Integration Requirements for Cross-Disciplinary Uncertainty Quantification turn Workflows into a Big-loop and Big-data Exercise?

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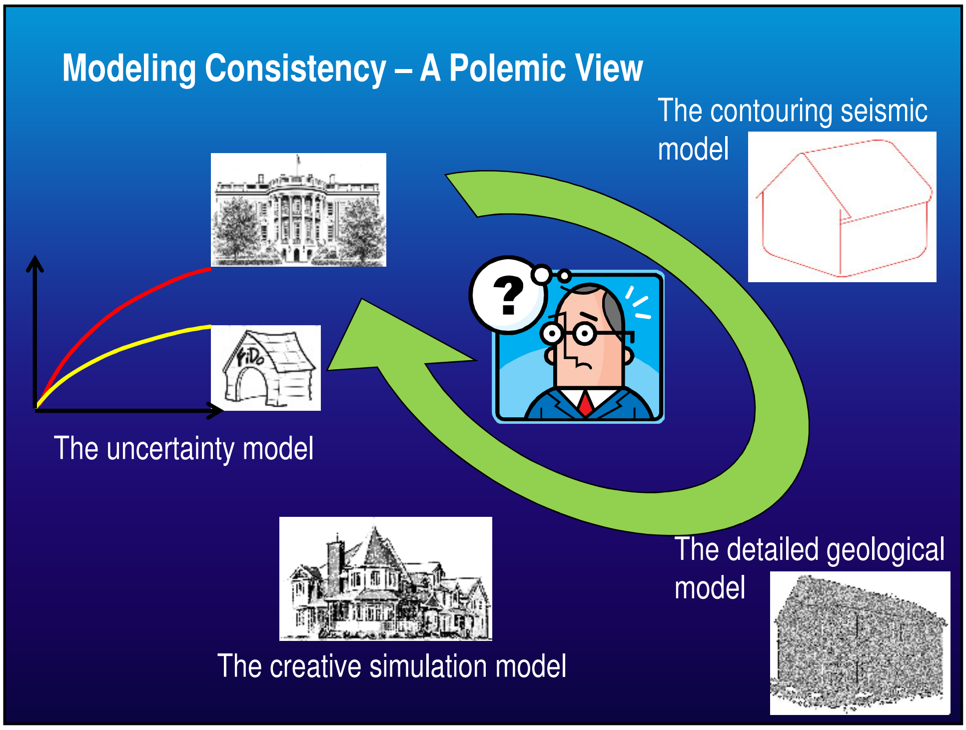
Abstract

- Reservoir model validation and uncertainty quantification workflows have significantly developed over recent years. Different optimization approaches were introduced and requirements for consistent uncertainty quantification workflows changed. Most of all integration requirements across multiple domains (big-loop) increase the complexity of workflow designs and amount of data (big-data) processed in the course of workflow execution. This triggers new requirements for the choice of optimization and uncertainty quantification methods in order to add value to decision processes in reservoir management. In this session we will discuss an overview on existing methods and perspectives for new methodologies based on parameter screening, proxy-based as well as analytical sensitivity approaches.

2



Modeling Consistency Challenges

What have we learnt from history matching?

How do we incorporate results into the detailed model?

How do we ensure next modeling pass is better?

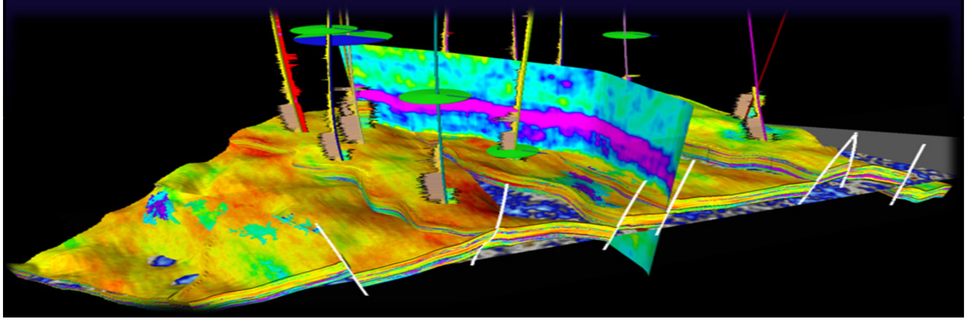
What are the pros and cons of different history matching approaches on behalf of providing information or models to 'close-the-loop'?

What should be best practice based on current technology?

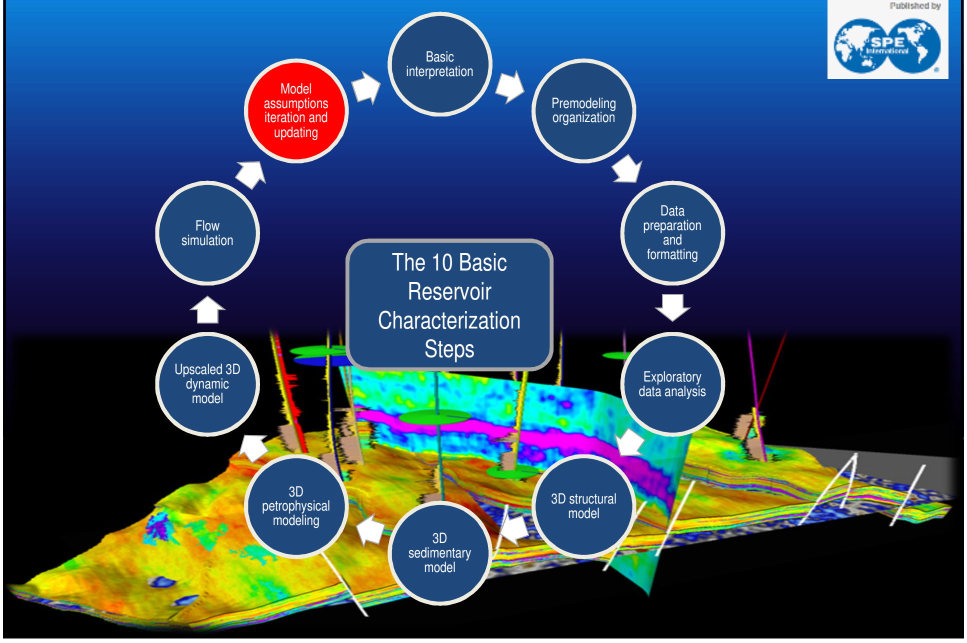
What is best practice in 5 years' time?

Shared Earth Model

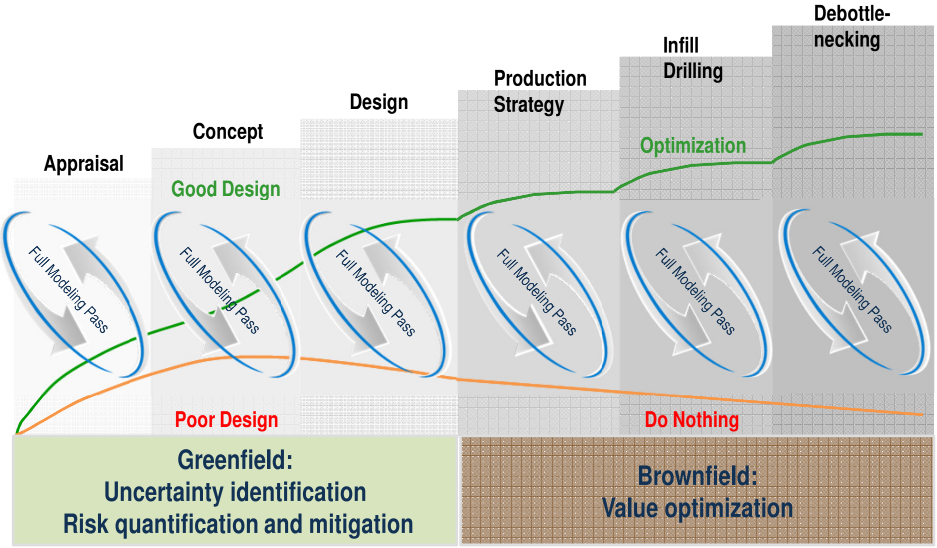
- The shared earth model is important in four ways:
  - It is a central part of the reservoir-characterization team's work
  - It ensures cross-disciplinary data consistency
  - It allows each discipline to measure how its own interpretation fits with other specialty models
  - It leads to a more-consistent global model



Shared Earth Model



Many Revisions of the Shared Earth Model



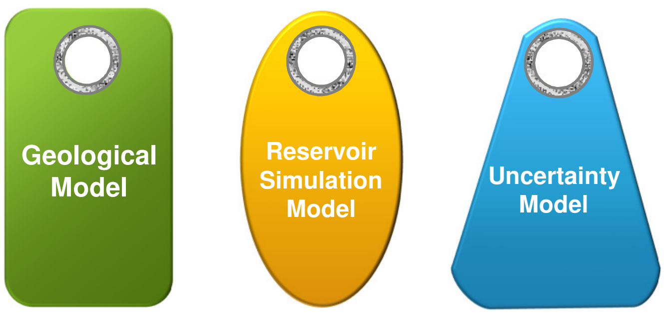
7

Challenge – Cross-disciplinary Modeling Consistency

Resolve the common challenge of having

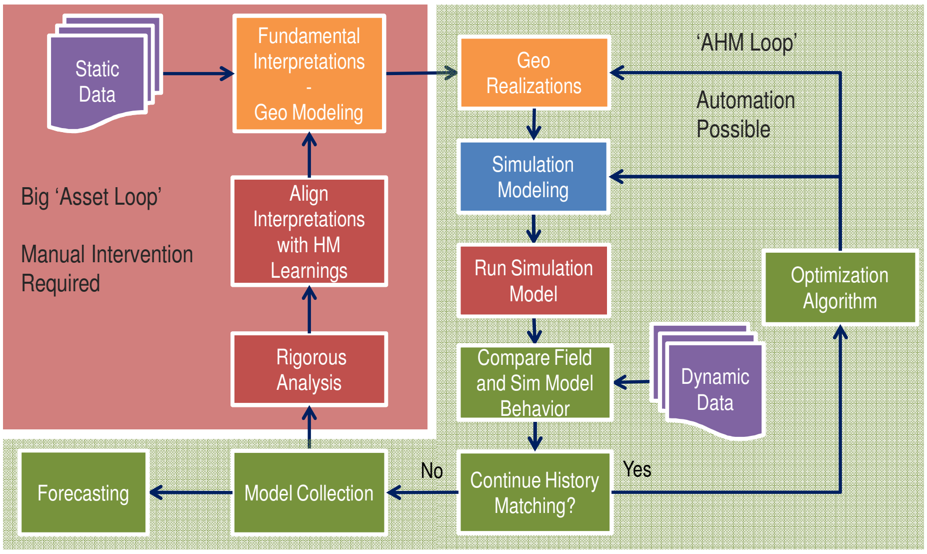
- geo models,
- reservoir simulations models and
- uncertainty estimates (p10, p50 and p90s etc)

out of sync after history matching projects

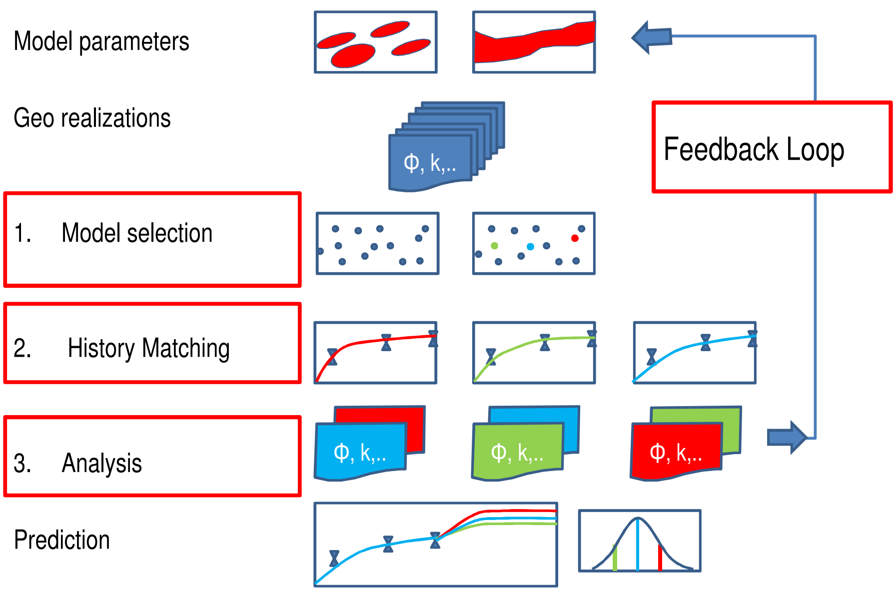


8

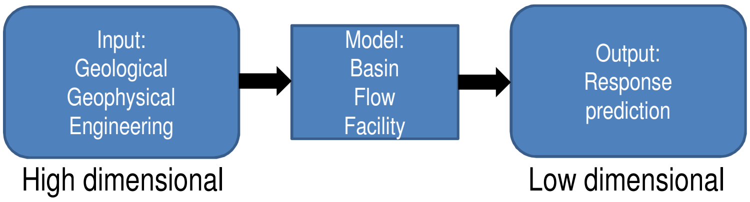
Workflow Framing



Workflow Design



Model Selection – Dimensional Reduction

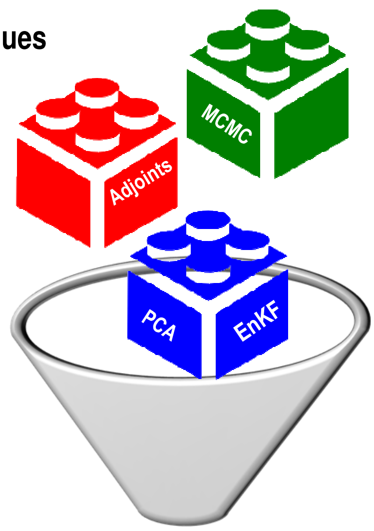


- Screening of geological models based on, e.g.,
  - Volumetric parameters
  - Connectivity
  - Field production rates, etc.
- Apply projection and clustering techniques for selecting set of distinct models.
- Combine existing and newly developed software components in an integrated workflow.

History Matching – Optimization Techniques

A variety of history matching optimization approaches with a particular requirements and target areas are available.
The future focus should be on the potential for successfully aligning the geo and reservoir simulation models

Criteria, e.g.:
- Reliability
- Universality
- Efficiency
- Automation
- Data storage requirements
- Feedback algorithms



History matching supporting modelling consistency

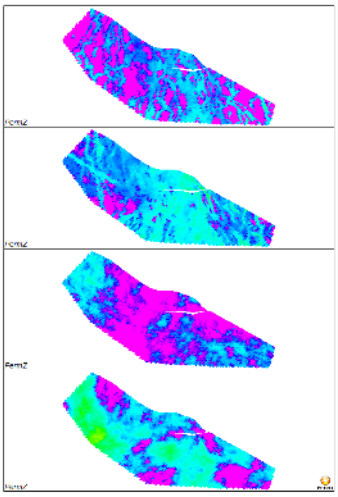
Principal Component Analysis

Optimization Approach

Ref: SDR
Geologically constrained history matching with PCA
Michael D. Prange, Thomas P. Dombrowsky and William J. Bailey
first break volume 30, November 2012

Principal Component Analysis

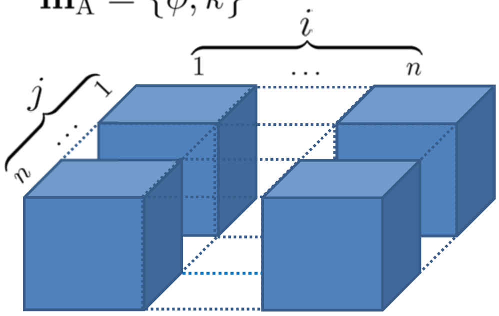
- (Linear) Principal Component Analysis (PCA) is often mentioned and discussed in connection with:
  - Multi-Dimensional Scaling (MDS)
  - Proper Orthogonal Decomposition (POD)
  - Karhunen–Loève Transform (K-LT)
- The main scope is to find key features in a number of models, i.e., components which can be included in an optimization approach, e.g., history matching.

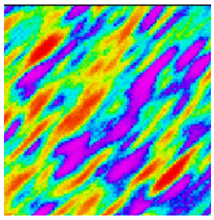


Source: SDR

Example: PCA Dimension Reduction

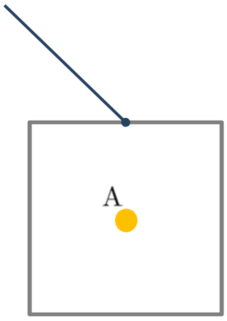
$$\mathbf{m}_{\mathrm{A}} = \{\phi, k\}$$





Think of this plot as representing the whole of $\mathbf{m}_{\mathrm{A}}$. We can only show two co-ordinates of 1 point.

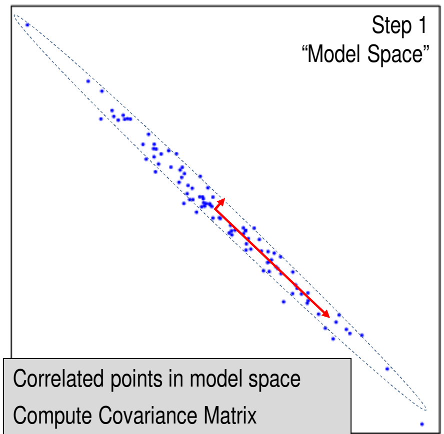
$$\mathbf{m}_{\mathrm{A}} = \begin{Bmatrix} \phi_{[1,1]} \\ \vdots \\ \phi_{[n,n]} \\ k_{[1,1]} \\ \vdots \\ k_{[n,n]} \end{Bmatrix}$$



This is a cartoon representing just one point – model ‘A’ – in a high-dimensional model space.

Source: SDR

How: Model Space → Feature Space



Correlated points in model space

Compute Covariance Matrix

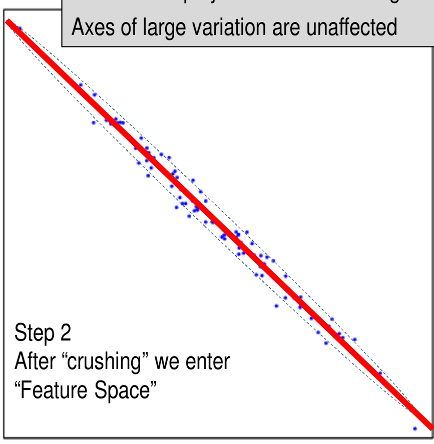
- Eigenvectors are axes of ellipse
- Eigenvalues are axis lengths

These axes define a new (optimal) rotated coordinate system

Axes of small variation are **crushed** to zero

- Dimension reduction
- Points are projected onto remaining axes

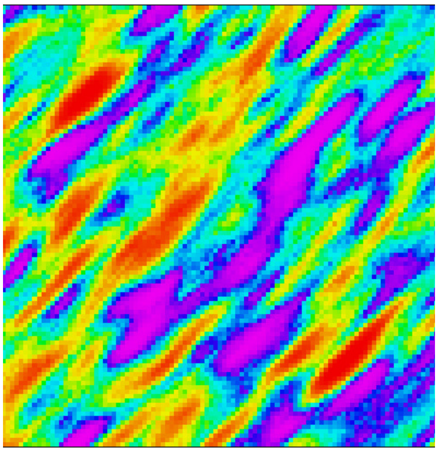
Axes of large variation are unaffected



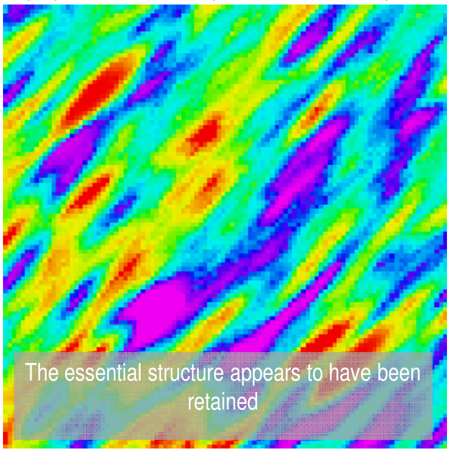
Source: SDR

How: PCA Retains Structure

Model Space



A projection of Model Space into Feature Space



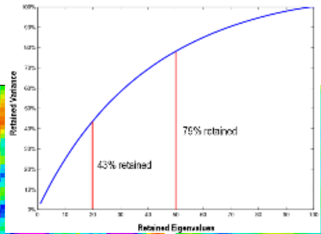
Example: $d = 10000$ (cells)
$n = 100$ (prior models)

$k = 50$ (principal components)

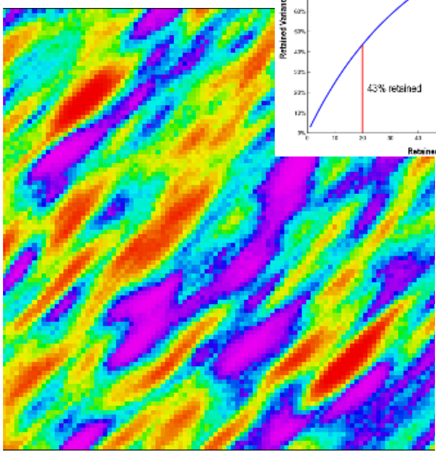
Source: SDR

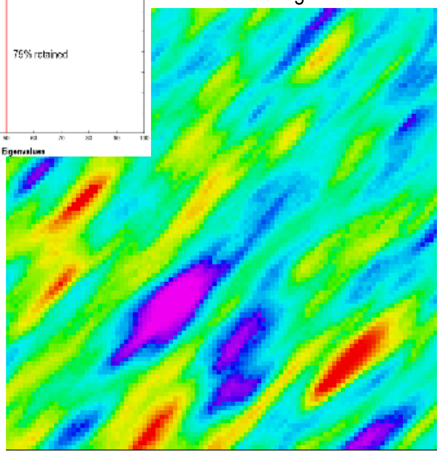
17

How: Some Structure Still Visible



Pre-compute "information loss" and evaluate degree to reduction





Example: $d = 10000$ (cells)
$n = 100$ (prior models)

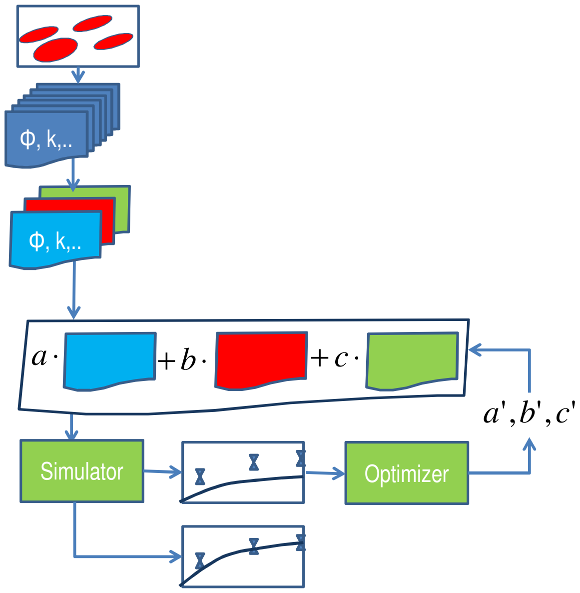
$k = 20$ (principal components)

Source: SDR

18

History Matching Loop using PCA

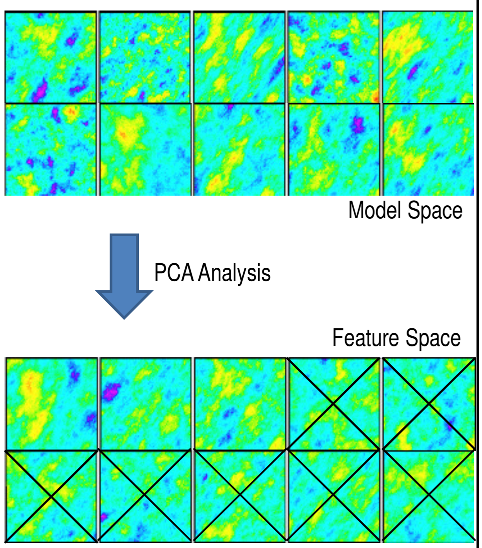
- Geological Concept
- Realization
- Dimensional Reduction
- History Matching
  - Parameterization
  - Small-scale Optimizer



19

Example: History Matching – Principal Component Analysis

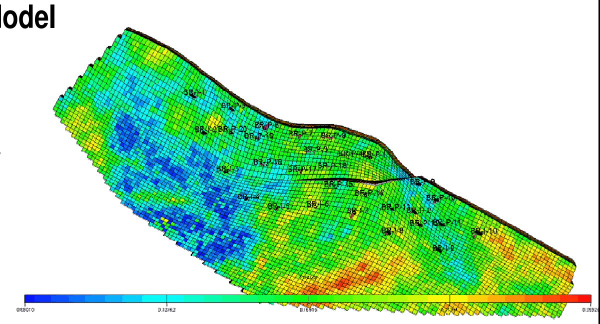
- Generate a number of realizations
- Find a projection from model to feature space which allows to decompose features
- Keep relevant features and reduce number of components
  → "dimension reduction"
- Use principal components in a history matching workflow
- Note:
  Key geological features are preserved; consistency is kept

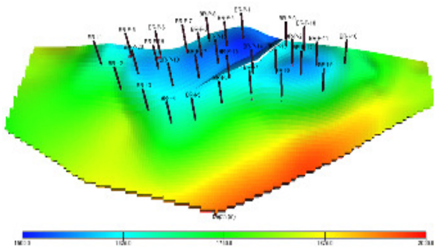


20

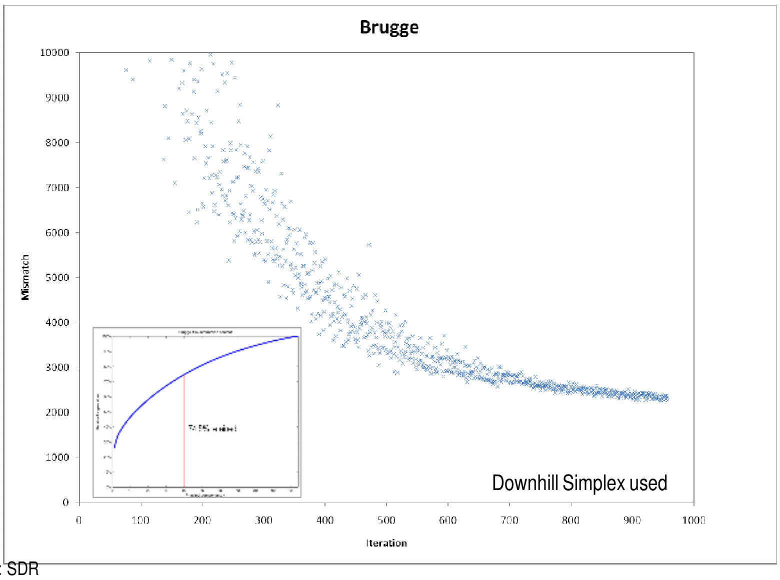
Application to Brugge Model

- Benchmark case prepared by TNO cf. Peters et al. SPE119094
- Scope
  - Closed loop production optimization workflow
- Delivery
  - 100+ realizations
  - 10 injectors, 20 producers
  - Production data for 10 years
  - Well constraints for future production scenarios





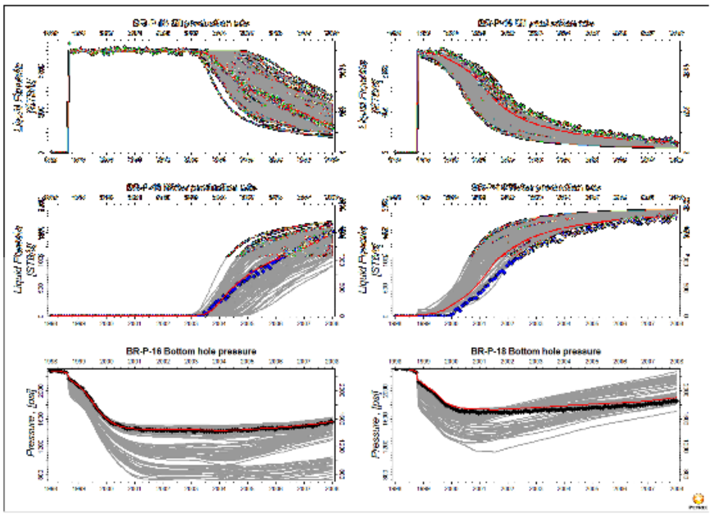
Optimization To Reduce Misfit



Source: SDR

Validation

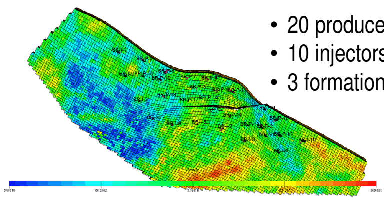
- PCA approach was capable to find a model which reproduces well production data.
- All implemented model changes conform with the geological information provided



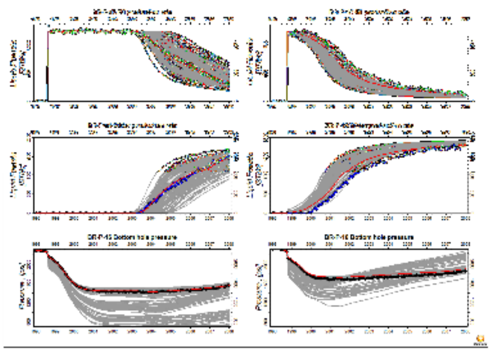
Source: SDR

23

Summary: PCA Application

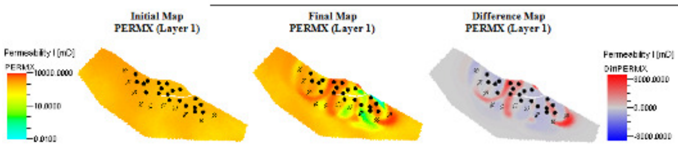


- 20 producers
- 10 injectors
- 3 formations



- Well-by-well match
- All conform with the geological information provided

Analysis



- Investigate modifications, differences, trends across multiple models.

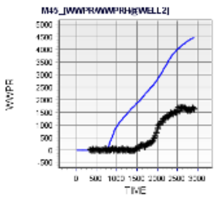
24

Adjoints

Deterministic Approach

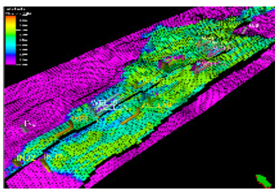
Deterministic Solution – e.g., Adjoint Approach

- The adjoint system is solved with the sole aim of finding a deterministic solutions to the problem
- The mismatch is quantified by an objective function, Q



$$Q(m) = \sum_i \frac{\left(d_i^{sim}(m) - d_i^{obs}\right)^2}{\sigma_i^2}$$

- Minimize Q; calculate $\frac{\partial Q(m)}{\partial d}$ on a cell-by-cell basis.



- Regression step

$$m^{l+1} = m^l + \alpha_l \delta m^{l+1}$$

- Optimization, e.g., Levenberg-Marquardt

$$\delta m^{l+1} = \frac{m^l - m_{prior}}{1 + \lambda_l} + K\left[\frac{G_l(m^l - m_{prior})}{1 + \lambda_l} - \left(g(m^l) - d_{obs}\right)\right]$$

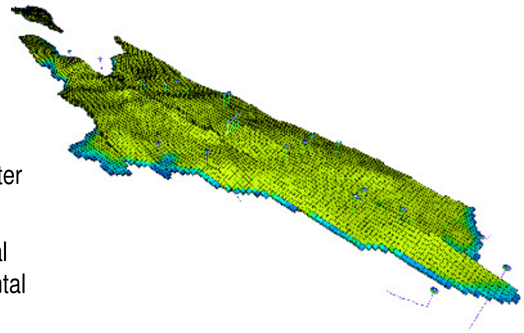
SPE 164337: Schulze-Riegert et al.,2013

26

Application/Results – History Matching

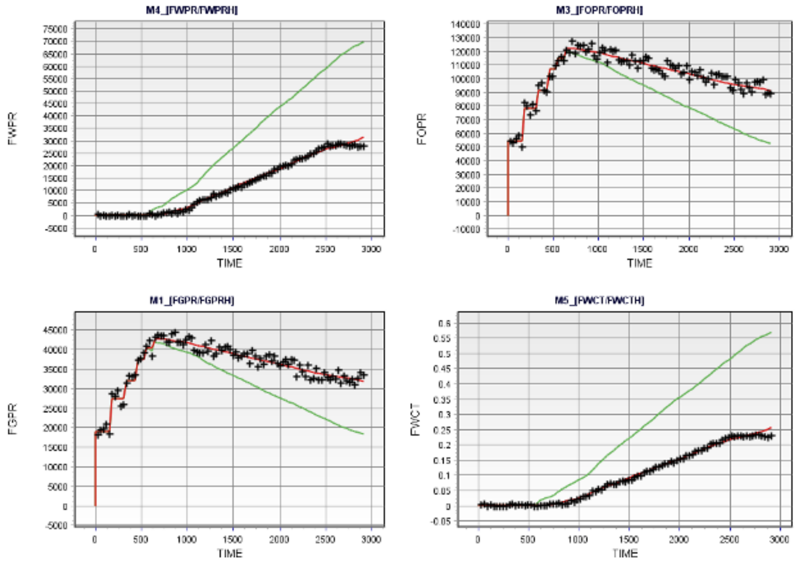
Watt Field Description

- A semi-synthetic reservoir model
- Base case model chosen from 81 realisations provided.
- Fluid phases present = Oil and Water
- Model grid size = 226 x 59 x 21
- 7 injectors consisting of 5 horizontal and 2 vertical wells and 16 horizontal producers
- Reservoir drive mechanism = Solution gas (depletion) drive
- Approximately 8 years of history



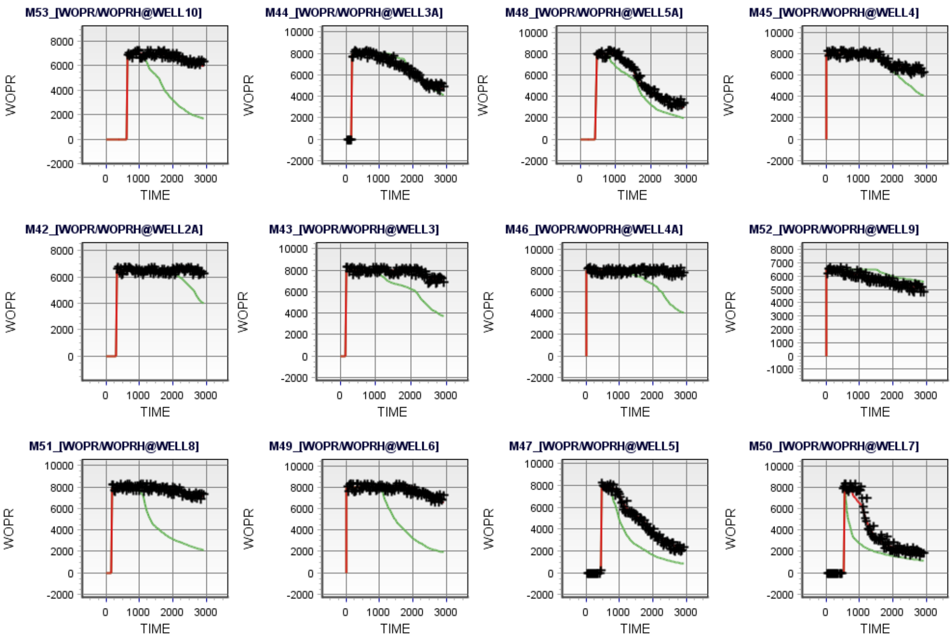
27

History Matching Result – Field Level

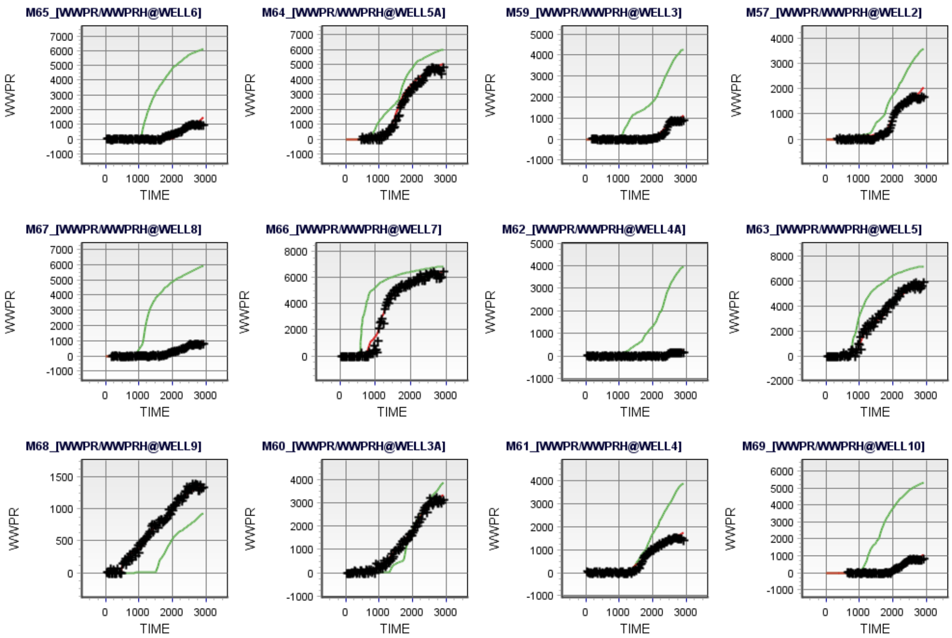


28

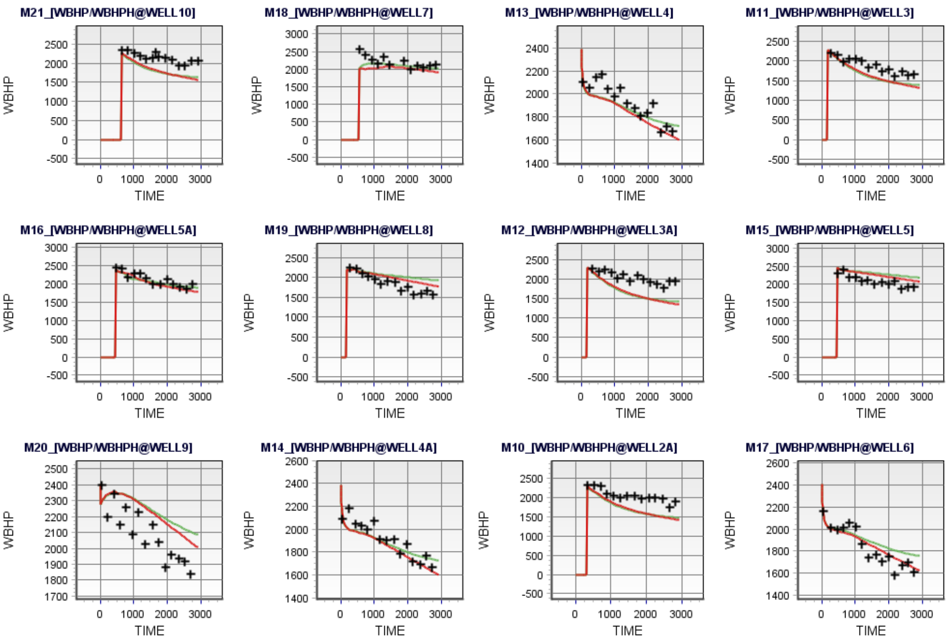
Results Well Oil Production Rate – Selected Profiles



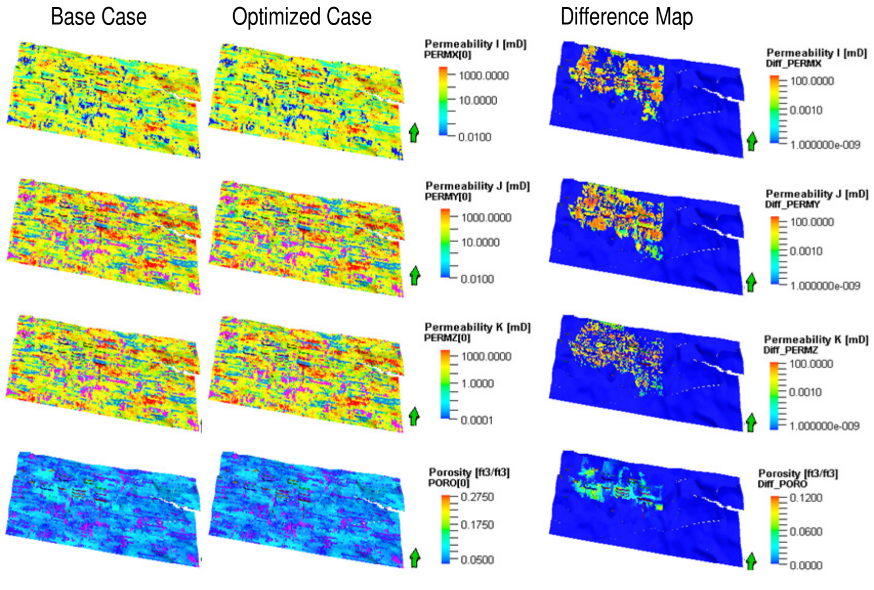
Results Well Water Production Rate – Selected Profiles



Results Well Bottom Hole Pressure – Selected Profiles

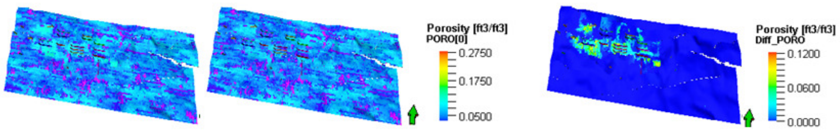


Results – Impact on Rock Property Distribution



Results – Impact on Volumetric Parameters

Base Case Optimized Case Difference Map

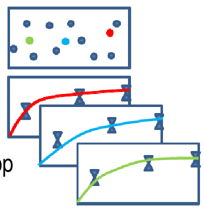


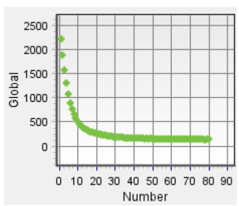
	Base Case	Optimized Case	Relative diff. [%]
PORV	**21.4**60.906.026	**21.5**00.848.891	0.2
OOIP [STB]	**3.84**1.092.109	**3.88**6.724.103	1
OWIP [STB]	**16.3**95.959.920	**16.3**79.474.077	0.1
OGIP [MSCF]	**1.34**4.382.238	**1.36**0.353.436	1

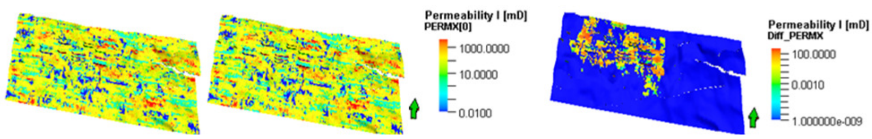
History Matching using Analytical Sensitivities

Application project

- Analytical sensitivity calculation (Adjoints) on grid cell level $\frac{\partial Q(m)}{\partial m}$
- Workflow – Extension
  - Ranking
  - History Matching of multiple models
  - Analysis and modeling consistency – feedback loop
- Visualization and Analysis





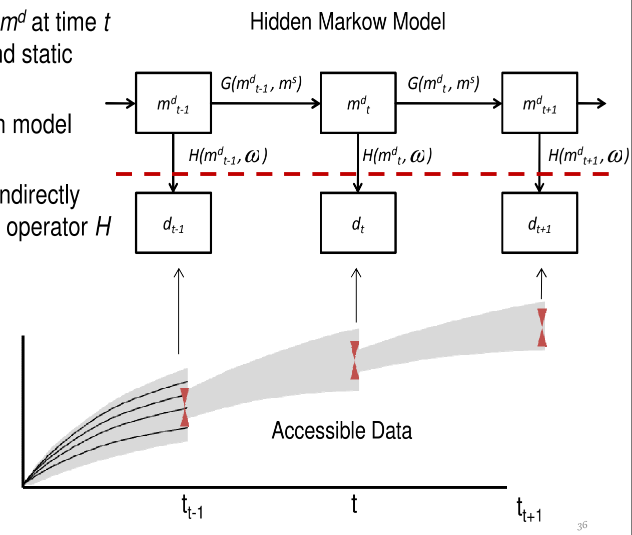


Ensemble Kalman Filter (EnKF)

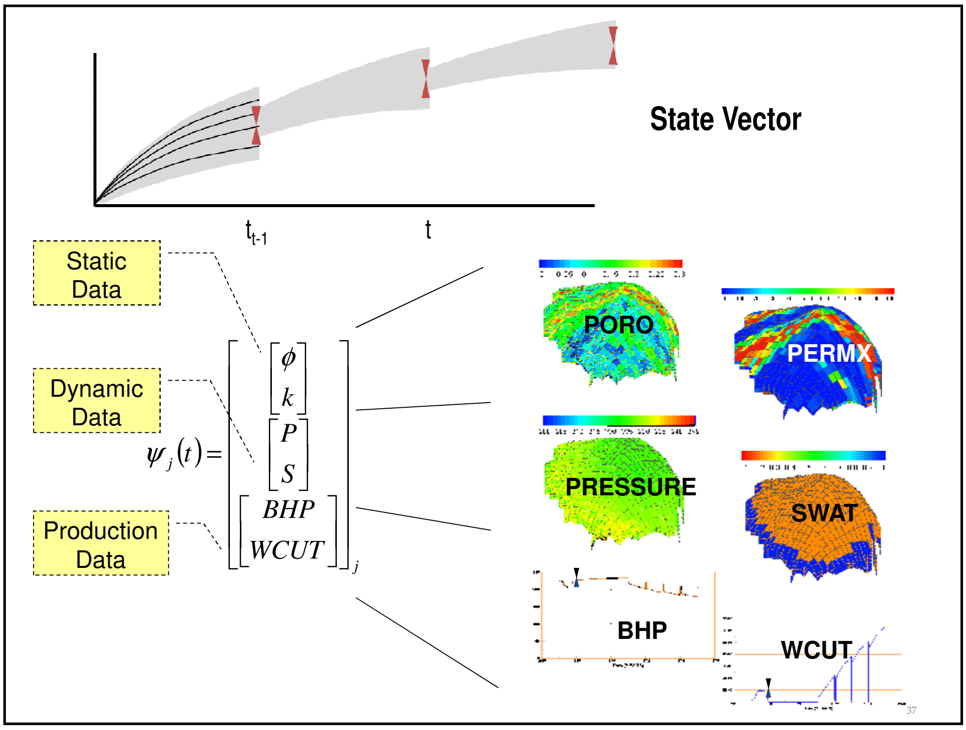
Bayesian Formulation

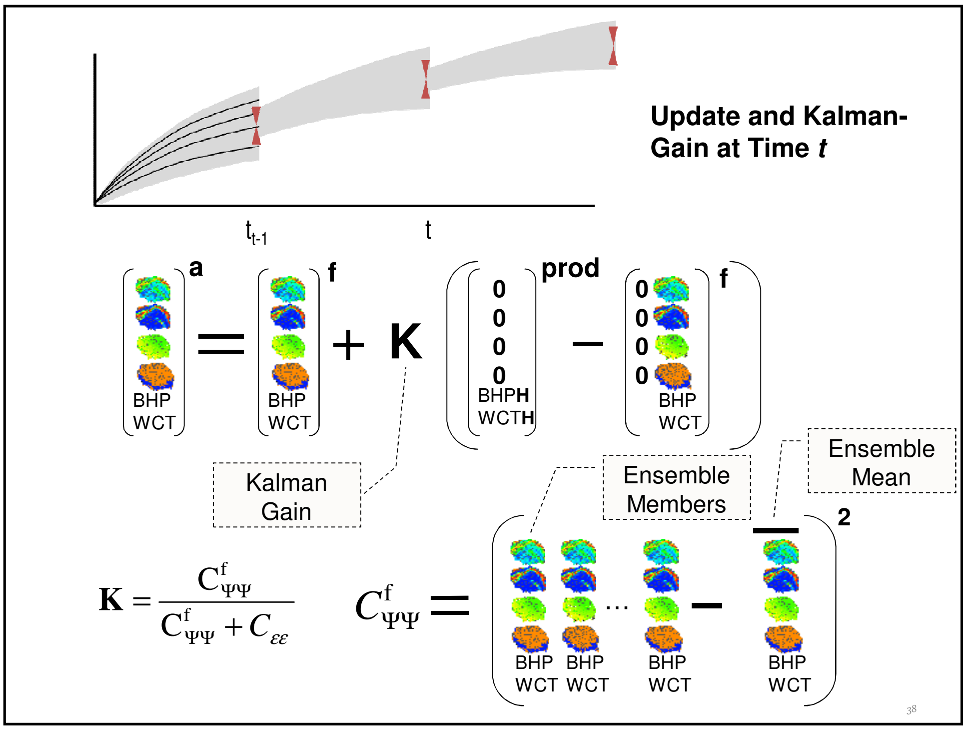
Ensemble Kalman Filter (EnKF): Central Assumptions

- Dynamic state variables m^d at time *t* depend on state at *t-1* and static model parameters m^s
- State evolves with known model operator *G*
- State is only accessible indirectly via (noisy) measurement operator *H*



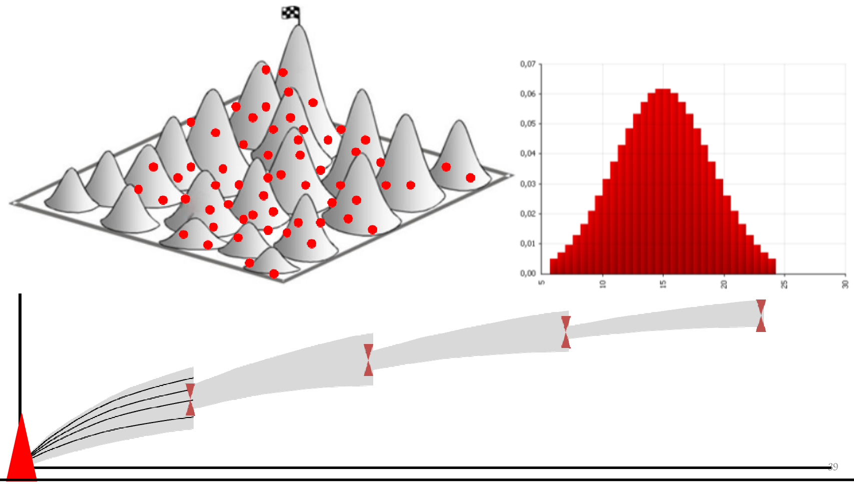
36


State Vector
t_{t-1}
t
Static Data
Dynamic Data
Production Data
$\psi_j(t) = \begin{bmatrix} \begin{bmatrix} \phi \\ k \end{bmatrix} \\ \begin{bmatrix} P \\ S \end{bmatrix} \\ \begin{bmatrix} BHP \\ WCUT \end{bmatrix} \end{bmatrix}_j$
PORO
PERMX
PRESSURE
SWAT
BHP
WCUT


Update and Kalman-Gain at Time *t*
t_{t-1}
t
a
f
prod
f
BHP
WCT
+ K
0
0
0
0
BHPH
WCTH
Kalman Gain
Ensemble Members
Ensemble Mean
2
$\mathbf{K} = \frac{C^f_{\Psi\Psi}}{C^f_{\Psi\Psi} + C_{\varepsilon\varepsilon}}$
$C^f_{\Psi\Psi} =$
BHP WCT
…

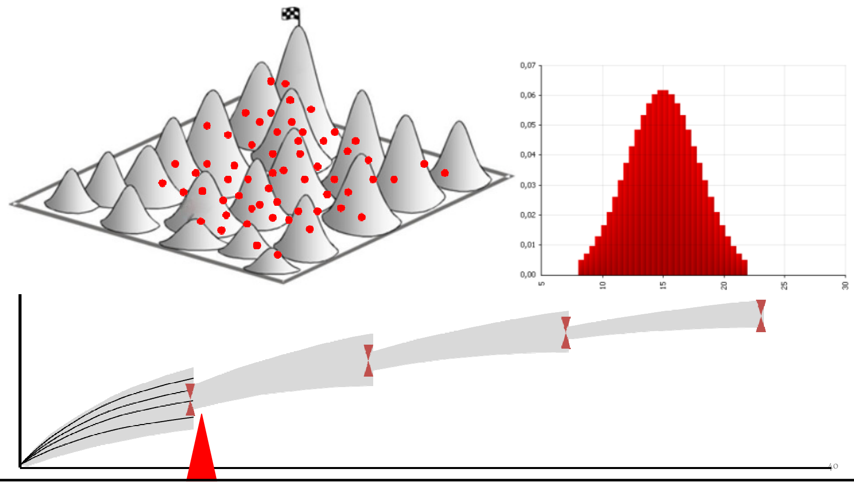
EnKF: Progression of static state

- State Initialization

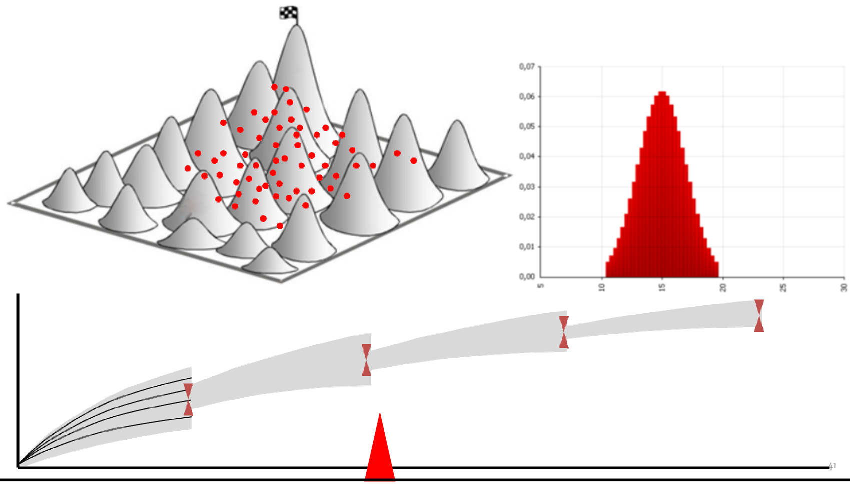


EnKF: Progression of static state

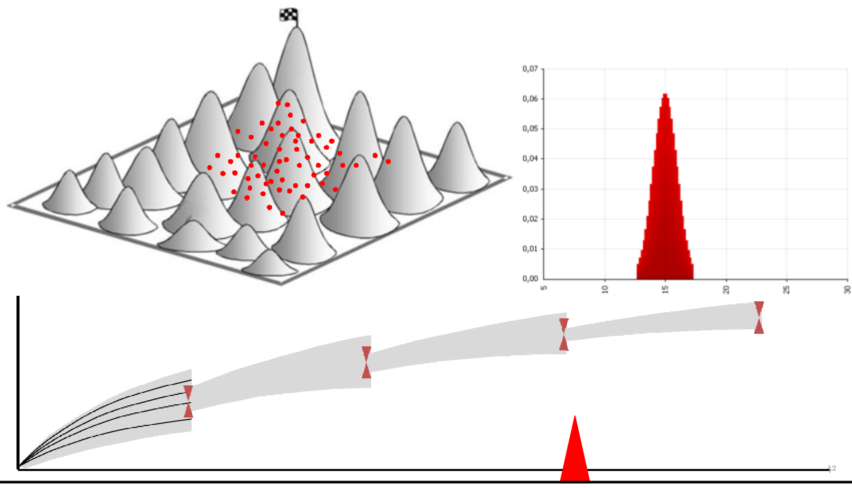
- Update and uncertainty reduction
  in uncertainty and response parameter space

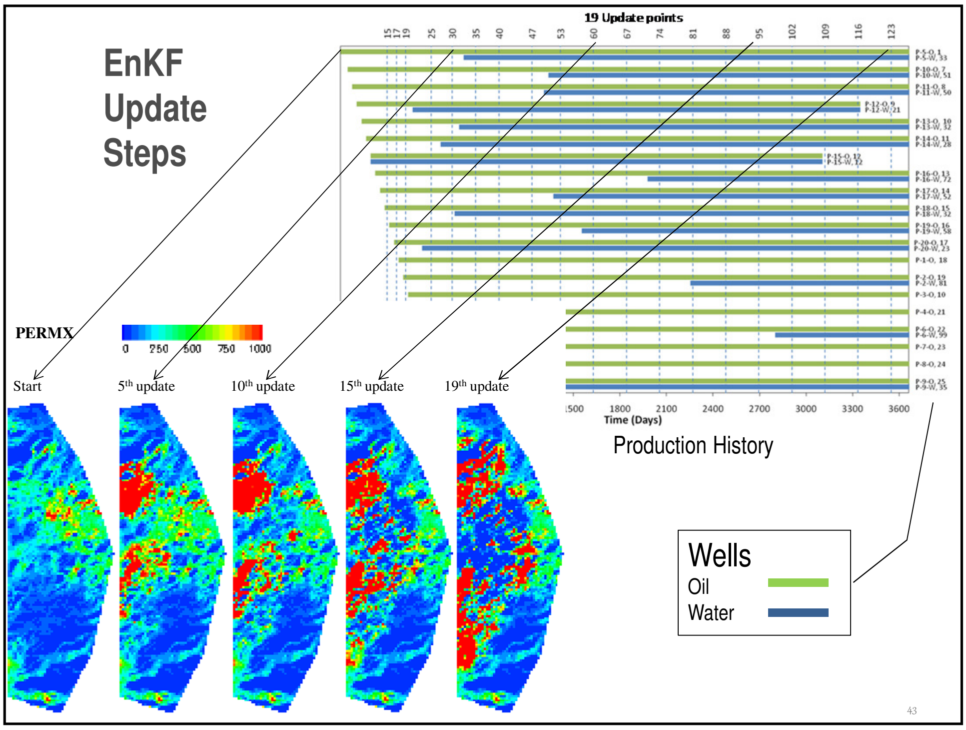


EnKF: Progression of static state



EnKF: Progression of static state





Summary EnKF

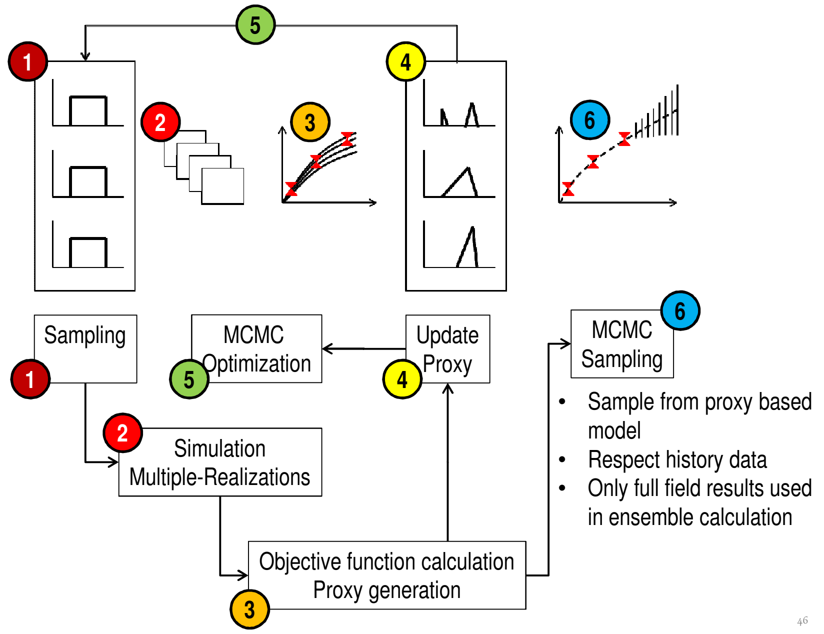
- Well suited for updating entire property fields
- Consistent uncertainty handling for establishing a posterior probability distribution
- Also used for updating alternative parameters
  (WOC, relative permability, fault transmissiblities)
  cf. A. Seidler 2009, Y. Chen 2009.
- Sequential update scheme supports closed-loop approaches.
- Changing dynamical parameters remains conceptually difficult.

44

MCMC + Proxy

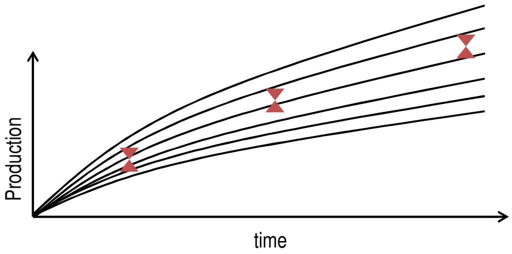
Bayesian Approach

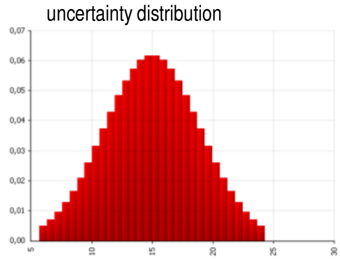
Bayesian Update using MCMC



46

Background: Bayesian Update using MCMC





- Mismatch

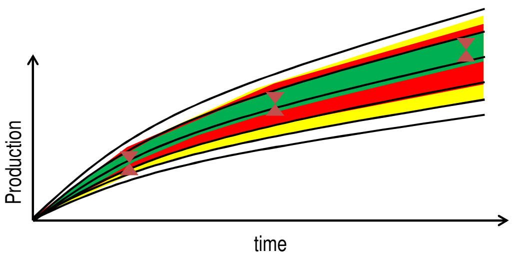
$$Q^L(d,m) = \sum_t \left(d_t^{\mathrm{m}} - d_t^{\mathrm{c}}(m)\right)^2$$

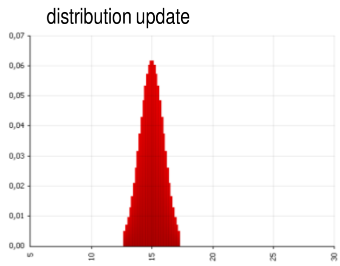
- Posterior weight

$$\mathrm{p}(m \mid d) \propto \exp\left(-Q^L(d,m)\right)$$

47

Background: Bayesian Update using MCMC





- Markow Chain, i.e., update depends on previous state only

$$m^{i+1} = m^i + \delta$$

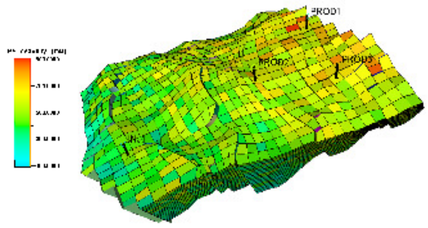
- Sampling new candidate

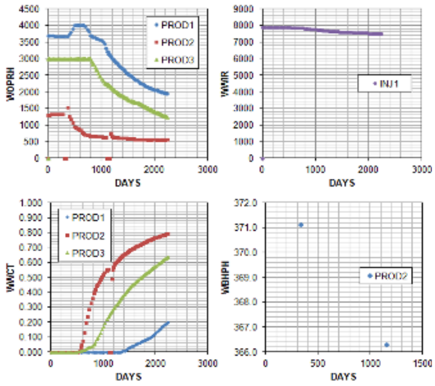
$$\alpha = \frac{\mathrm{p}(m^{i+1} \mid d)}{\mathrm{p}(m^i \mid d)} \quad \begin{cases} \alpha \geq 1 & \text{accept} \\ \alpha < 1 & \begin{cases} \text{accept if } \alpha > \mathrm{rnd}(0,1) \\ \text{reject} \end{cases} \end{cases}$$

48

Application Case

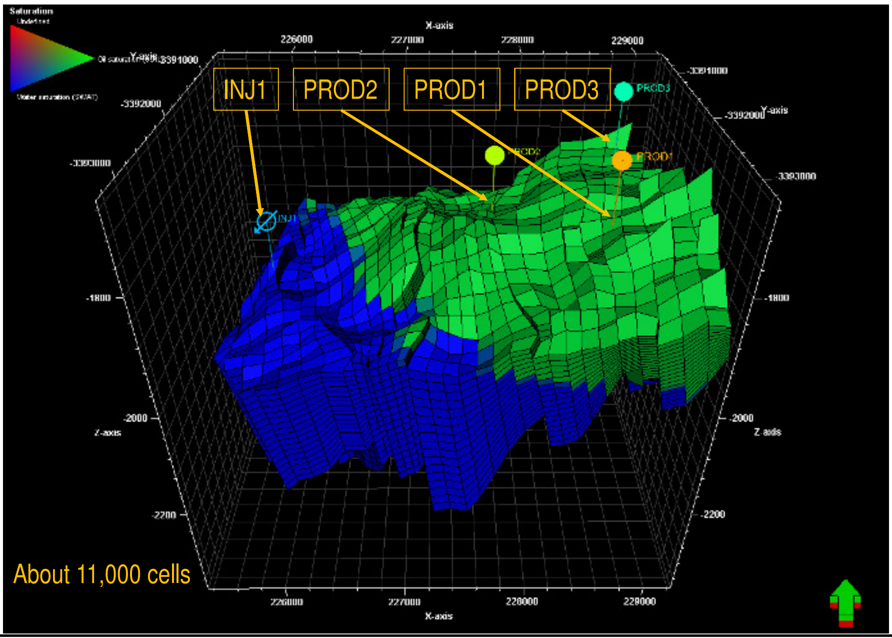
- Under-saturated oil reservoir
- 3 main sand layers and 4 main faults
- Porosity varies between 0.027 to 0.280
- Permeability varies between 90 to 1000 [mD].
- Aquifer acts at the North-western edge flank of the reservoir
- 3 production wells, 1 injector
- 7-year production history .





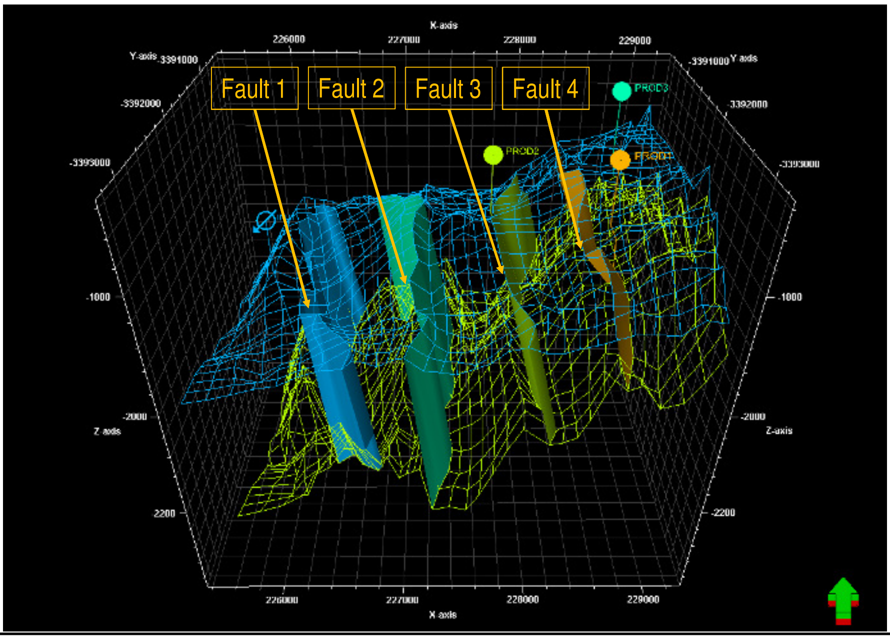
49

Wells

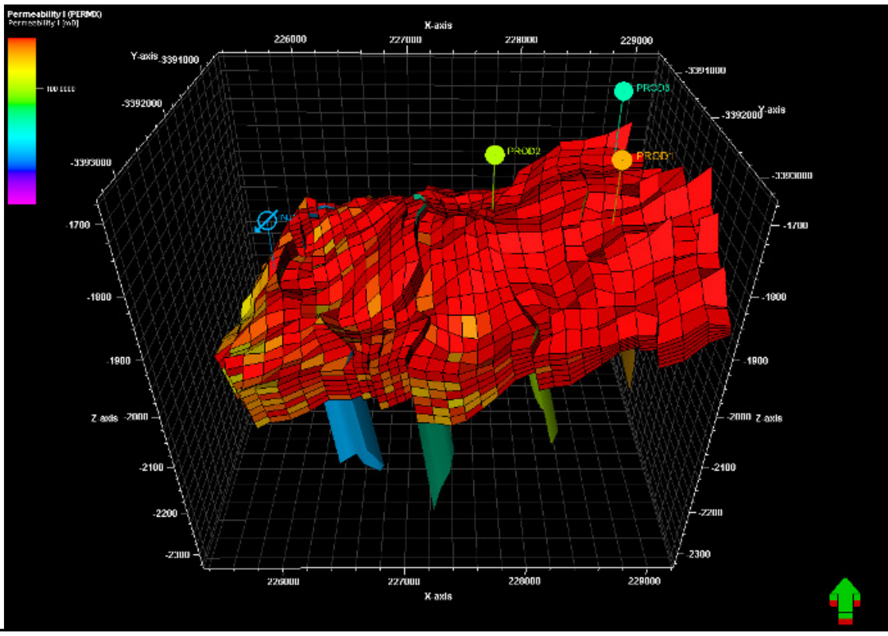


50

Faults



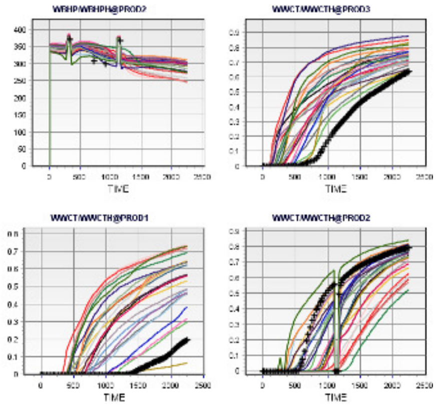
Layer A

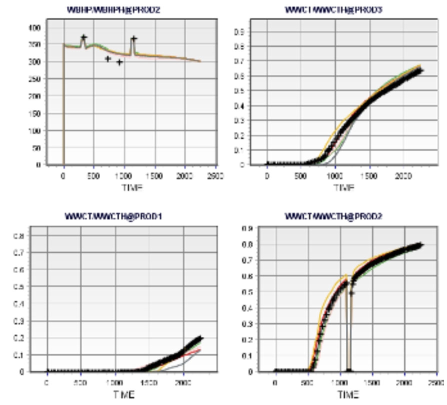


History Matching using MCMC Optimization

Screening

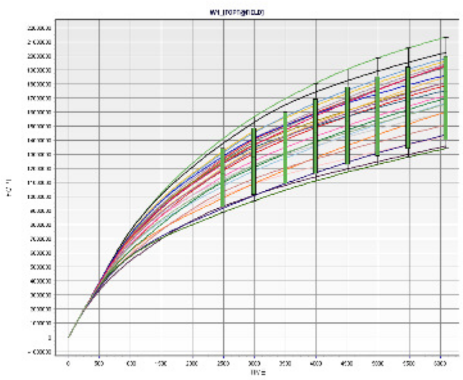
History Match

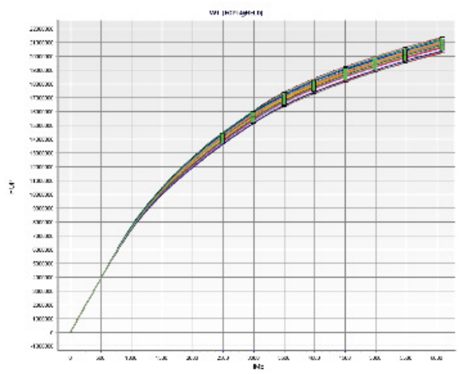




53

Results: Ensemble Improvement



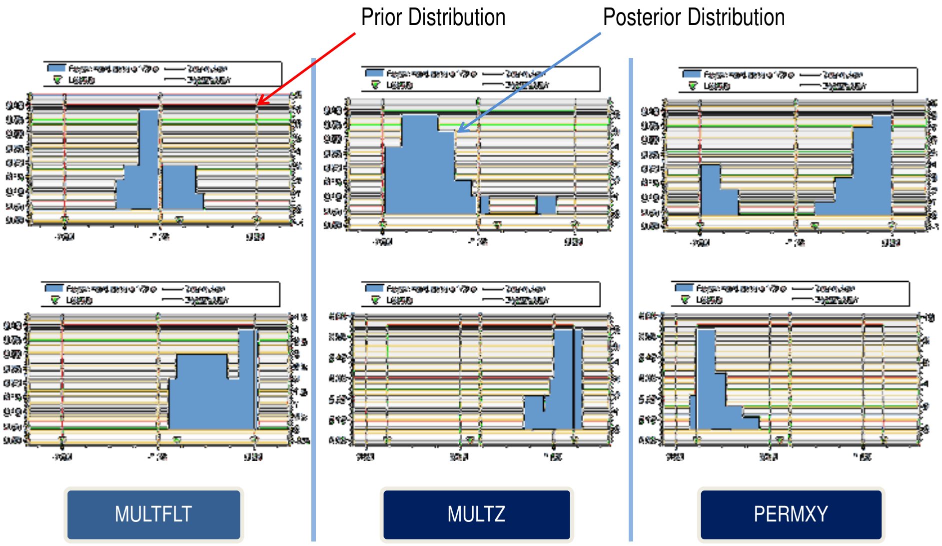


Prior Ensemble

Posterior Ensemble

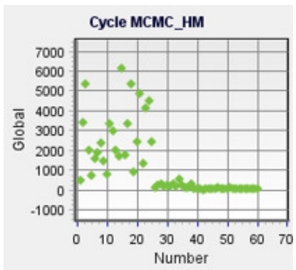
54

Results: Prior and Posterior Parameter Distribution



Summary MCMC-Proxy – Workflow

- Proxy-based approach allows to apply MCMC workflows to complex simulation cases
- Fast convergence behavior
- Efficient technique for global optimization, i.e., calibration of global uncertainty parameters
- Technique offers extensive potential for accelerating optimization workflows based on proxy modelling support.

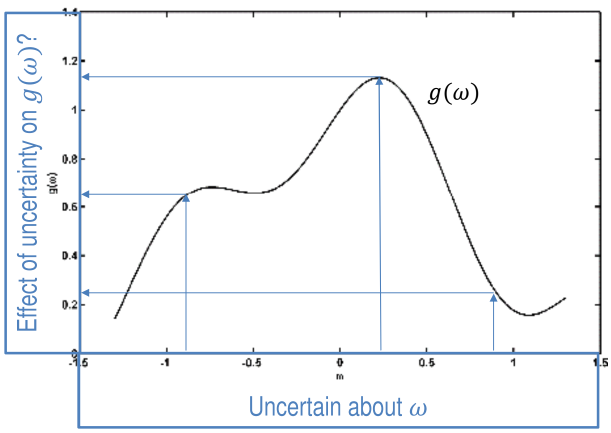


- Method is less well prepared to handle large multi-dimensional parameter spaces in the current application scenario
- Variance estimation may still be expensive

Stochastic Proxy Modeling - gPCE

Probabilistic Formulation

Make Expensive Uncertain Parametric Problems Cheap



- g *expensive* to evaluate (e.g. reservoir model) → replace by "cheaper version" $\tilde{g}$: a proxy model

Improved Proxy Modeling

Problem:

- *Monte Carlo* (MC) is often used to perform uncertainty propagation for random variables (RVs)
  - is flexible and
  - independent on the dimensionality of the problem
- but exhibits very slow convergence ($\sqrt{N}$), where *N* is the sample size.

Scope:

- Find "better" proxy model for probabilistic uncertainty quantification & optimization of hydrocarbon reservoirs

Requirements on proxy model:

- Fast creation, fast evaluation
- Use in existing workflows, with existing simulators
- Precision advantages for probabilistic workflows

Methodology

- We choose (generalized) Polynomial Chaos Expansion (gPCE)

59

gPCE: A Probabilistic Proxy Model

- Introduce parameter ω for uncertainty, use *probability theory* to quantify it
- Primary quantities: **random variables** (RVs):

$$\boldsymbol{r}(\omega) \in L_2(\Omega; V; P)$$

  - Ω: sample space of possible outcomes,
    V: vector space,
    P : probability measure.

- Generalized Polynomial Chaos Expansion (gPCE):
  spectral representation of RVs:

$$\boldsymbol{r}(\omega) = \sum_{\alpha \in J} \boldsymbol{r}^{\alpha}\, \boldsymbol{P}_{\alpha}(\xi_1(\omega), \xi_2(\omega), \dots)$$

  - Here: Series of known Legendre polynomials and uniform basis RVs; unknown "spectral" coefficients
- How to compute the coefficients?

60

Computing the gPCE Coefficients: NISP

- Legendre polynomials are orthogonal w.r.t. $P = U(-1,1)$
  → Use *projection* to compute coefficients $\boldsymbol{r}^{\alpha}$:

$$\forall \alpha: \ \boldsymbol{r}^{\alpha} = \frac{\langle \boldsymbol{r}, \boldsymbol{P}_{\alpha} \rangle}{\langle \boldsymbol{P}_{\alpha}, \boldsymbol{P}_{\alpha} \rangle}$$

Reminder:

$$\langle u, v \rangle = \int_{\Omega} u(\omega) v(\omega) dP(\omega)$$

- Cannot change simulator code, problem possibly high-dimensional
  → Use *sparse*-grid numerical cubature:
  "Sparse tensorization" of 1D Gauss-Legendre quadrature
  [**Warning:** Sparse grids come with assumptions]
  → Simulations computationally feasible, projection *cheap*

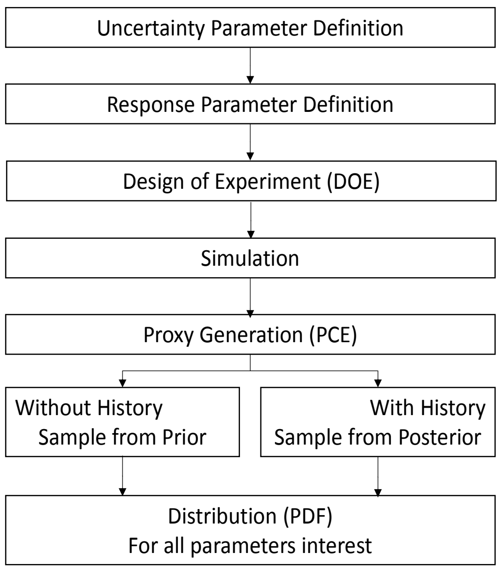
$$\int_{\Omega} f(x)dx \approx \sum_i w_i f(x_i)$$

- gPCE + "non-intrusive spectral projection" (NISP) using sparse grids: good fulfilment of requirements! How well does it work?

61

Workflow Design for gPCE Application

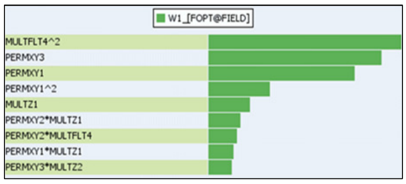
- Zander, E. (2014, May 19). SGLib - A Matlab/Octave toolbox for stochastic Galerkin methods. doi:10.5281/zenodo.9966



62

Create gPCE Proxies For Production Quantities

- 9 uncertain parameters of varying type & importance
- Distributions selected from prior knowledge, some with high uncertainty, some with low



- Methods for comparison:
  - gPCE (different max. orders)
  - Monte Carlo (50k samples)
  - "Automatic Regression" method
    - 2nd order polynomial regression
    - Adaptively selects terms (expensive)
    - Uses gPCE experimental designs (to be fair)

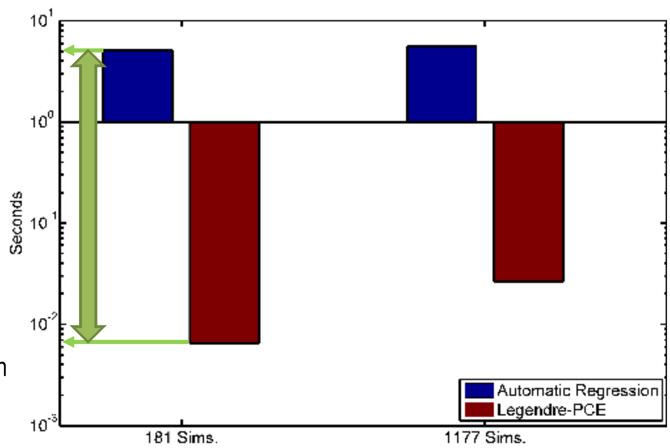
Parameter	Distribution	% of Range
MULTFLT 1	U(0.316, 1.000)	68.4 %
MULTFLT 2	U(0.013, 0.080)	6.7 %
MULTFLT 3	U(0.056, 1.000)	94.4 %
MULTFLT 4	U(0.794, 1.000)	20.6 %
MULTZ 1	U(0.500, 1.000)	50.0 %
MULTZ 2	U(0.003, 0.018)	1.5 %
PERMXY 1	U(0.400, 0.620)	-
PERMXY 2	U(0.200, 0.280)	-
PERMXY 3	U(0.200, 0.210)	-

63

Results: gPCE vs "Automatic Regression"

- gPCE builds 100 proxies in parallel → major performance gain (per proxy)
- gPCE computes more coefficients than "Automatic Regression"!

➢ gPCE is *orders of magnitude* faster when constructing multiple proxies (due to projection)

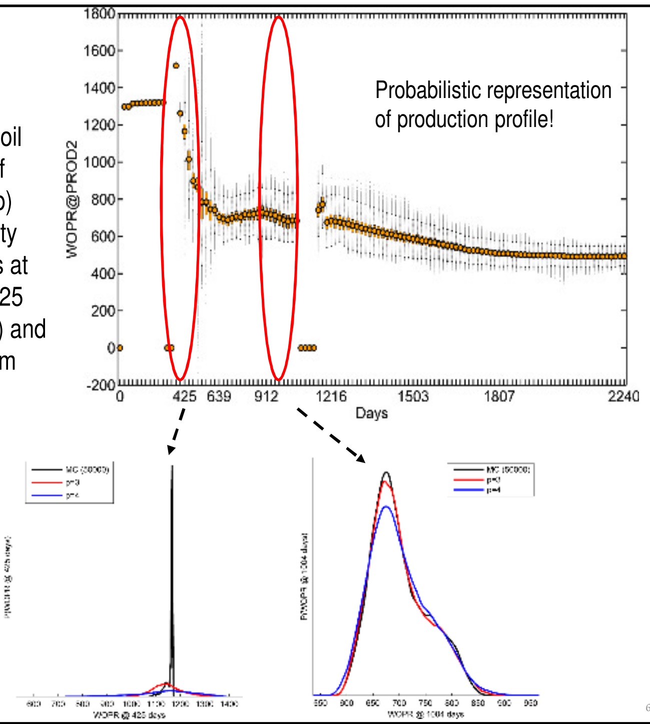


Performance: Average Time for Building One Proxy for FOPT @ 2240 Days

64

gPCE Accuracy

- Boxplots for well oil production rate of producer two (top) and two probability density estimates at key time points 425 days (bottom left) and 1004 days (bottom right)



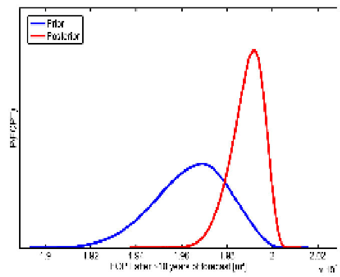
Summary gPCE

gPCE is:

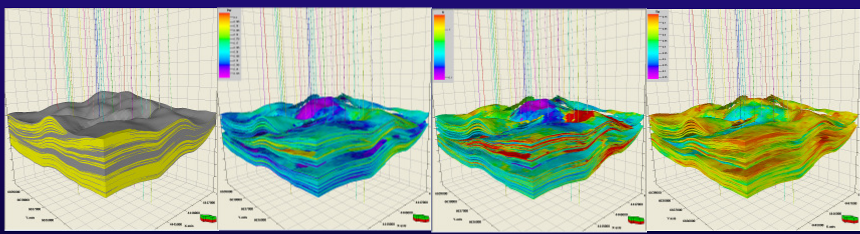
- A probabilistic proxy model for smooth model responses
- Compatible with existing workflows, parameterizations
- Equal or better than "automatic regression"
- Significantly faster (projection) and very exact for certain responses

Challenges remain:

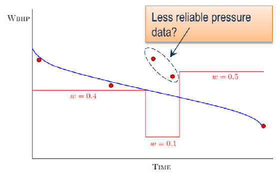
- Difficulties with non-linear response
- Adaptive basis selection; other basis functions

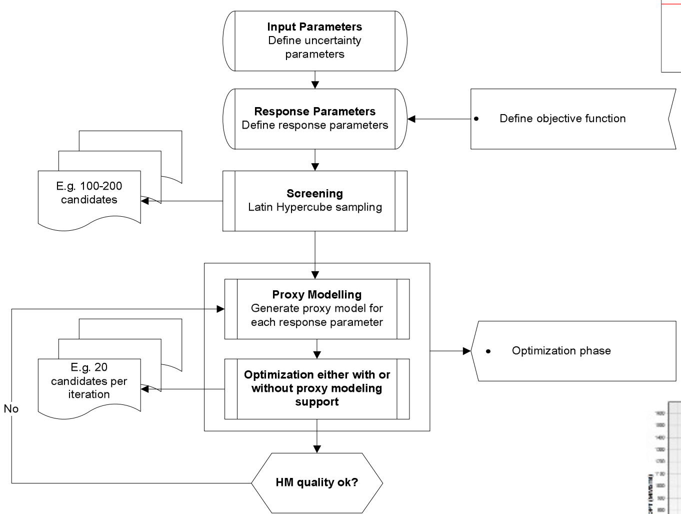


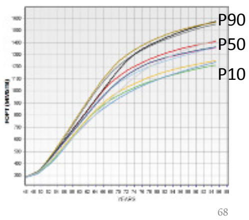
Practical Conclusion



History Matching

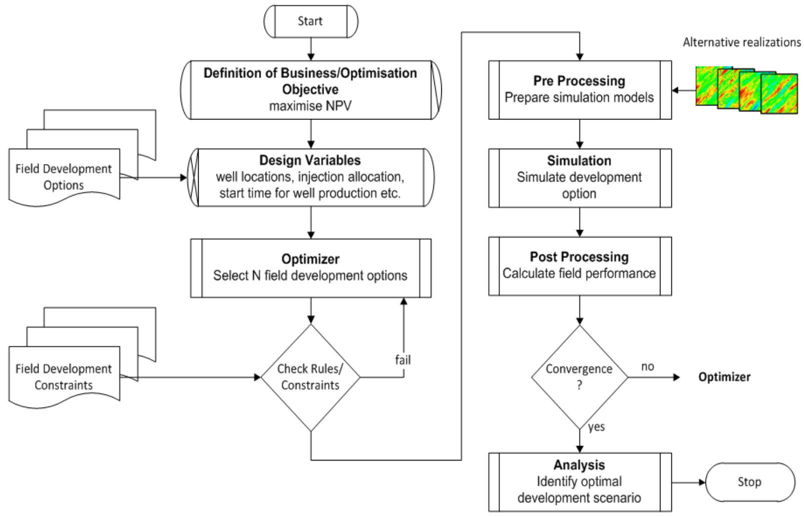






Practical reproducible workflow design

Field Development Optimization under Uncertainty



Practical reproducible workflow design

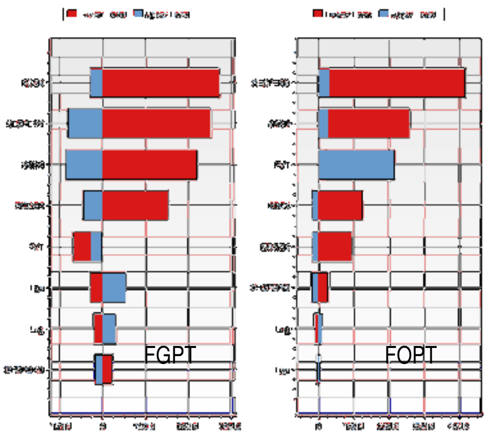
Uncertainty Matrix – Parameterization

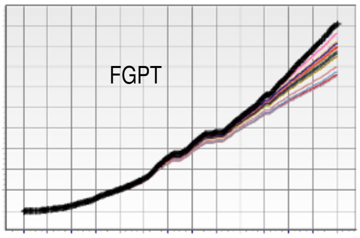
Domain	Description	Type	Form	Parameter(s)	Input properties
Geology	Structure	Discrete	Set	N/A	N/A
Geology	Lateral continuity	Continuous	Function	TBD	TBD
Petrophysics	Porosity	Continuous	Algorithm	phi_mean	PORO
Petrophysics	Permeability	Continuous	Algorithm	k_mean	PERMX
Petrophysics	Vertical anisotropy	Continuous	Function	kvkh_mult	PERMX
SCAL	Saturation end points	Continuous	Constant	SOGCR	SOGCR
SCAL	Saturation end points	Continuous	Constant	sgcr1	SOGCR, SWL
SCAL	Permeability end points	Continuous	Constant	KRORG	KRORG
SCAL	Permeability end points	Continuous	Constant	KRGR	KRGR
SCAL	Curvature	Continuous	Function	Lg	N/A
SCAL	Curvature	Continuous	Function	Eg	N/A
SCAL	Curvature	Continuous	Function	Tg	N/A
SCAL	Curvature	Continuous	Function	Lo	N/A
SCAL	Curvature	Continuous	Function	Eo	N/A
SCAL	Curvature	Continuous	Function	To	N/A
PVT	PVT model	Discrete	Set	N/A	N/A
PVT	PVT model	Discrete	Set	N/A	N/A
Fracturing	HF parameters	Discrete	Constant	Xf	N/A
Fracturing	HF parameters	Discrete	Constant	wf	N/A
Fracturing	HF parameters	Discrete	Constant	Fc (Kprop)	N/A
Fracturing / SCAL	Curvature	Continuous	Function	n (Corey-like)	N/A
ROCK	Compressibility table - porosity	Continuous	Function	poro_sl (slope in log scale)	N/A
ROCK	Compressibility table - permeability	Continuous	Function	perm_sl (slope in log scale)	N/A

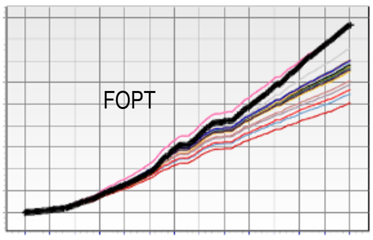
Complex uncertainty definitions (continuous, discrete) require practical customizations of the workflow.

Sensitivity Analysis

- Sensitivity results based on reference case:







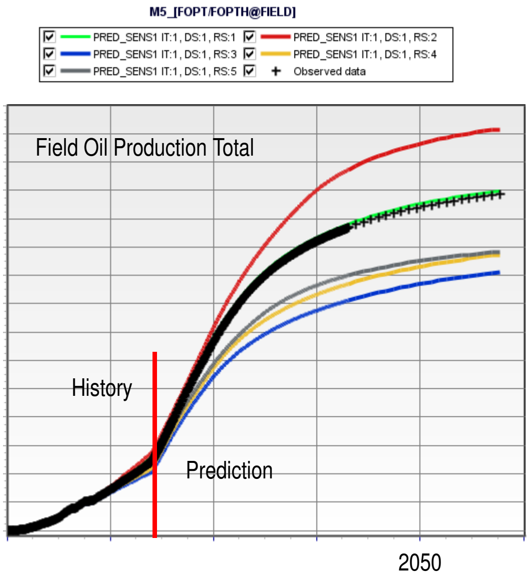
Workflow Scenario

1
- **<u>Sensitivity analysis for production performance forecast</u>**
- Define prediction scenario
- Test performance

2
- Run full uncertainty matrix on prediction scenario
- Identify realizations which meet rate match within tolerance level

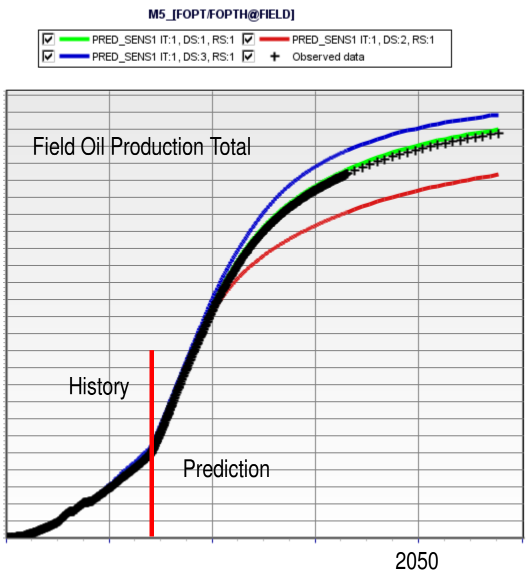
3
- Run prediction scenario for selected realizations and test impact of local parameter adjustments

Realizations which meet rate match within tolerance level



73

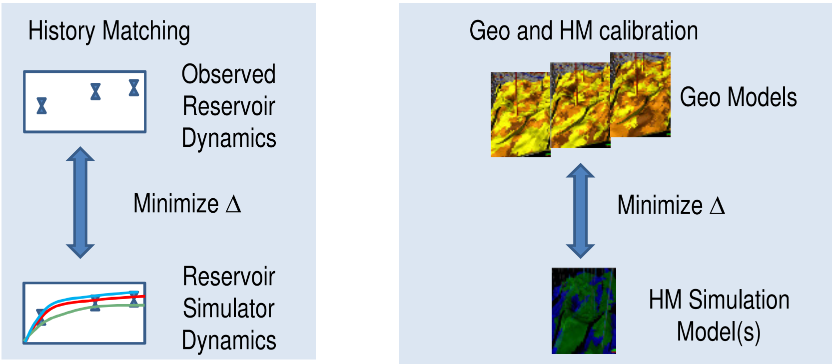
Selected realization with impact of local parameter adjustments



74

Conclusion: Analysis and Closing Feedback-Loop

- The complexity of aligning the geo and simulation models depends on the parameterization and optimization approach
- A methodology should ideally be designed to analyze parameter trends which influence a minimum discrepancy between geo modeling scenarios/realizations and the history matched models



References

- Michael D. Prange, Thomas P. Dombrowsky and William J. Bailey, “Geologically constrained history matching with PCA”, first break volume 30, November 2012
- Oliver Pajonk, Franck Yanou Ngongang , Ralf Schulze-Riegert,
  Evaluation of Non-Intrusive Generalised Polynomial Chaos Expansion in the Context of Reservoir Simulation,
  ECMOR XIV – 14th European Conference on the Mathematics of Oil Recovery
  Catania, Sicily, Italy, 8-11 September 2014 – Extended Version 19.12.2014
- R. Schulze-Riegert, I. Ajala, D. Awofodu, H. Almuallim, J. Baffoe, F. Chataigner, N. Kueck, O. Pajonk, A. Shiromizu, “Cross-Verification of Adjoint and MCMC Workflows for Estimation of Prediction Uncertainties including Historical Data”,
  This extended abstract was accepted for presentation
  at the 76th EAGE Conference & Exhibition 2014 in Amsterdam, the Netherland
- Ralf Schulze-Riegert, Markus Krosche, Oliver Pajonk, Hassan Mustafa, Data Assimilation Coupled to Evolutionary Algorithms - A Case Example in History Matching, paper SPE 125512 was prepared for presentation at the 2009 SPE/EAGE Reservoir Characterization and Simulation Conference held in Abu Dhabi, UAE, 19–21 October 2009

76