Uncertainty Quantfication for Medical Engineering

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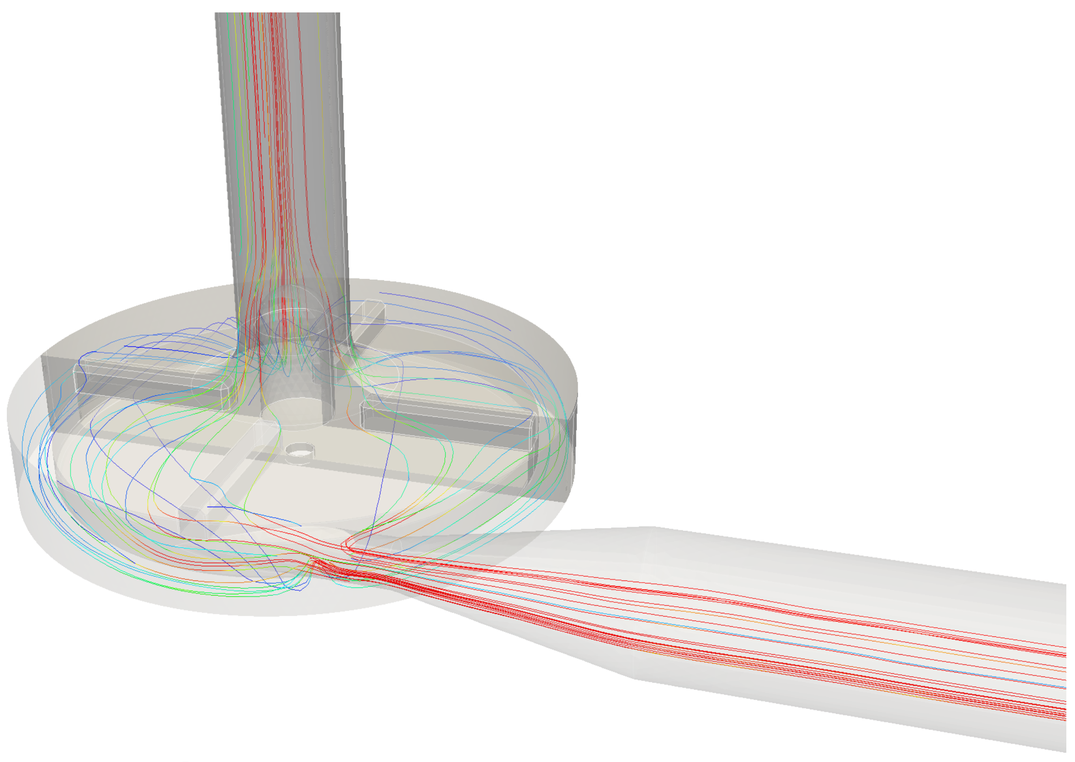
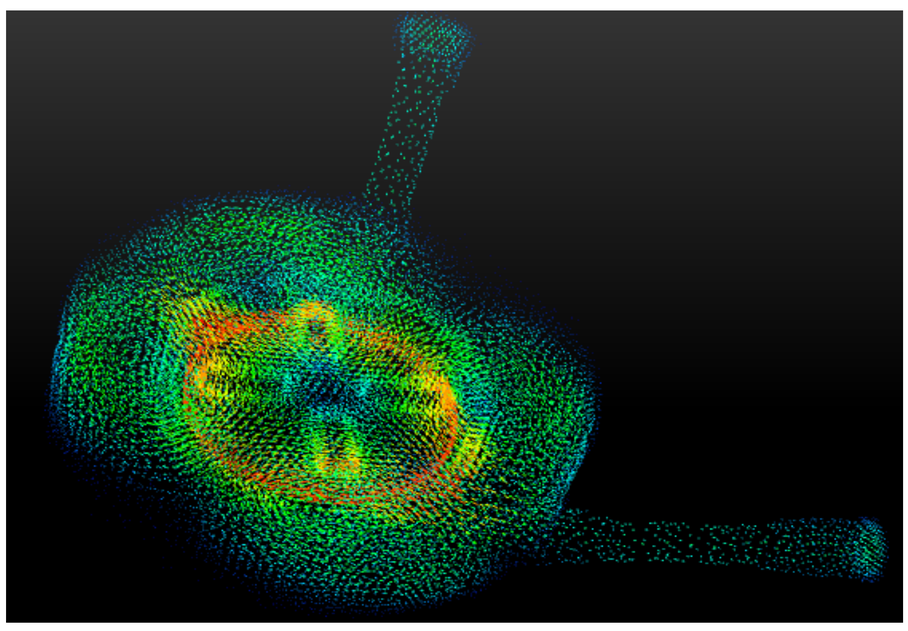
Uncertainty Quantification (UQ) FEM blood pump simulation by using Galerkin projection method (intrusive)

Navier-Stokes equation with uncertain parameters

$$
\begin{aligned}
\rho u(x,\xi)\cdot\nabla u(x,\xi) &= -\nabla P(x,\xi)+\mu\nabla^2 u(x,\xi) && x\in D_{fix}\\
\nabla\cdot u(x,\xi) &= 0 && x\in D_{fix}\\
\rho u_r(x,\xi)\cdot\nabla u_r(x,\xi)+ & && \\
\rho(2\omega\wedge u_r(x,\xi)+\omega\wedge\omega\wedge r) &= -\nabla P(x,\xi)+\mu\nabla^2 u_r(x,\xi) && x\in D_{rot}\\
\nabla\cdot u_r(x,\xi) &= 0 && x\in D_{rot}\\
u(x,\xi) &= g(x,\xi) && x\in\Gamma_{in}\\
u(x,\xi) &= 0 && x\in\Gamma_{out}\\
u(x,\xi) &= f(x,\xi) && x\in\Gamma_{rot}\\
(\mathbb{I}P+\frac{\mu}{\rho}u(x,\xi))\cdot n &= 0 && x\in\Gamma_{rest}
\end{aligned}
$$





numerical result with Multiple Reference Frame(MRF) mehtod

Furture works

- High Performance Computing (HPC)
- Rotating effects need to be coupled for instationary state
- Sliding meshes model
- Improvement of stochastic solvers
- Scability studies
- Analysing with experimental results

Quantifying medical data by using Collocation sampling method (no-intrusive)

Clinical cooperation workflow

- Collection patient imaging data (daily)
- Selecting anatomical landmarks
- Deformation modeling
  * landmarks-wise analysis
  * combined analysis
- Uncertainty modeling and simulation
  * head posture variation
  * neck posture variation

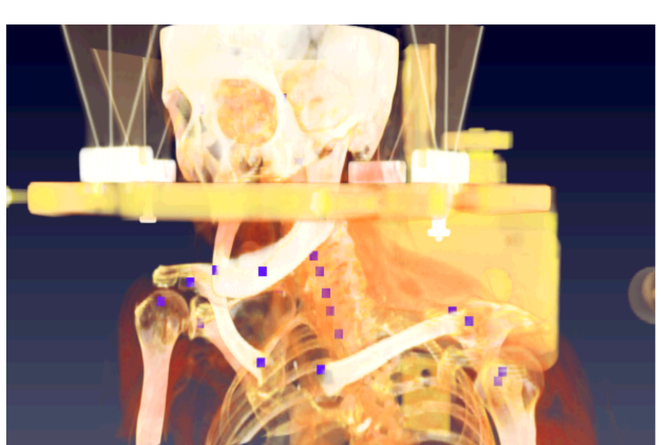
Tumor metastasis data studies

- kV - control images from CT-on-rails
- 19 head neck patient data
- 20-30 fraction images
- 24 anatomical landmarks (shoulders, skull, backbones ...)
- Manually propagated to all planning CTs

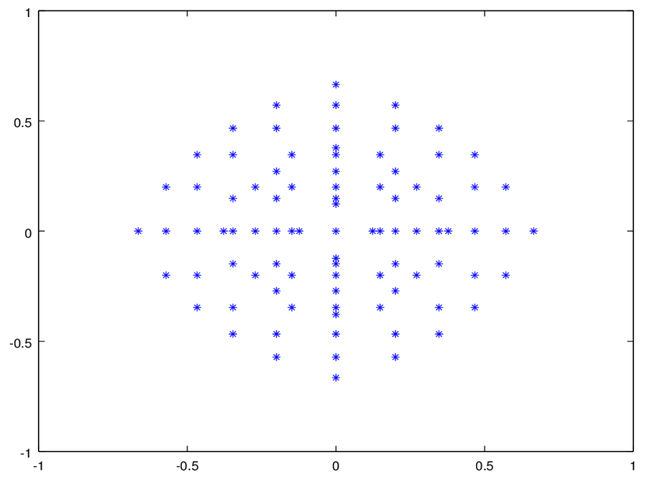
Collocation method

- Sample random inputs according to probablity distribution
- Perform a sequence of independent simulations
- Analyzing simulation results regard to the quantity of interest
- Different polynomials correspond to different probability behaviors

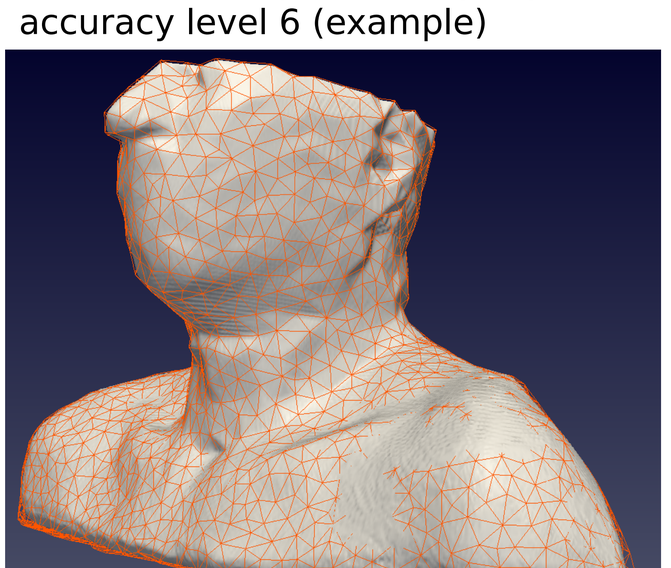
$$< Q >= \int Q(\xi)d\xi = \sum_{i=1}^{N} Q(\xi^i)w_i$$



Volume rendering, single patient, multiple fraction CT images with landmarks



2 dimensions sparse sampling with accuracy level 6 (example)



preliminary result: solid color : Monte-Carlo method. gridlines : collocation method

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