Fast Simulation Tools for Flow in Heterogeneous Porous Media

Knut–Andreas Lie

SINTEF ICT, Dept. Applied Mathematics, Oslo, Norway

SimTech 2011, Stuttgart, 14–17 June 2011

Application: petroleum production and CO_2 storage

Simulation support for two main areas:

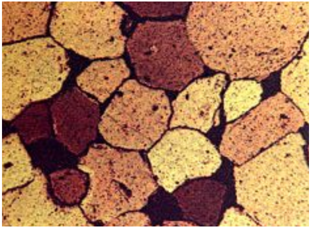
- Increase recovery of petroleum resources (planning and management): understand reservoir and fluid behavior, test hypotheses and scenarios, assimilate data, optimize production, etc.
- Ensure storage of carbon: how fast can one inject, will the injected CO_2 leak, where will the CO_2 move?

$\longrightarrow$ robust, efficient, and accurate simulation methods for partial differential equations with highly heterogeneous parameters on complex grids

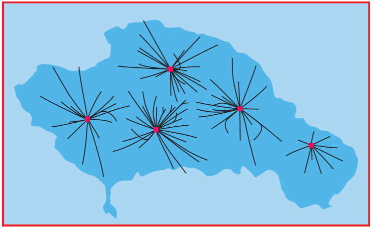
Porous media flow – a multiscale problem

The scales that impact fluid flow in subsurface rocks range from

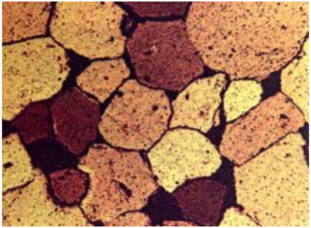
- the micrometer scale of pores and pore channels
- via dm-m scale of well bores and laminae sediments
- to sedimentary structures that stretch across entire reservoirs



$\longrightarrow$



Porous media flow – a multiscale problem



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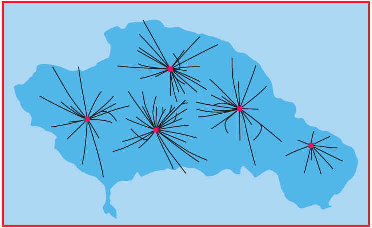
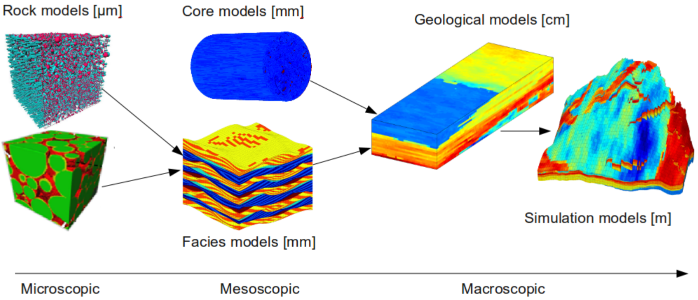
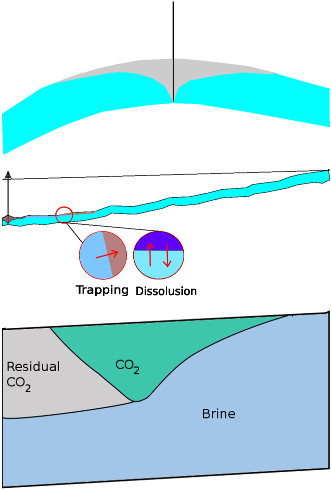




Example: injection and migration of CO_2

Physical process:

- supercritical CO_2 injected into an aquifer or abandoned reservoir
- forms a liquid phase that is lighter, less dense, and weakly soluble in water
- the CO_2-phase will migrate upward in the formation, limited above by the caprock, displacing the resident brine
- the displacement front is mainly driven by gravity (but also processes like dissolution, vaporization, salt precipitation, drying, etc)



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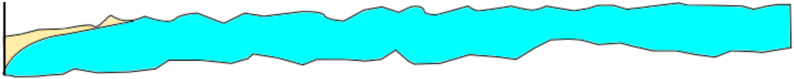
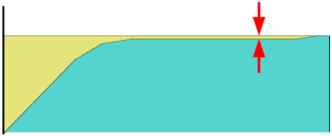




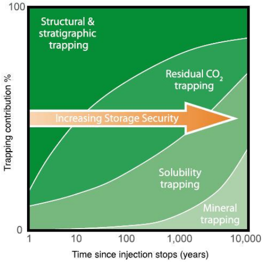
Spatial scales:

- horizontal extent of geological formation: 10–100 km
- height of formation: 10–200 m
- the tip of the CO_2-plume: 0.1–1 m

Time scales:

- pressure buildup: hours
- injection period: 20–50 years
- migration: 100–10000 years

See plenary talk by Prof. M. Celia.



Macroscopic models of flow in porous media

- Single-phase, incompressible flow: conservation of mass + Darcy's law:

$$\vec{v} = -\mu^{-1}\mathbf{K}\nabla p, \qquad \nabla \cdot \vec{v} = q$$

- Multiphase, compressible flow:

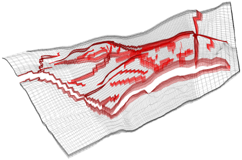
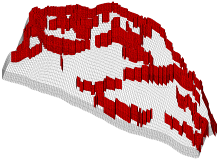
$$\vec{v} = -\lambda \mathbf{K}\big(\nabla p - \sum_j \rho_j f_j \vec{g}\big)$$

$$\nabla \cdot \vec{v} = q - c_t \frac{\partial p}{\partial t} + \Big(\sum_j c_j f_j \vec{v} + \alpha(p)\mathbf{K}\vec{g}\Big) \cdot \nabla p$$

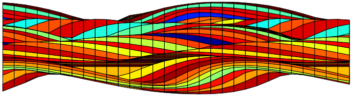
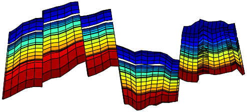
$$\phi \frac{\partial s_j}{\partial t} + \nabla\Big(f_j\big(\vec{v} + h_j \mathbf{K}\vec{g}\big)\Big) = q_j$$

Grid – volumetric representation of the reservoir

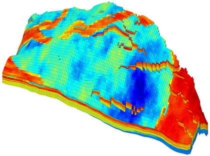
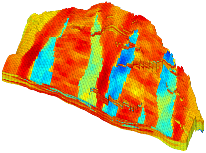
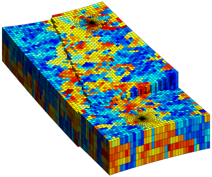
The structure of the reservoir (geological surfaces, faults, etc)

The stratigraphy of the reservoir (sedimentary structures)

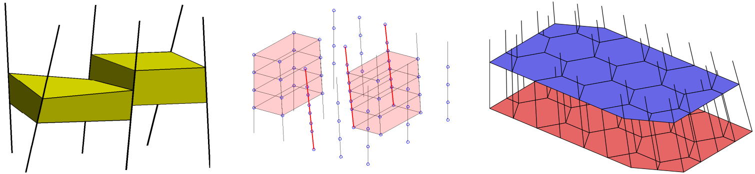



Petrophysical parameters (permeability, porosity, net-to-gross, . . .)

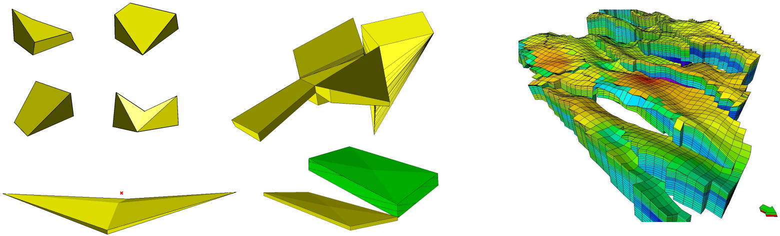




Grid – volumetric representation of the reservoir

Industry standard: stratigraphic grids (corner-point, 2.5D PEBI, etc)



Geometrical and numerical challenges: high aspect ratios, unstructured connections, many faces/neighbors, small faces, ...

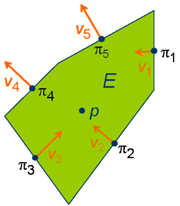


Research challenge: consistent discretizations

Poisson type problem:

$$\nabla \cdot \vec{v} = q, \qquad \vec{v} = -\mu^{-1}\mathbf{K}\nabla p$$

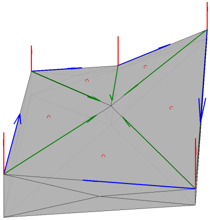
Design of methods capable of handling anisotropic (full-tensor) **K** on general polyhedral grids with curved faces



Basic discretization – relation between flux and pressure on a single cell E

$$\boldsymbol{M}\boldsymbol{v}_E = p\boldsymbol{e} - \boldsymbol{\pi}$$

$$\boldsymbol{M} = \frac{1}{|E|}\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{C}^\mathsf{T} + \boldsymbol{Q}_N^\perp \boldsymbol{S}_M \boldsymbol{Q}_N^{\perp\,\mathsf{T}}$$

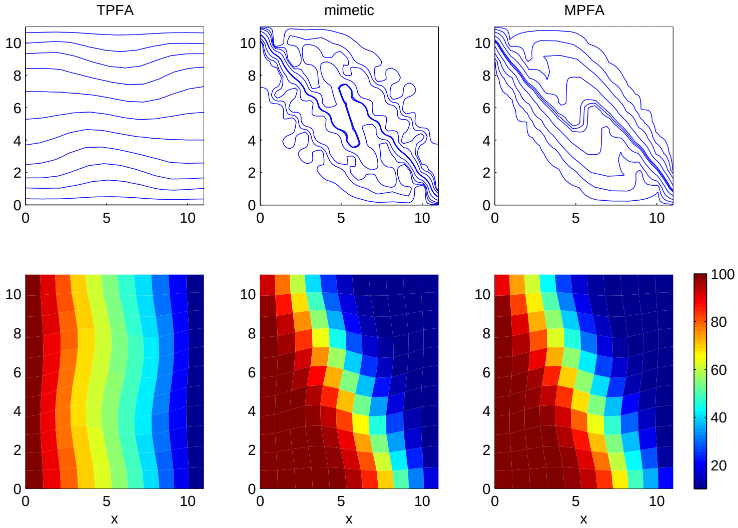


General class: TPFA, MPFA, mixed, mimetic, . . .

Mixed (hybrid) formulation:

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D} \\ \boldsymbol{C}^\mathsf{T} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{D}^\mathsf{T} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ -\boldsymbol{p} \\ \boldsymbol{\pi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{q} \\ \boldsymbol{0} \end{bmatrix}$$

Research challenge: consistent discretizations



Homogeneous **K** $= \mathrm{diag}([1, 1000])$ rotated 30°, pressure drop from left to right

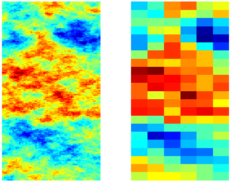
Research challenge: computationally efficient/tractable

Simulators incapable of handling required model detail. Example:

- geological models: 10^7–10^9 cells
- simulators: 10^5–10^6 cells

Demand for complexity is continuously increasing.

Particular challenge: lack of scale separation



Upscaling (homogenization): bottleneck in workflow, inefficent and not sufficiently robust

Research challenge: computationally efficient/tractable

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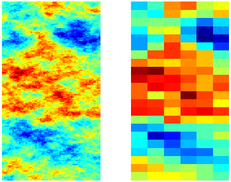
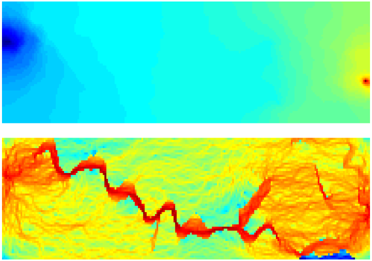


Upscaling (homogenization): bottleneck in workflow, inefficent and not sufficiently robust

Multiscale methods



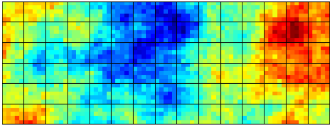
- Up-/downscaling in one step
- Pressure on coarse grid
- Fluxes on fine grid

Incorporate impact of subgrid heterogeneity in approximation spaces

Advantages: utilize more geological data, more accurate solutions, geometrical flexibility

Multiscale methods

Coarse partitioning:



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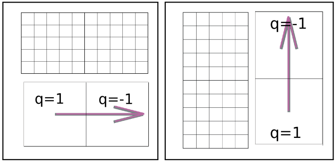
Local flow problems:



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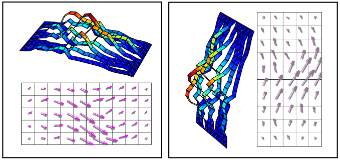
Flow solutions → basis functions:



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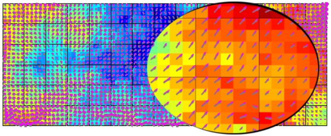
Flow field with subresolution:

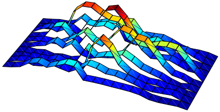


Multiscale mixed finite elements

Make the following assumption

$$\boldsymbol{v} = \boldsymbol{\Psi} \boldsymbol{v}_c + \tilde{\boldsymbol{v}}$$

$$\boldsymbol{p} = \boldsymbol{\mathcal{I}} \boldsymbol{p}_c + \tilde{\boldsymbol{p}}$$



$\boldsymbol{\Psi}$ – matrix with basis functions
$\boldsymbol{\mathcal{I}}$ – prolongation from blocks to cells

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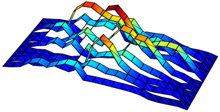

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Reduction to coarse-scale system:

$$\begin{bmatrix} \boldsymbol{\Psi}^\mathsf{T} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\mathcal{I}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\mathsf{T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Psi} \boldsymbol{v}_c + \tilde{\boldsymbol{v}} \\ -\boldsymbol{\mathcal{I}} \boldsymbol{p}_c - \tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\mathcal{I}}^\mathsf{T} \boldsymbol{q} \end{bmatrix}$$

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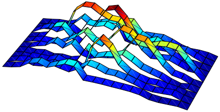

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$$\begin{bmatrix} \boldsymbol{\Psi}^\mathsf{T} B \boldsymbol{\Psi} & \boldsymbol{\Psi}^\mathsf{T} C \boldsymbol{\mathcal{I}} \\ \boldsymbol{\mathcal{I}}^\mathsf{T} C^\mathsf{T} \boldsymbol{\Psi} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_c \\ -\boldsymbol{p}_c \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\Psi}^\mathsf{T} B \tilde{\boldsymbol{v}} + \boldsymbol{\Psi}^\mathsf{T} C \tilde{\boldsymbol{p}} \\ \boldsymbol{q}_c - \boldsymbol{\mathcal{I}}^\mathsf{T} C^\mathsf{T} \tilde{\boldsymbol{v}} \end{bmatrix}$$

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Multiscale basis function:

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\mathsf{T} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Psi} \\ \boldsymbol{\Phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{w} \end{bmatrix}$$

Set of equations located to coarse blocks. Flow driven by weight w

Reduction to coarse-scale system:

$$\begin{bmatrix} \boldsymbol{\Psi}^\mathsf{T} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\mathcal{I}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\mathsf{T} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Psi}\boldsymbol{v}_c + \tilde{\boldsymbol{v}} \\ -\boldsymbol{\mathcal{I}}\boldsymbol{p}_c - \tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\mathcal{I}}^\mathsf{T}\boldsymbol{q} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\Psi}^\mathsf{T}\boldsymbol{B}\boldsymbol{\Psi} & \boldsymbol{\Psi}^\mathsf{T}\boldsymbol{C}\boldsymbol{\mathcal{I}} \\ \boldsymbol{\mathcal{I}}^\mathsf{T}\boldsymbol{C}^\mathsf{T}\boldsymbol{\Psi} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_c \\ -\boldsymbol{p}_c \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\Psi}^\mathsf{T}\boldsymbol{B}\tilde{\boldsymbol{v}} + \boldsymbol{\Psi}^\mathsf{T}\boldsymbol{C}\tilde{\boldsymbol{p}} \\ \boldsymbol{q}_c - \boldsymbol{\mathcal{I}}^\mathsf{T}\boldsymbol{C}^\mathsf{T}\tilde{\boldsymbol{v}} \end{bmatrix}$$

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Additional assumptions:

1. Since p is immaterial, assume $\boldsymbol{w}^\mathsf{T}\tilde{\boldsymbol{p}} = 0$. Hence, $p_c^i = \int_{\Omega_i} wp\,dx$

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Additional assumptions:

1. Since p is immaterial, assume $\boldsymbol{w}^\mathsf{T}\tilde{\boldsymbol{p}} = 0$. Hence, $p_c^i = \int_{\Omega_i} wp\,dx$
2. Assume that $\boldsymbol{\Psi}$ spans velocity space, i.e., $\tilde{\boldsymbol{v}} \equiv \boldsymbol{0}$.

Constructing multiscale basis functions

Example: Velocity basis function ψ_{ij} solves a local system of equations in Ω_{ij}:

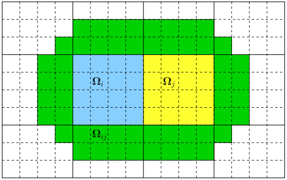
$$\vec{\psi}_{ij} = -\mu^{-1}\mathbf{K}\nabla\varphi_{ij}$$

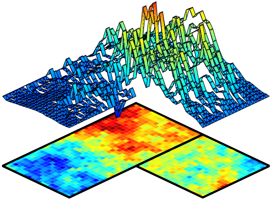
$$\nabla\cdot\vec{\psi}_{ij} = \begin{cases} w_i(\vec{x}), & \text{if } \vec{x}\in\Omega_i, \\ -w_j(\vec{x}), & \text{if } \vec{x}\in\Omega_j, \\ 0, & \text{otherwise.} \end{cases}$$

with no-flow conditions on $\partial\Omega_{ij}$

Source term: $w_i \propto \mathrm{trace}\,(K_i)$ drives a unit flow through Γ_{ij}.

If there is a sink/source in T_i, then $w_i \propto q_i$.





Alternative: use good approximation to set 'global' boundary conditions for each block

Residual correction

To get a convergent method, we need to also account for variations that are not captured by the basis functions[1] $\longrightarrow$ solve a residual equation

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^{\mathsf{T}} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Psi} \boldsymbol{v}_c + \tilde{\boldsymbol{v}} \\ -\boldsymbol{\mathcal{I}} \boldsymbol{p}_c - \boldsymbol{D}_\lambda \boldsymbol{\Phi} \boldsymbol{v}_c - \tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{q} \end{bmatrix}$$

[1] The term $\boldsymbol{C}\boldsymbol{D}_\lambda \boldsymbol{\Phi} \boldsymbol{v}_c$ corresponds to subscale pressure variations modelled by the numerically computed basis functions for pressure, which should scale similar to $\boldsymbol{\Psi}$ since $\boldsymbol{B}\boldsymbol{\Psi} - \boldsymbol{C}\boldsymbol{\Phi} = \boldsymbol{0}$.

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To get a convergent method, we need to also account for variations that are not captured by the basis functions[1] $\longrightarrow$ solve a residual equation

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\mathsf{T} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Psi v}_c + \tilde{\boldsymbol{v}} \\ -\boldsymbol{\mathcal{I} p}_c - \boldsymbol{D}_\lambda \boldsymbol{\Phi v}_c - \tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{q} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\mathsf{T} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{v}} \\ -\tilde{\boldsymbol{p}} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{C D}_\lambda \boldsymbol{\Phi} - \boldsymbol{B \Psi}) \boldsymbol{v}_c + \boldsymbol{C \mathcal{I} p}_c \\ \boldsymbol{q} - \boldsymbol{C}^\mathsf{T} \boldsymbol{\Psi v}_c \end{bmatrix}$$

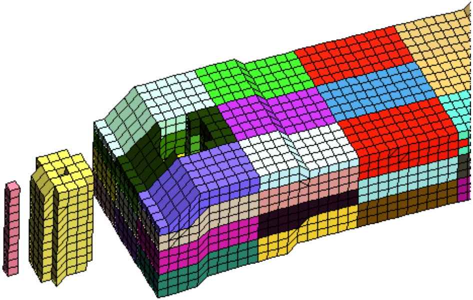
To solve this equation, we will typically use a (non)overlapping domain-decomposition method.

[1] The term $\boldsymbol{C D}_\lambda \boldsymbol{\Phi v}_c$ corresponds to subscale pressure variations modelled by the numerically computed basis functions for pressure, which should scale similar to $\boldsymbol{\Psi}$ since $\boldsymbol{B\Psi} - \boldsymbol{C\Phi} = \boldsymbol{0}$.

Advantage: flexible generation of coarse grids

(Unique) grid flexibility:

Given a method that can solve local flow problems on the subgrid, the MsMFE method can be formulated on any coarse grid in which the coarse blocks consist of a connected collection of fine-grid cells



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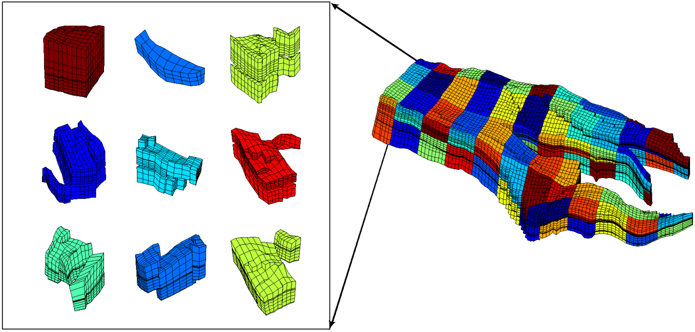

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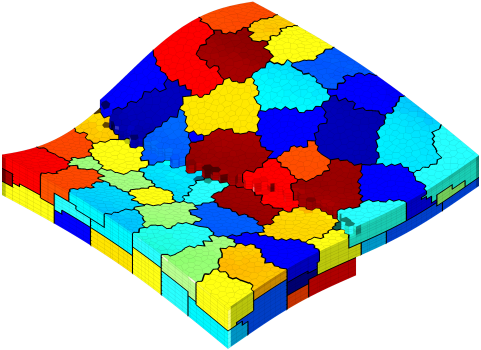


Advantage: computational efficiency

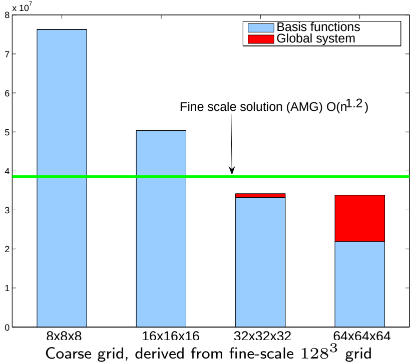
Multigrid will often be more efficient when computing pressure once.

Why bother with multiscale pressure solvers?

- Full multiphase simulation: $\mathcal{O}(10^2)$ time steps.
- Basis functions need not be recomputed or be updated infrequently

Also:

- Lower memory requirements – possible to solve very large problems
- Easy parallelization – computation of basis functions



Where can multiscale methods be used?

Typical applications

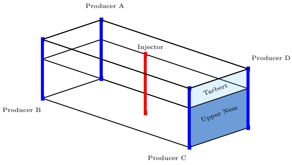
- 'Interactive' screening of flow patterns during geological modelling
- Simulation of multiple realizations to quantify uncertainty
- Production optimization: well rates, well placement, ...
- History matching

Key ideas:

- Having 80–90% of the answer in 5–10% of the time enables geologists and engineers to explore more modelling choices
- 'Full physics' is seldom needed early in the modelling workflow, focus on the *important* effects

Example: highly efficient streamline simulation

SPE 10, Model 2:



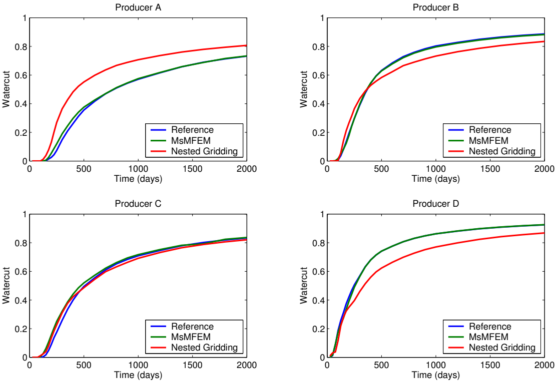
Fine grid: $60 \times 220 \times 85$
2000 days production
25 time steps

Inhouse code from 2005:
multiscale: 2 min and 20 sec
multigrid: 8 min and 36 sec

Fully unstructured Matlab/C code from 2010:

mimetic : 5–6 min

Water-cut curves at the four producers



upscaling/downscaling, multiscale, fine grid

Example: highly efficient streamline simulation

Computational efficiency of a prototype code fine-scale mimetic versus a multiscale mimetic solver in a commercial solver. Neither prototypes have been optimized

Three versions of the SPE10 model (upscaled, original, 3×3 repeat)

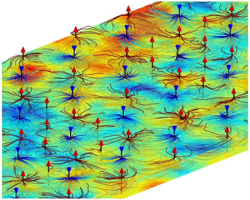
Model	Solver	Grid	Steps	Init	Basis	Assembly	Pressure	Transp	Total
56 k	AMG	30×110×17	13	3	—	26	96	2	129
			50	8	—	89	261	14	373
	M-S	6× 22×17	13	3	13	2	2	5	27
			50	8	11	2	4	18	44
1.1 M	AMG	60×220×85	13	46	—	525	1,787	38	2,424
	M-S	12× 44×17	13	46	350	27	14	45	514
10 M	AMG	180×660×85	13	470	—	4,803	25,538	398	31,401
	M-S	36×132×17	13	470	2,597	193	169	305	3,925

Example: history-matching a million-cell model

Assimilation of production data to calibrate model

- 1 million cells, 32 injectors, and 69 producers
- 2475 days $\approx$ 7 years of water-cut data

Generalized travel-time inversion (quasi-linearization of misfit functional) with analytical sensitivities along streamlines, Datta–Gupta et al.



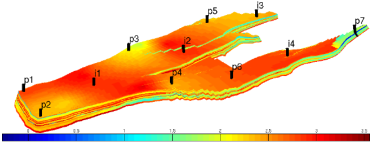
	CPU-time (wall clock)		
Solver	Total	Pres.	Transp.
Multigrid	39 min	30 min	5 min
Multiscale	17 min	7 min	6 min

Computer: 2.4 GHz Core 2 Duo, with 2 GB RAM
History match: 7 forward simulations, 6 inversions

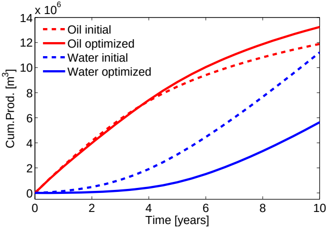
No parallelization of basis functions, streamline tracing, and 1D transport solves

Example: rate optimization of water-flood

Adjoint-based multiscale method:



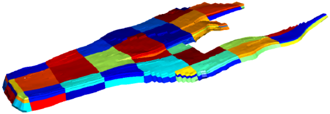
Grid model: from offshore Norway



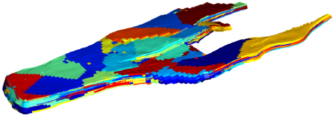
Forward simulations:
44 927 cells, 20 time steps,
$<$ 5 sec in Matlab, $\sim 100\times$ speedup

Specialized simulator with different grid for pressure and transport solvers

Pressure grid:



Transport grid:



In addition: efficient communication between the two coarse grids

Current research: MsMFE for compressible flow

Simplest approach – four key components:

1. Elliptic basis functions, constructed with $w(x) \propto \phi(x)$
2. Coarse-scale system

$$\begin{bmatrix} \boldsymbol{\Psi}^\mathsf{T} \boldsymbol{B} \boldsymbol{\Psi} & \boldsymbol{\Psi}^\mathsf{T} \boldsymbol{C} \boldsymbol{\mathcal{I}} \\ \boldsymbol{\mathcal{I}}^\mathsf{T} (\boldsymbol{C}^\mathsf{T} \boldsymbol{\Psi} - \boldsymbol{P}_\nu \boldsymbol{D}_\lambda \boldsymbol{\Phi}) & \boldsymbol{\mathcal{I}}^\mathsf{T} \boldsymbol{P}_\nu \boldsymbol{\mathcal{I}} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_c^{\nu+1} \\ -\boldsymbol{p}_c^{\nu+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Psi}^\mathsf{T} \boldsymbol{f}_\nu \\ \boldsymbol{\mathcal{I}}^\mathsf{T} \boldsymbol{g}_\nu \end{bmatrix}$$

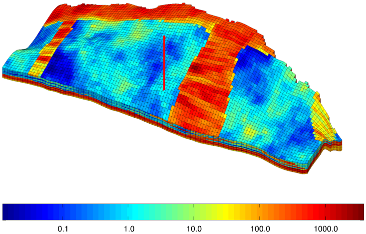
3. Residual equation

$$\begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\mathsf{T} & \boldsymbol{P} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{v}}^{\nu+1} \\ -\hat{\boldsymbol{p}}^{\nu+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_c - \boldsymbol{\Psi}^\mathsf{T} \boldsymbol{B} \boldsymbol{\Psi} \boldsymbol{v}_c + \boldsymbol{\Psi}^\mathsf{T} \boldsymbol{C} \boldsymbol{\mathcal{I}} \boldsymbol{p}_c \\ \boldsymbol{g}_c - \boldsymbol{\mathcal{I}}^\mathsf{T} (\boldsymbol{C}^\mathsf{T} \boldsymbol{\Psi} - \boldsymbol{P}_\nu \boldsymbol{D}_\lambda \boldsymbol{\Phi}) \boldsymbol{v}_c + \boldsymbol{\mathcal{I}}^\mathsf{T} \boldsymbol{P}_\nu \boldsymbol{\mathcal{I}} \boldsymbol{p}_c \end{bmatrix}$$

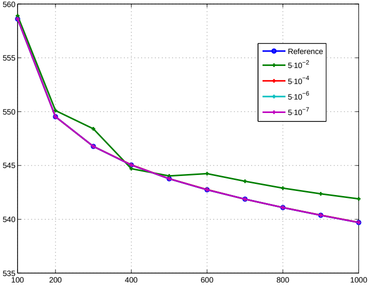
4. Iterations over multiscale and residual equations

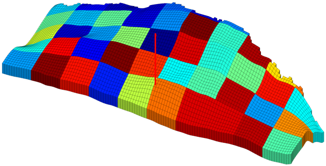
Example: primary production

- Shallow-marine reservoir (realization from SAIGUP)
- Model size: $40 \times 120 \times 20$
- Initially filled with gas, 200 bar
- Single producer, bhp=150 bar
- Multiscale solution for different tolerences compared with fine-scale reference solution.



Rate in well perforation (m^3/day)





Summary

Presented a multiscale framework that can be used to reduce computational complexity by

- resolving effects on different scales
- utilizing sparsity
- (systematically) reusing computations

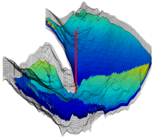
Well tested for two-phase, incompressible flow. Research needed for more complex flow physics:

- basis function dictionary by bootstrapping
- model reduction
- better error control

Matlab Reservoir Simulation Toolbox (MRST)

MRST core

- routines for creating and manipulating grids and physical properties
- basic incompressible flow and transport solvers



Modules

Add-on software that extends, complements, and overrides existing MRST features. Presently implements more advanced solvers and tools:

- adjoint methods, experimental multiscale, fractures, MPFA, upscaling
- black-oil models, three-phase flow, vertically integrated models, ...
- streamlines, (flow-based) coarsening, ...
- Octave support, C-acceleration, ...

Download

`http://www.sintef.no/MRST/`

Version 2011a was released on the 22nd of February, 2011, and can be downloaded under the terms of the GNU General Public License (GPL)