Enhancing 4D phase-contrast MRI in an aortic phantom considering numerical simulation

Jonas Kratzke [1], Vincent Heuveline [1]
1 Engineering Mathematics and Computing Lab (EMCL), Interdisciplinary Center for Scientific Computing (IWR), Heidelberg University, Germany

Motivation: Risk assessment of cardiovascular disease

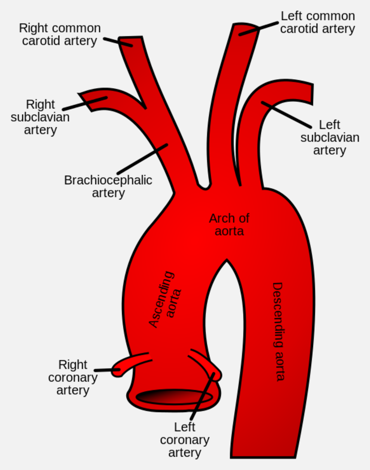


Figure 1: Aortic arch anatomy [en.wikipedia.org/wiki/Aortic_arch]

Anatomical aspects of aortic disease have been thoroughly investigated in the last decades by means of CT, MRI and ultrasound. To date, morphologic variations can be determined individually and with high sensitivity. In general clinical routine, aortic anatomy and pathology is represented and surveyed statically. There is high potential for techniques, that quantify dynamic morphology and physiology of the aorta during full cardiac cycle.
With respect to the coherences between bio-mechanical behavior and aortic disease various open tasks exist such as a more extensive acquisition of risk factors for atherosclerosis, aneurysm formation or aortic dissection. Within the scope of increasing understanding of vascular pathology, development of functional imaging of the aorta becomes more and more important.

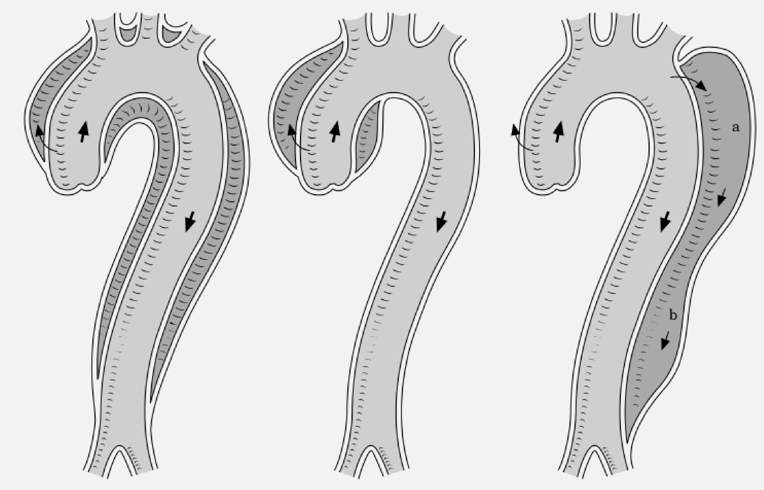


Figure 2: Aortic dissection DeBakey types I, II and III [Erbel2001]

4D phase-contrast MRI measurement of aortic phantoms

Figure 3: Aortic phantoms, Department of Cardiac Surgery, University Hospital Heidelberg.

Yet, modern medical imaging techniques help to understand the underlying morphological and physiological dy- namics and to improve medical diagnosis. Among them, phase-contrast MRI represents a non-invasive technique to measure time-resolved velocity fields of cardiovascular blood flow.
Comprehensive 4D PC-MRI studies can be realized with aortic phantoms enabling investigations in a controlled environment. Entirely made of non-metallic components, blood-like fluids can be flown through aortic phantoms and the time-resolved velocity field can be measured by PC-MRI technology.

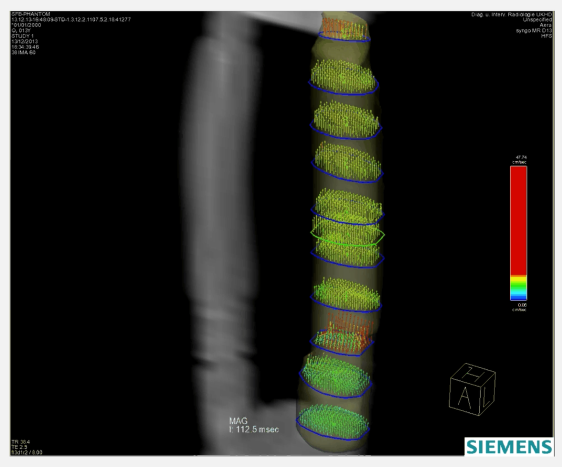


Figure 4: 4D PC-MRI measurement of a prototypic aortic phantom.

Mathematical model and calibration

In this work, we propose a mathematical model of an aortic silicon phantom. As the elasticity of the silicon phantom wall plays a significant role and is reflected in the Windkessel effect in the case of the aortic bow, we model the wall as elastic structure.
The physical dynamics for fluid flow and elastic deformation can be modeled by means of partial-differential equations derived by basic laws of continuum mechanics, namely, the conservation of mass and momentum:

$$\frac{d}{dt}\rho = 0, \quad \frac{d}{dt}\rho v - \nabla \cdot \sigma = \rho f.$$

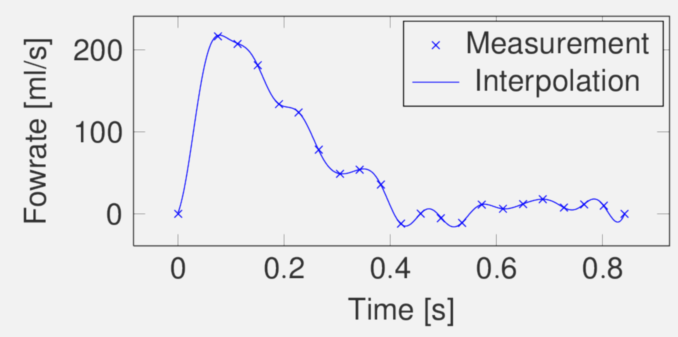
In the case of the aortic phantom, a suitable constitutive law for the fluid stress-strain-rate relation $\sigma_f(v)$ is given by the incompressible Newtonian fluid model, leading to the incompressible Navier-Stokes equations. The structure stress-strain relation $\sigma_s(F)$ can be stated by means of the St. Venant Kirchhoff material model.

Together with according boundary conditions we get the following fluid-structure interaction initial boundary value problem:

$$\begin{aligned}
\partial_t v + (v \cdot \nabla) v - \nu \Delta v + \frac{1}{\rho}\nabla p &= f, && \text{in } \Omega_f \times I,\\
\nabla \cdot v &= 0, && \text{in } \Omega_f \times I,\\
\partial_t^2 u - \frac{1}{\rho}\nabla \cdot (F\lambda_1(\mathrm{tr}(E)I + 2\lambda_2 E)) &= f, && \text{in } \Omega_s \times I,\\
v_f &= v_s, && \text{on } \Gamma_i \times I,\\
\sigma_f \cdot n_f + \sigma_s \cdot n_s &= 0, && \text{on } \Gamma_i \times I,\\
u_s = g_s, \quad v_f &= g_f, && \text{on } \Gamma_D,\\
(\nu \nabla v - pI) \cdot n &= 0, && \text{on } \Gamma_{out},\\
u(x,0) = u_0(x), \quad v(x,0) &= v_0(x), && \text{in } \Omega.
\end{aligned}$$

Calibration

- Structure parameters from silicon tensile test
- MRI measured flowrates for boundary conditions

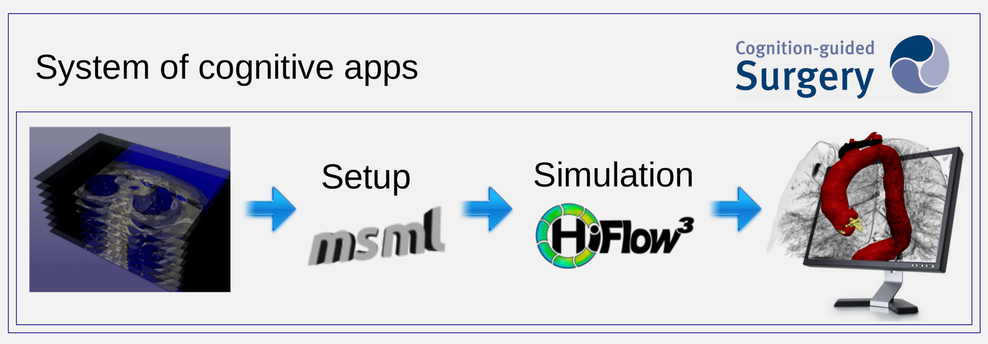


- RC-type outflow boundary condition

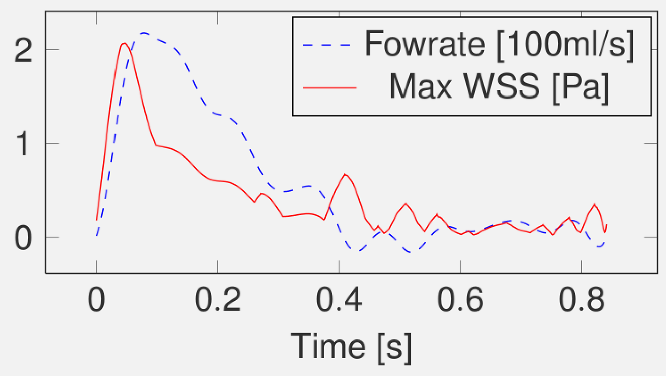
$$P_{in} - P_{out} + RC\frac{d}{dt}P_{in} = RQ.$$

Numerical simulation, framework embedding and evaluation

- Monolithic ALE solver
- P2/P2/P1 and/or Q2/QP2/Q1 finite elements
- θ-step time discretization



- Stress and Wall shear stress visualization
- Flow characteristics calculation
- Common MRI and simulation post-processing analysis



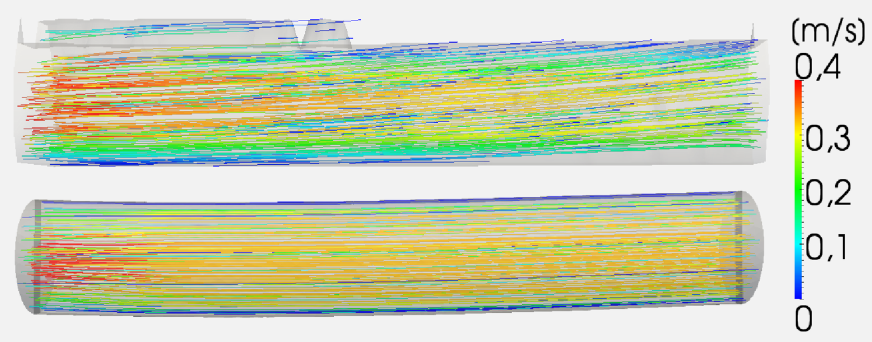


Figure 5: 4D PC-MRI and CFD simulation, prototypic aortic phantom.

Outlook

Jonas Kratzke, Nicolai Schoch, Christian Weis, Matthias Müller-Eschner, Stefanie Speidel Mina Farag, Carsten Beller, Vincent Heuveline, *Enhancing 4D PC-MRI in an aortic phantom considering numerical simulations* , SPIE medical imaging (2015), accepted.

- Validation with common MRI and simulation post-processing analysis
- Constitutive law for arterial wall
- Uncertainty Quantification [Schick2014] for
  - MRI-callibrated in- and outflow boundary conditions
  - Fluid viscosity and structure parameters

References

[1] Erbel, R., et al., *Diagnosis and management of aortic dissection Task Force on Aortic Dissection*, European Society of Cardiology. European Heart Journal, 22(18), 1642-1681 (2001).

[2] Formaggia, L., Quarteroni, A. M., and Veneziani, A., *Cardiovascular mathematics.* Milan: Springer (2009).

[3] Grinberg, L. and Karniadakis, G. E., *Outflow boundary conditions for arterial networks with multiple outlets*, Annals of biomedical engineering 36(9), 1496âĂŞ1514 (2008).

[4] Markl, M., et al., *4D flow MRI*, Journal of Magnetic Resonance Imaging 36(5), 1015âĂŞ1036, (2012).

[5] Schick, M., Heuveline, V. and Le MaÃőtre, O. P., *A Newton-Galerkin method for fluid flow exhibiting uncertain periodic dynamics*, SIAM/ASA Journal on Uncertainty Quantification (2)1, pp. 153-173 (2014).