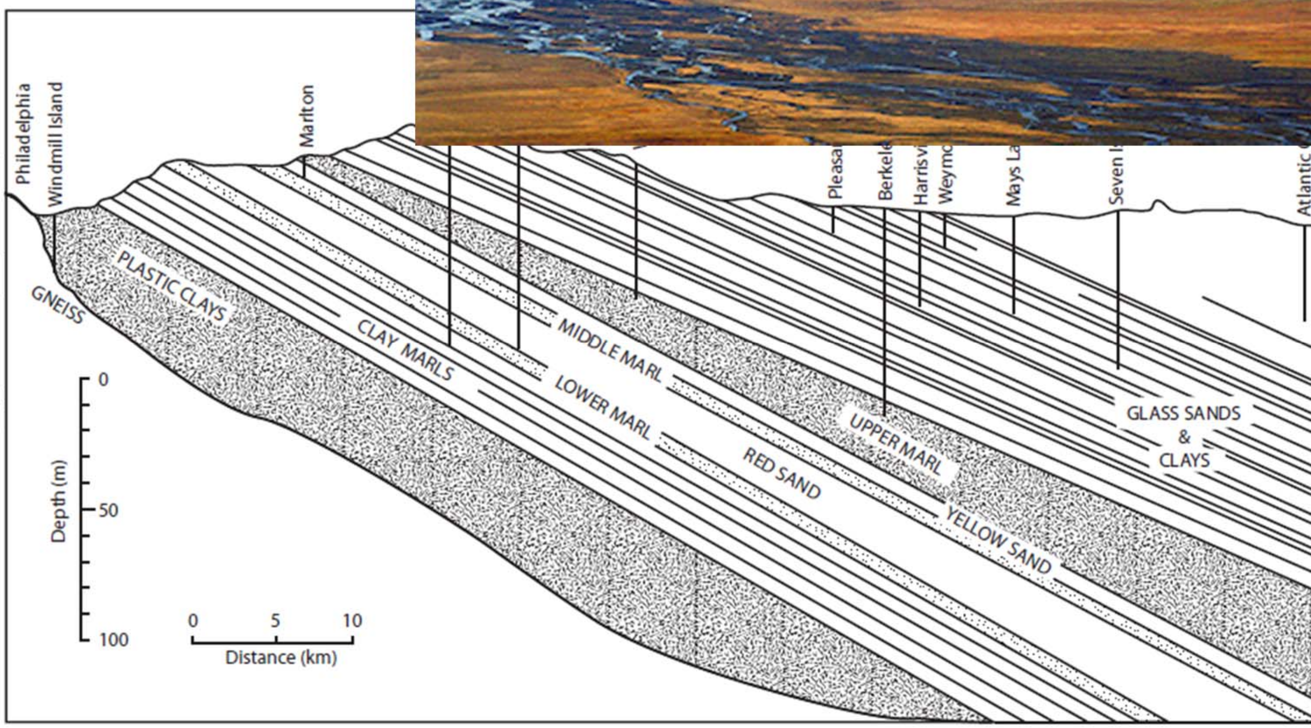

| Technology | Effort Required to Achieve One Wedge (from [4]) |
|---|---|
| Automobile Efficiency | Increase fuel efficiency from 30 miles-per-gallon (mpg) to 60 mpg for 2 billion cars (currently there are fewer than 1 billion cars worldwide). |
| Nuclear Power | Add twice the currently installed nuclear capacity. |
| Solar Power | Add 700 times the currently installed solar power capacity. |
| Wind Power | Add 40 times the currently installed capacity of wind-generated electricity. |
| Carbon Capture and Storage | Install carbon capture and storage at 800 large-scale coal-fired power plants. |

# Geological Storage Options



# Challenges

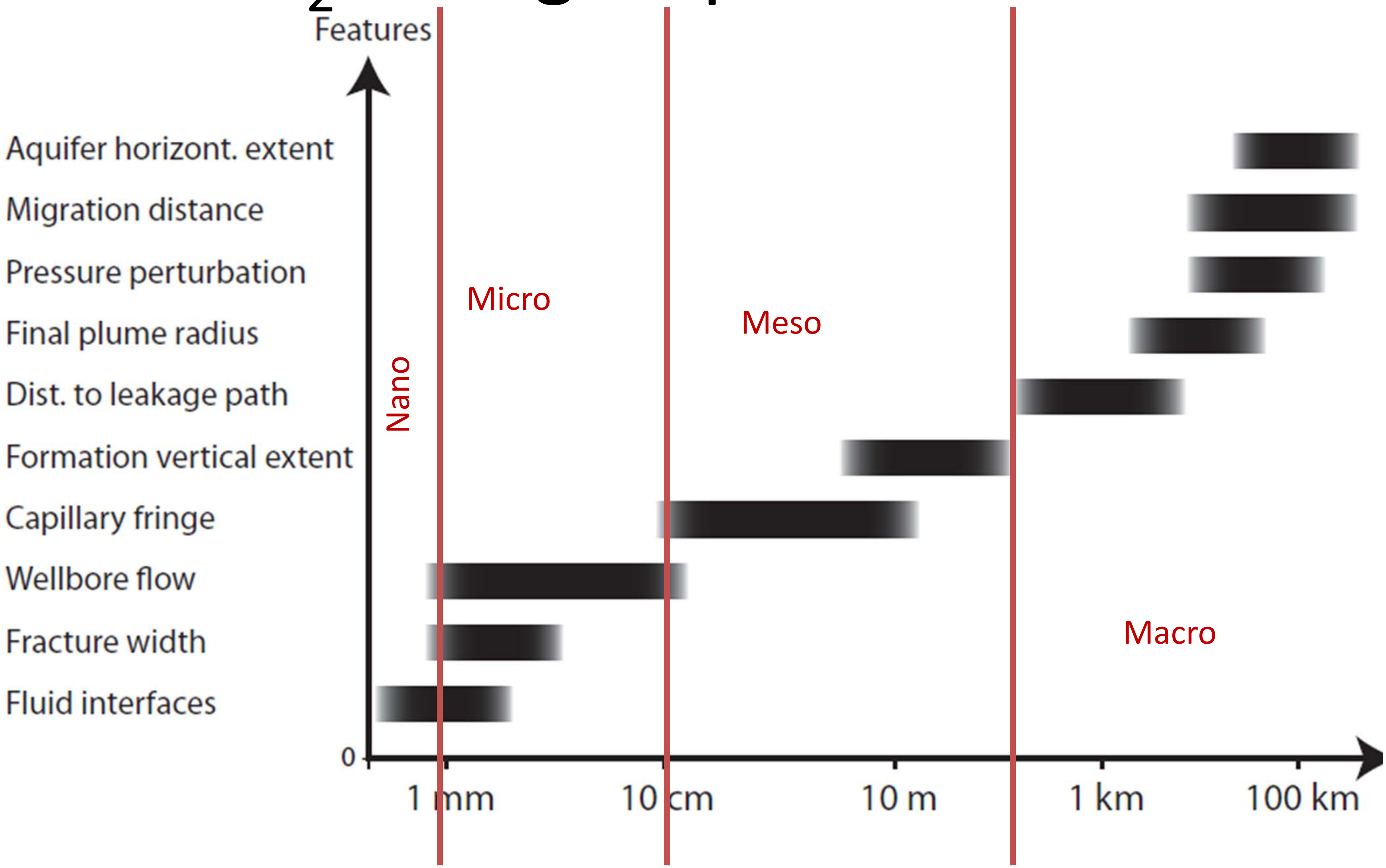
- Large lateral extent of
- Small vertical extent.
- Regulatory framework analysis.
- Highly het parameter
- Potentially



# Large scale models for $CO_2$ storage

- Dependent on length and time scales.
- Dependent on physical processes.
- Need to include aquifer topology and heterogeneity.
- Should have transparent derivation and interpretation.

# $CO_2$ storage: Spatial scales

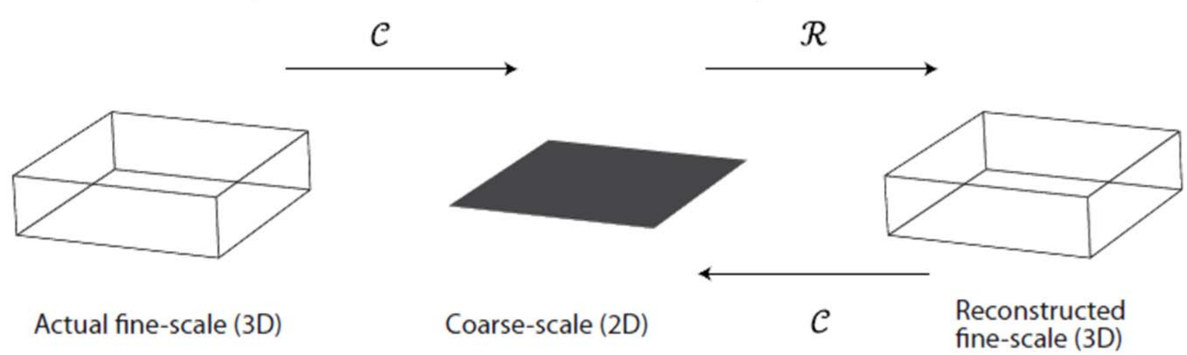


# Aquifer models for $CO_2$ storage

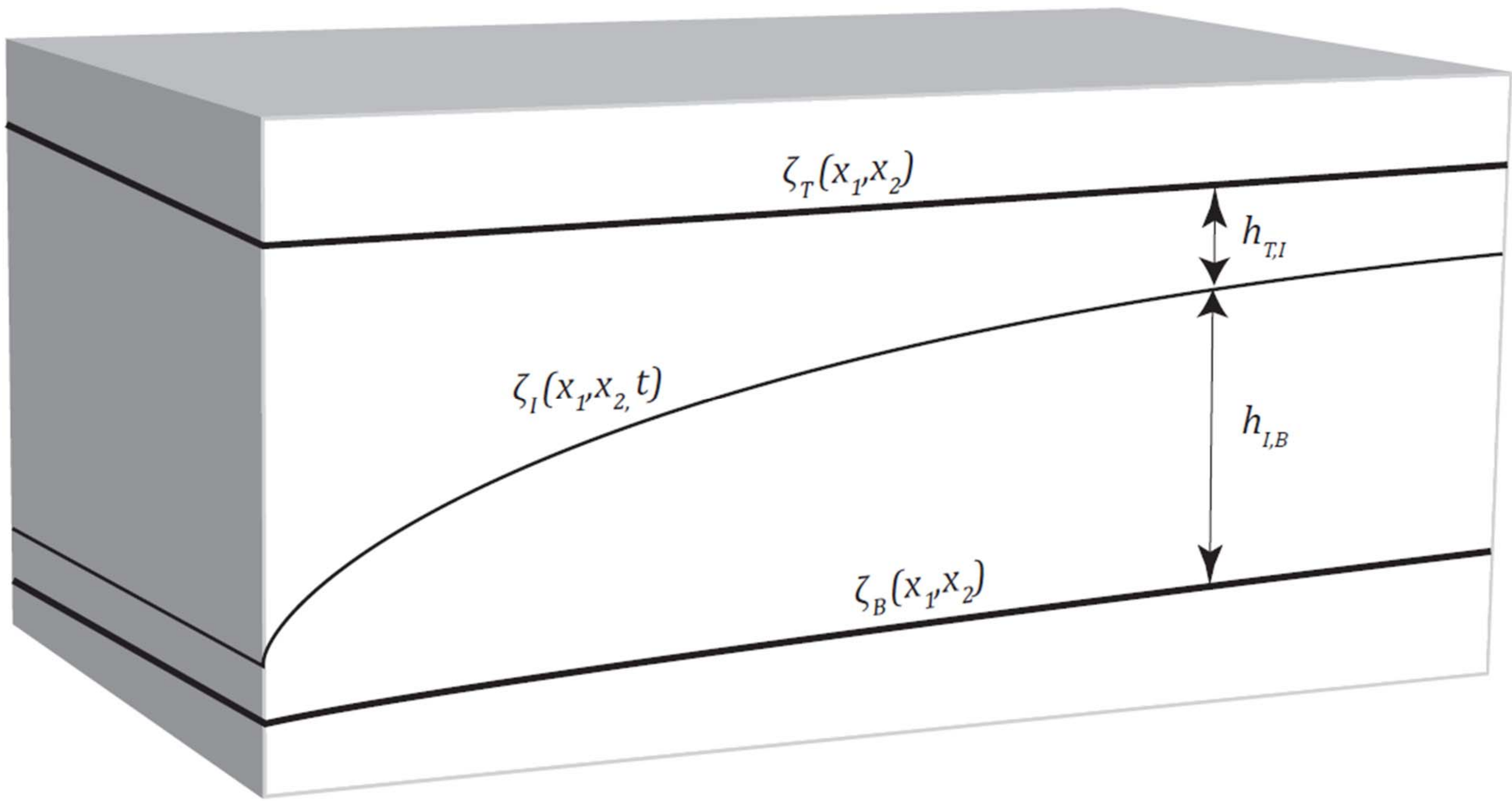
- Upscale equations vertically: 3D -> 2D
- Key requirements:
  1. Assumptions on distribution of fine-scale variables
  2. Component masses, pressure, saturations, …
- Traditional models:
  Sharp interface model for saltwater intrusion;
  Sharp interface model in oil and gas recovery;
  Models for unconfined aquifers
- Horizontal upscaling?

# Multiscale framework

- Coarsening operator (denoted $\mathcal{C}$) represents e.g. integration, subsampling, or other.
- Reconstruction operator (denoted $\mathcal{R}$) is required for upscaling of constitutive relationships.
- Consistency is enforced by $U = \mathcal{C}\mathcal{R}\, U$



# Upscaling two-phase flow

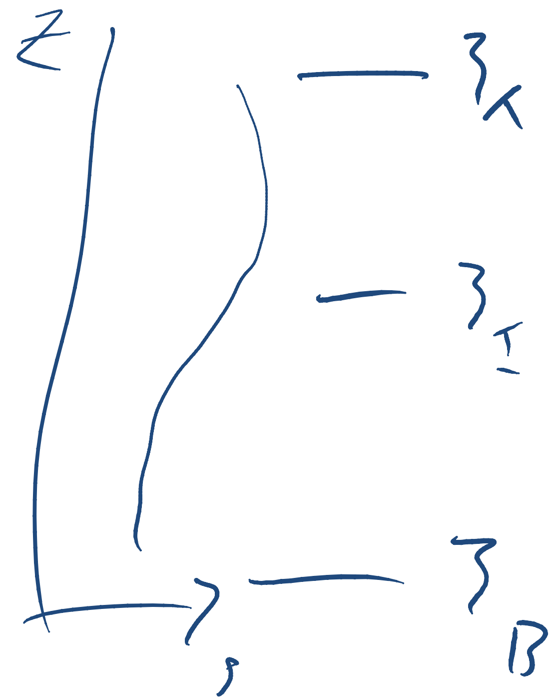


$\xi_i \; ; P$

# Coarsening operators

$$P = C p \equiv p(\xi_p)$$

$$\xi_I = C s$$



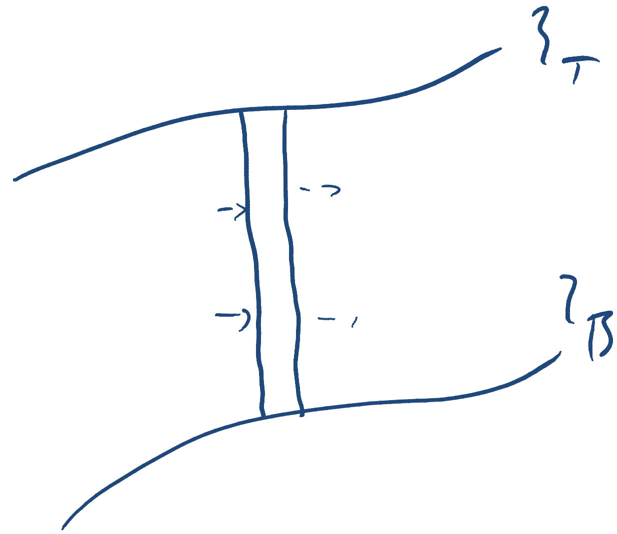
# Coarse model

- Mass/Volume conservation

$$\nabla_{\parallel} \cdot \underline{U}_T = 0$$

- Coarse flux law

$$\underline{U}_\alpha = \int_{\zeta_B}^{\zeta_T} \underset{\sim}{e}_{\parallel} \cdot \underline{u}_\alpha \, dz$$



# Reconstructing pressure

$$P = C p = p(\zeta_p)$$

$$P = P(x_1, x_2, t)$$

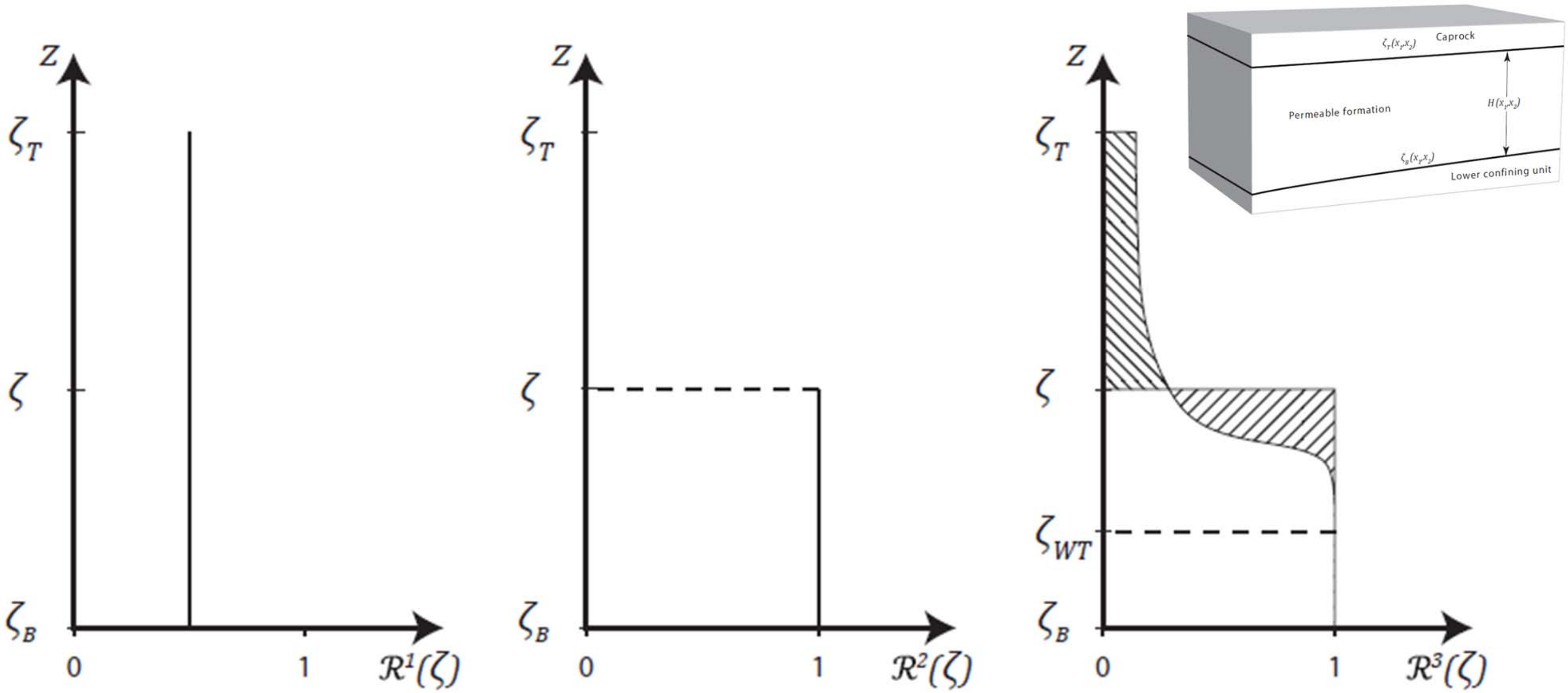
$$p = p(x_1, x_2, x_3, t)$$

$$0 = u_z = -\frac{k}{\mu}\frac{\partial}{\partial z}(p - \rho g z)$$

$$\hat{p} = \underbrace{p(\zeta_p)}_{P} - \rho g (x_3 - \zeta_p) \equiv R P$$

$$C \hat{p} = C R P = P$$

# Reconstructing saturation




- Same average saturation can be modeled as different vertical distributions!

# Incompressible two-phase flow

- Mass (volume) Conservation: $\Phi \dfrac{\partial S_\alpha}{\partial t} + \nabla_{||} \cdot \boldsymbol{U}_\alpha = \Upsilon_\alpha$
- Darcy's Law

$$\boldsymbol{U}_\alpha = \int_{\zeta_B}^{\zeta_T} \boldsymbol{e}_{||} \cdot \boldsymbol{u}_\alpha \, dx_3 = -\int_{\zeta_B}^{\zeta_T} \boldsymbol{k}_{||} \lambda_\alpha \left(\mathcal{R}^{II}_{s_\alpha} S_\alpha, \widehat{s_c}^{\,t}\right)\left(\nabla_{||} \mathcal{R}^{D}_{p_\alpha} P_\alpha - \rho_\alpha \boldsymbol{e}_{||} \cdot \boldsymbol{g}\right) dx_3$$

- Coarse scale forms:

$$\Phi \frac{\partial S_\alpha}{\partial t} - \boldsymbol{\nabla}_{||} \cdot \left(\boldsymbol{K} \boldsymbol{\Lambda}_\alpha \left(S_\alpha, \widehat{s_c}^{\,t}\right)\left(\boldsymbol{\nabla}_{||} P_\alpha - \varrho_\alpha \boldsymbol{G}\right)\right) = \Upsilon_\alpha$$

$$\boldsymbol{K} = \int_{\zeta_B}^{\zeta_T} \boldsymbol{k}_{||} dx_3, \quad \boldsymbol{\Lambda}_\alpha \left(S_\alpha, \widehat{s_c}^{\,t}\right) = \boldsymbol{K}^{-1} \int_{\zeta_B}^{\zeta_T} \boldsymbol{k}_{||} \lambda_\alpha \left(\mathcal{R}^{II}_{s_\alpha} S_\alpha, \widehat{s_c}^{\,t}\right) dx_3,$$

$$\boldsymbol{G} = \boldsymbol{e}_{||} \cdot \boldsymbol{g} + (\boldsymbol{g} \cdot \boldsymbol{e}_3) \boldsymbol{\nabla}_{||} \zeta_P$$

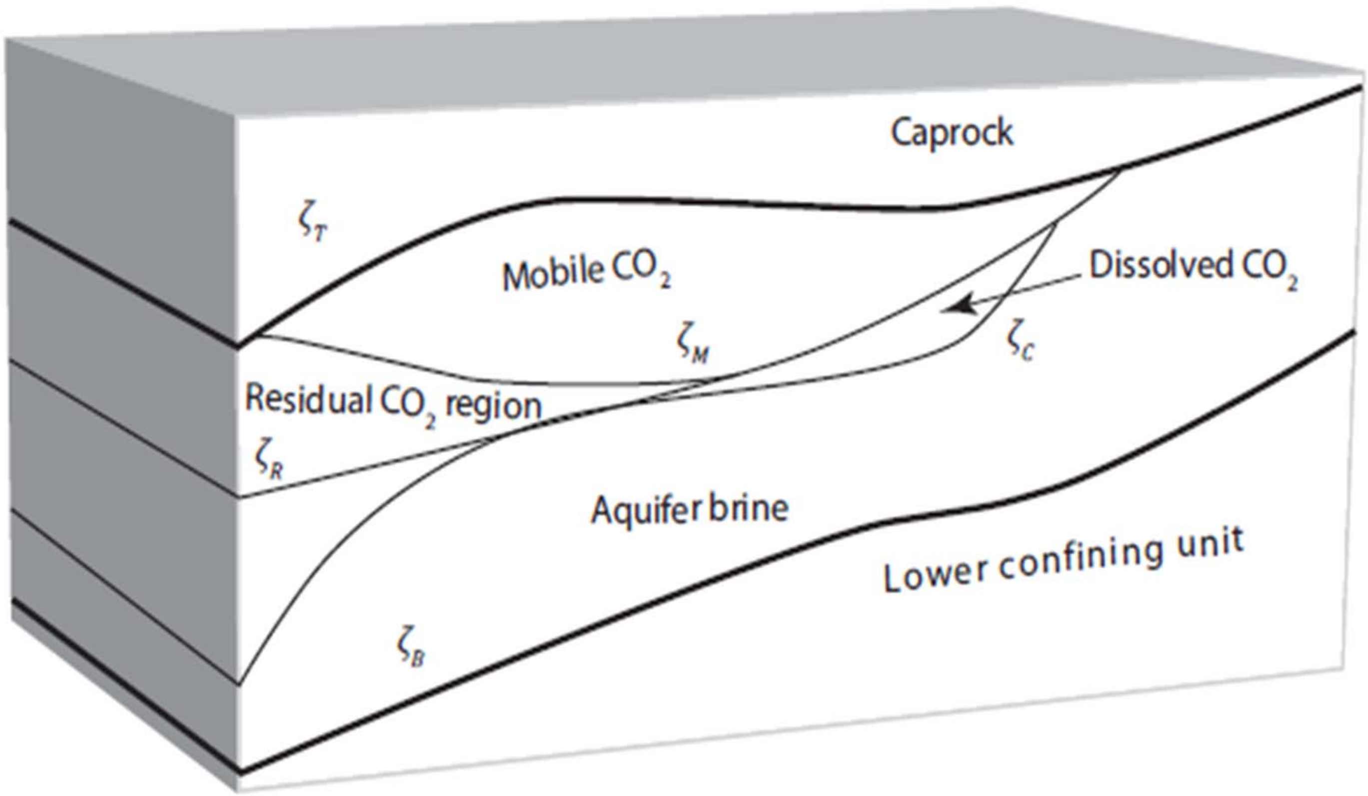
# Dependence on sat. reconstruction

- Uniform saturation:
  - «No upscaling»: Same as coarse numerical grid
  - Accurate for disperse systems
- Sharp transition:
  - «Sharp interface model»: Traditional, old-fashioned
  - Accurate for gravity dominated problems
- Capillary zone:
  - Unconventional model (although from 1970's).
  - Accurate when capillarity and gravity balance.

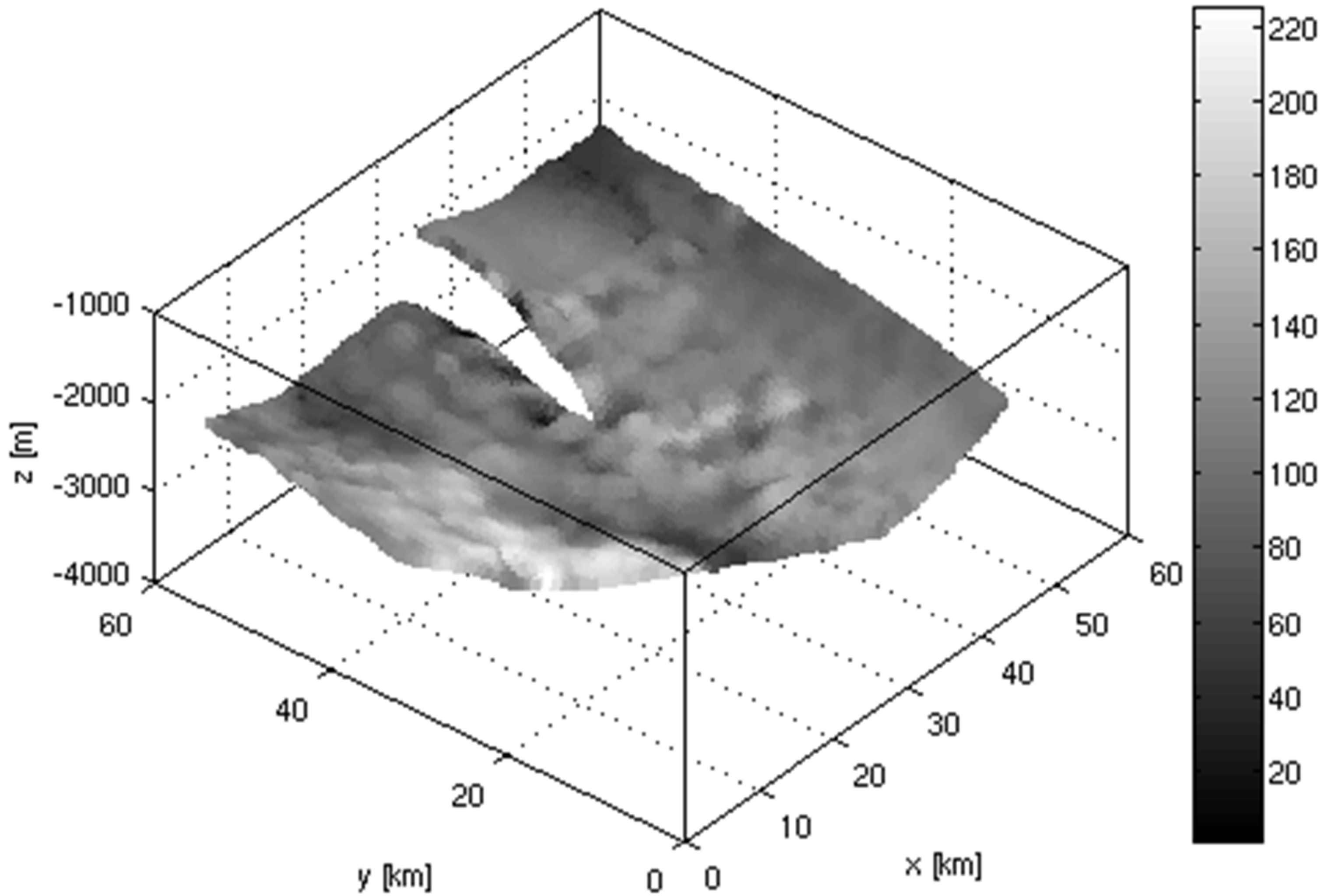
# What does this mean?

- A mathematically consistent family of coarse models.
- Complete transparancy with respect to modeling assumptions.
- Model family includes three classical models from literature.
- Framework easily extends to account for more complex phenomena.

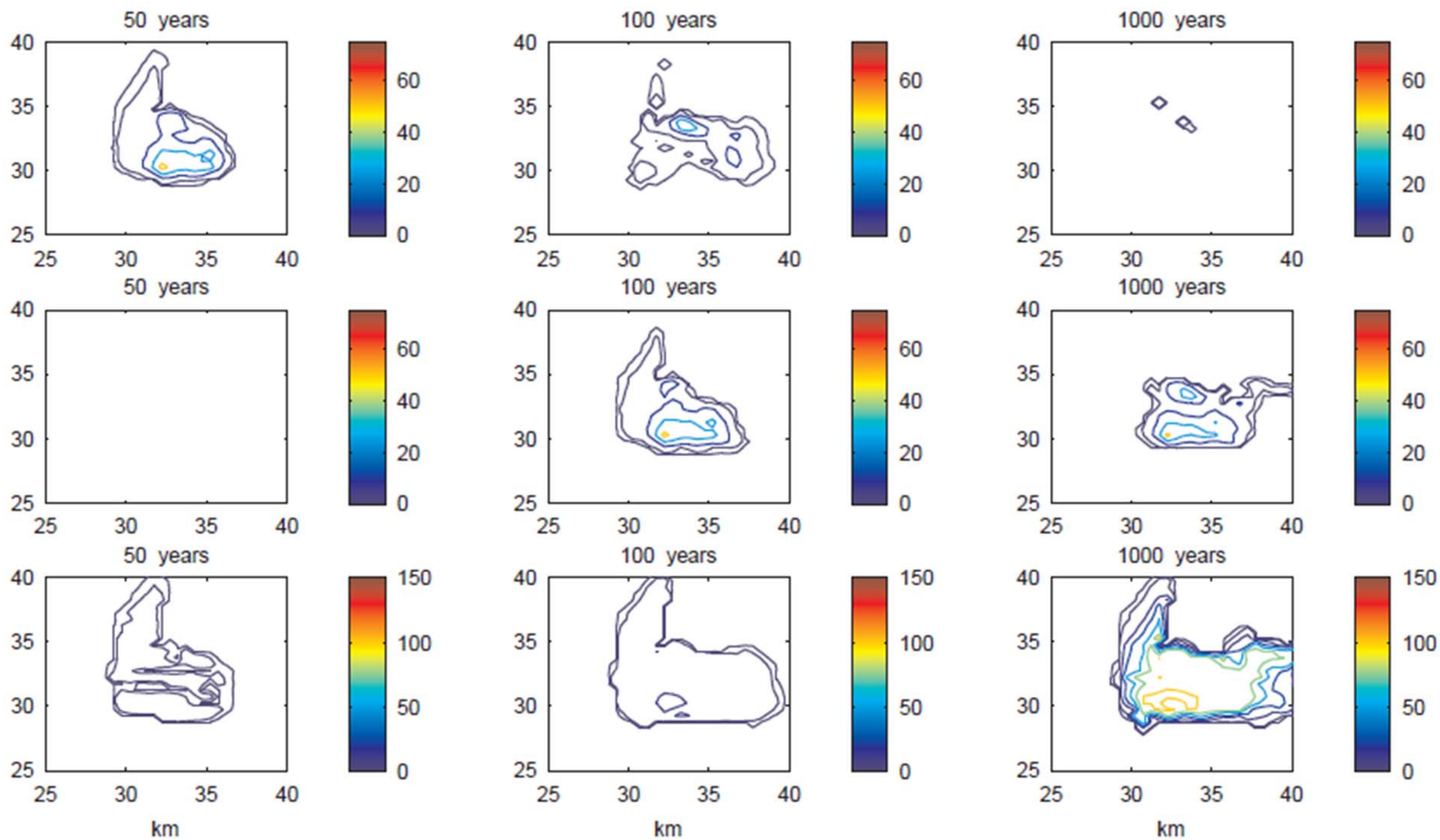
# Examples – CO2 storage



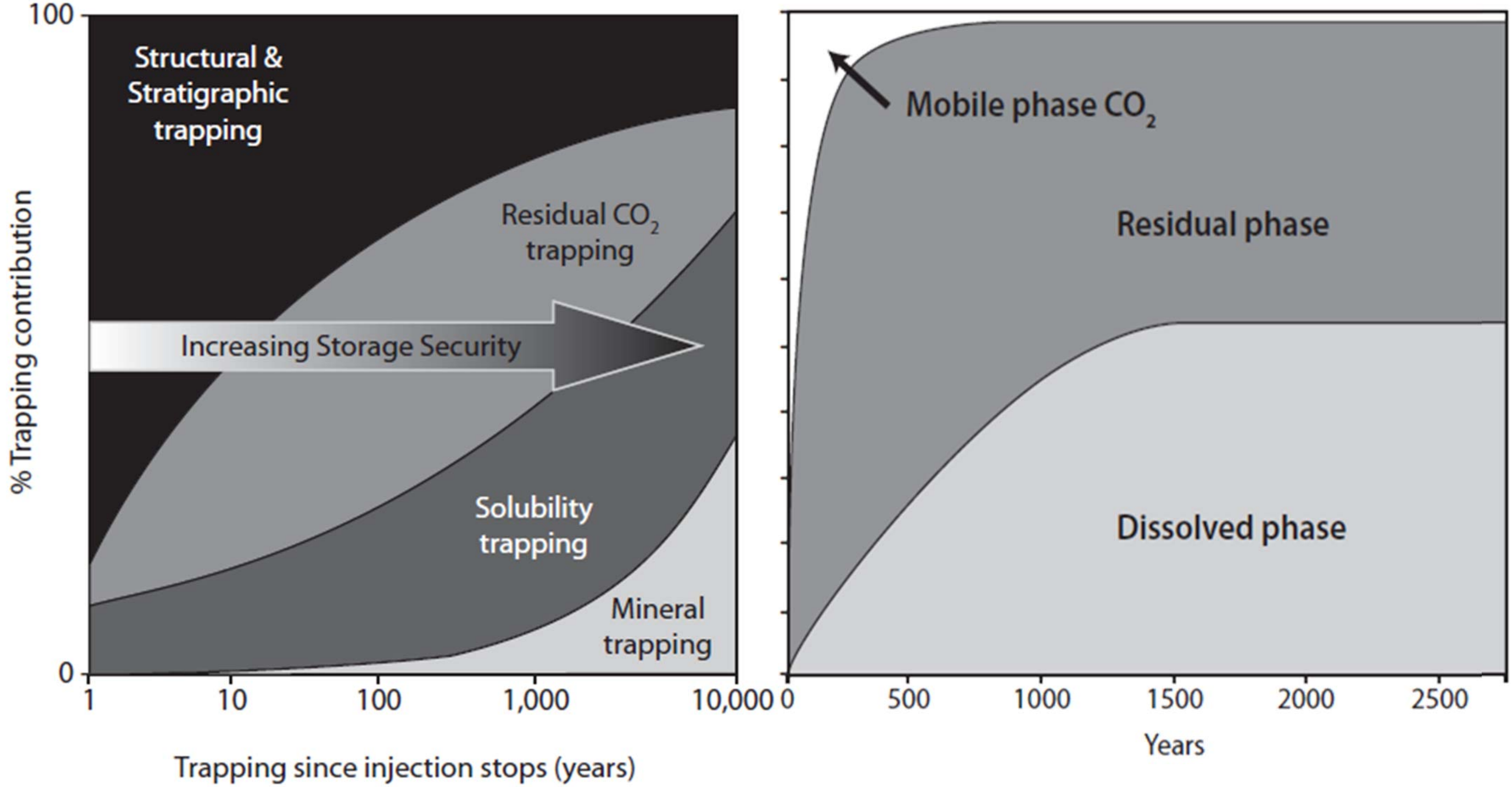
# Case study: Johansen formation



# Extent of $CO_2$ migration



# Storage security



# Thoughs for skiing

1. Think of a familiar application. Can you understand it in a new way by thinking in terms of multiscale modeling?
2. What is the difference between upscaling and multiscale modeling?
3. When can multiscale be used as a preconditioner?
4. What kind of non-linear problems are possible/impossible to consider with multiscale approaches?