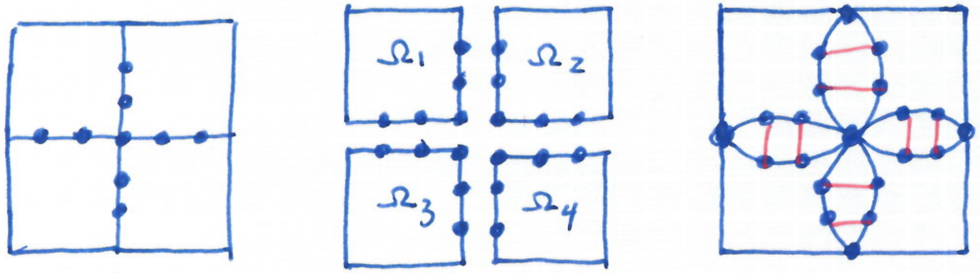
FETI – DP

(dual-primal)

Farhat – Lesoinne – et.al (2001)

Mandel – Tezaur (2001)



$\hat{W}$ $\qquad$ $W = \prod W_i$ $\qquad$ $\tilde{W}$

Introduce $\tilde{W}$

$$\hat{W} \subset \tilde{W} \subset W$$

relatively small number of continuity constraint.

Idea:

- continuity of the primal dofs by subassembly
- continuity constraints on dual dofs by Lagrange multp.

e.g. continuity at vertices / continuity of average over Γ_{ij}

classification:

Π : primal dofs

Δ : dual dofs.

Schur complement system on $\widetilde{W}$

$$A = \begin{pmatrix} A^{(1)} & & \\ & \ddots & \\ & & A^{(N)} \end{pmatrix} \longrightarrow \widetilde{A} = \begin{pmatrix} A_{II} & A_{I\Pi} & A_{I\Delta} \\ A_{I\Pi}^T & A_{\Pi\Pi} & A_{\Pi\Delta} \\ A_{I\Delta}^T & A_{\Pi\Delta}^T & A_{\Delta\Delta} \end{pmatrix}$$

No longer block-diagonal

stiffness matrix w.r.t. $\prod_i W^h(\Omega_i)$

stiffness matrix after gluing the primal dofs.

Schur complement a.w. Δ (dual dofs):

$$\widetilde{S} = A_{\Delta\Delta} - \begin{pmatrix} A_{I\Delta}^T & A_{\Pi\Delta}^T \end{pmatrix} \begin{pmatrix} A_{II} & A_{I\Pi} \\ A_{I\Pi}^T & A_{\Pi\Pi} \end{pmatrix}^{-1} \begin{pmatrix} A_{I\Delta} \\ A_{\Pi\Delta} \end{pmatrix}$$

And the system:

$$\boxed{\widetilde{S} u_\Delta = \widetilde{f}_\Delta}$$

$\widetilde{S}$: Sym. pos. def.

The constraint minimization

Find $u_\Delta \in \widetilde{W}$ s.t.

$$\left.\begin{aligned} J(u_\Delta) = \tfrac{1}{2} \langle \widetilde{S} u_\Delta, u_\Delta \rangle - \langle \widetilde{f}_\Delta, u_\Delta \rangle &\to \min \\ B_\Delta u_\Delta &= 0 \end{aligned}\right\}$$

Lagrange multiplier $\lambda \in U = \mathrm{range}(B_\Delta)$.
to enforce continuity.

$$\boxed{\begin{aligned} \widetilde{S} u_\Delta + B_\Delta^T \lambda &= \widetilde{f}_\Delta \\ B_\Delta u_\Delta &= 0 \end{aligned}}$$

Now $u_\Delta = \widetilde{S}^{-1}(\widetilde{f} - B_\Delta^T \lambda)$. Hence

$$\boxed{F \lambda = d}$$

Dual system

where $F = B_\Delta \widetilde{S}^{-1} B_\Delta^T$ and $d = B_\Delta \widetilde{S}^{-1} f_\Delta$

Once λ is found
$\downarrow$
u_Δ
$\downarrow$
u_I and u_Π
are found.

Preconditioner for FETI-DP

$D_\Delta^{(i)}$ contains $\delta_i^\dagger$ (weighted counting function)

$$M^{-1} = \sum_i D_\Delta^{(i)} B_\Delta^{(i)} S_\Delta^{(i)} B_\Delta^{(i)T} D_\Delta^{(i)T}$$

restriction of $S^{(i)}$ to the dual dofs of Ω_i

Preconditioned FETI-DP

$$M^{-1} F \lambda = M^{-1} d$$

$$\boxed{\kappa\left(M^{-1}F\right) \leq c\left(1 + \log \frac{H}{h}\right)^2}$$

Implementation

- $S_\Delta^{(i)}$ can be found from $S^{(i)}$ by deleting rows and cols of Π dofs.

- Applying $S^{(i)}$ is done in the standard way (Dirichlet prob)

- $\tilde{S}^{-1} w_\Delta$ can be found from Δ component of:

$$\tilde{A}^{-1} \begin{pmatrix} 0 \\ 0 \\ w_\Delta \end{pmatrix}$$

(Neumann prob)

$\ominus$ In order to save the cost of factoring $\tilde{A}$, it is more convenient to order the unknowns as (I, Δ, Π).

Eliminate I and Δ first.

$$\hat{S}_\Pi = A_{\Pi\Pi} - \begin{pmatrix} A_{I\Pi}^T & A_{\Delta\Pi}^T \end{pmatrix} \begin{pmatrix} A_{II} & A_{I\Delta} \\ A_{I\Delta}^T & A_{\Delta\Delta} \end{pmatrix}^{-1} \begin{pmatrix} A_{I\Pi} \\ A_{\Delta\Pi} \end{pmatrix}$$

which corresponds to a $\boxed{\text{Coarse problem}}$.

One-level FETI vs FETI-DP

- FETI-DP does not require Null spaces of local Neumann
- Enforcing primal constraints
  - make local-problems non-singular
  - provides a coarse global problem

- FETI-DP can start from an arbitrary λ_0
- one-level FETI needs $G\lambda_0 = e$