Heidelberg Institute for Theoretical Studies | HITS | Universität Heidelberg – Zukunft. Seit 1386. | IWR Interdisciplinary Center for Scientific Computing | KIT Karlsruhe Institute of Technology | DFG

Optimal Economic Power Flow

Philipp Gerstner[1,2], Vincent Heuveline[1,2], Michael Schick[1]

Goal

Optimize operation state of a given electrical power grid

Minimize fuel costs of conventional power plants while maintaining network stability

$\Rightarrow$

Optimization problem

$$\min_X \sum_t \sum_i f_i(X_i^t)$$
$$g(X^t, V^t) \leq 0$$
$$g(X^t, X^{t+1}) \leq 0$$
$$AC(X^t, V^t) = 0 \quad \forall t$$

X_i^t: injected power at node i, time t
V_i^t: voltage at node i, time t

$\Leftarrow$

Physical model of network flows

$$AC(X^t, V^t)_i = X_i^t - D_i^t - P_i(V^t)$$
$$P_i(V^t) = \sum_{k \in adj(i)} p_{ki}(V_k^t, V_i^t)$$

D_i^t : power demand at node i, time t
p_{ki}: power flow from node i to k

Solve large-scale nonconvex nonlinear optimization problem with an Interior Point Method (IPM)
Current work: Development of specialized linear solver adopted to the underlying physical model

The main computational effort lies in the solution of the arising linear system
Matrix dimension is proportional to number of nodes in the power grid and number of time steps.
Typical matrix size for German transmission grid with 3 day time horizon: ~1.500.000

Characterization of arising linear systems

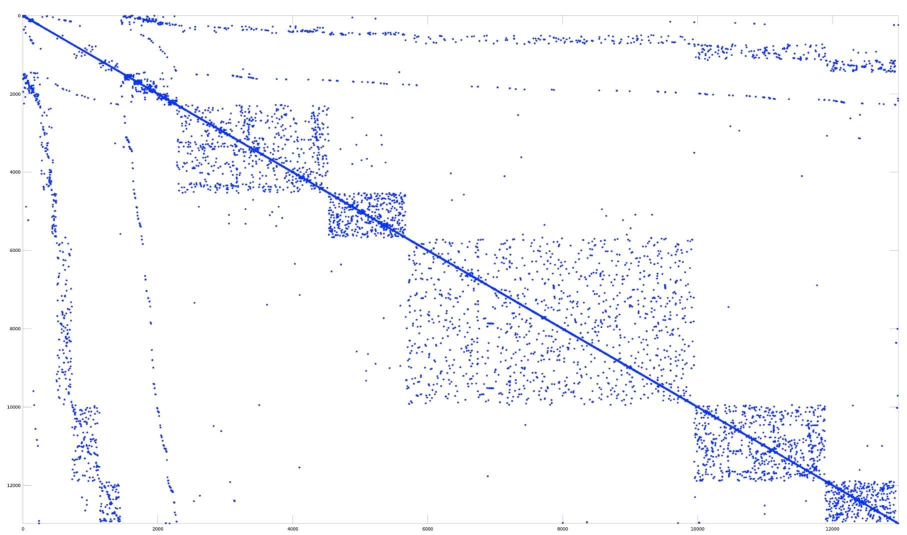
- Symmetric and indefinite
- Increasing condition number as IPM evolves
- Sparsity structure corresponds to topography of underlying power grid

Physical grid

~ 1000 nodes
~ 200 power plants
~ 2500 lines

Locality of variables

Group variables corresponding to same physical node and permute Newton matrix.

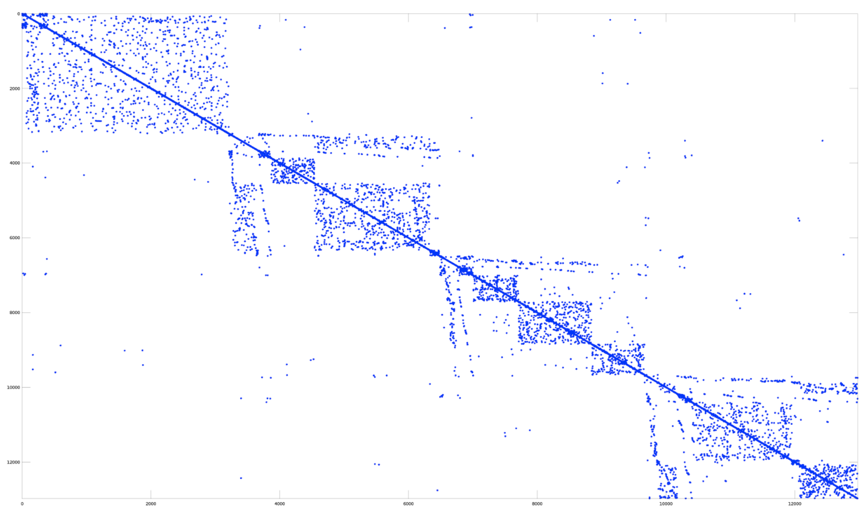


Observation

Newton matrix has same sparsity structure as adjacency matrix of physical grid

Partitioned network graph

Determine q subdomains of similar size and minimal number of cutting edges.



Overlapping Schwarz methods

Restriction operator for subdomain i

$$R_i : \mathbb{R}^N \to \mathbb{R}^{n_i}$$

Solution operator for subdomain i

$$B_i = R_i^T (R_i A R_i^T)^{-1} R_i$$

Additive Schwarz

$$x_+ = x + \sum_i B_i(b - Ax)$$

Multiplicative Schwarz

$$x_i = x_{i-1} + B_i(b - Ax_{i-1})$$
$$x_* = x_q$$

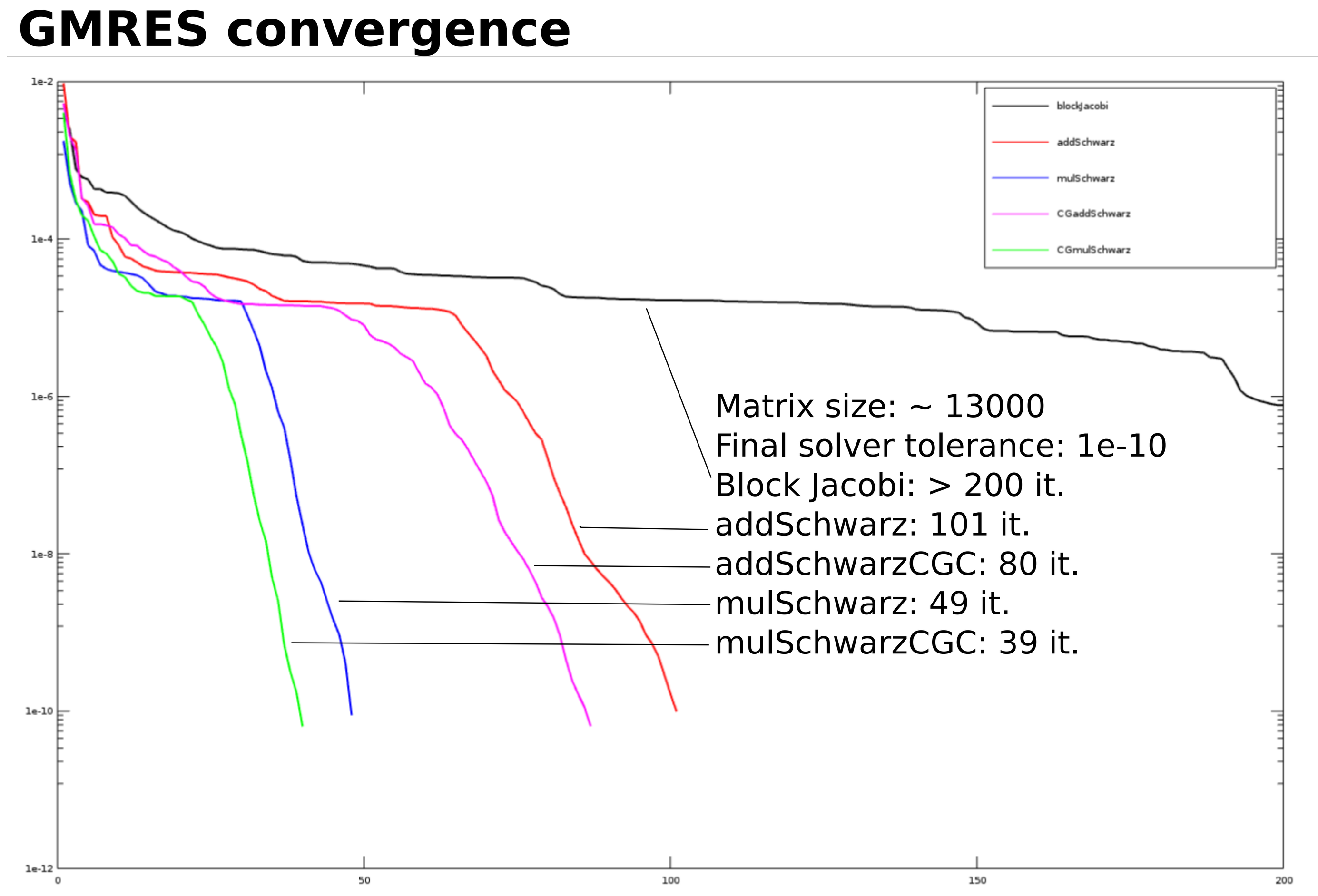
Additional coarse grid correction

$$B_c = R_c^T (R_c A R_c^T)^{-1} R_c$$
$$x_{\cdot,c} = x_\cdot + B_c(b - Ax_\cdot)$$

Preconditioner

$$P : x \mapsto x_{\cdot,c}$$

GMRES convergence



Conclusions

- Linear systems arising in power grid optimization are well suited for domain decomposition methods
- Schwarz preconditioners significantly improve GMRES convergence
- Stable convergence rate, despite increasing condition number
- Allows parallelization on distributed memory systems

Future work: Take into account uncertainties in input data
Leads to 2-stage stochastic program

Uncertain demand: $D_i^t \to D_i^t(\omega)$

Uncertain renewable energy: $g(X^t, V^t) \to g(X^t, V^t, \omega)$

$\Rightarrow$

$$\min_{X^1} \sum_i f_i(X_i^1) + \mathbb{E}_\omega \left[\sum_{t \geq 2} \sum_i f_i(X_i^t(X^1, w)) \right]$$
$$g(X^1, V^1) \leq 0$$
$$AC(X^1, V^1) = 0$$

with $X^t(X^1, \omega), t \geq 2$ solving

$$\min_{X^t, t \geq 2} \sum_{t \geq 2} \sum_i f_i(X_i^t)$$
$$g(X^t, V^t, \omega) \leq 0$$
$$g(X^t, X^{t-1}) \leq 0$$
$$AC(X^t, V^t, \omega) = 0 \quad \forall t \geq 2$$

1 Heidelberg Institute for Theoretical Studies, Germany
2 Interdisciplinary Center for Scientific Computing, Heidelberg University, Germany
3 Karlsruhe Institute of Technology, Germany