Identification of reservoir parameters using flexible representations

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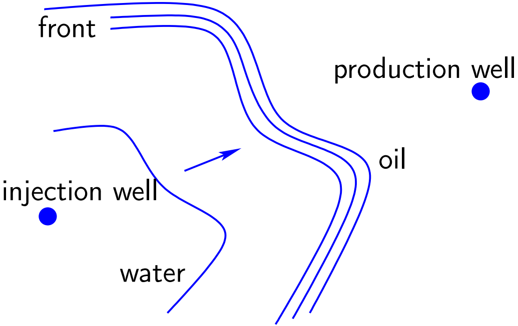
Bergen Workshop, October 2010

Two-phase flow

Forward model

$$\phi \frac{\partial S}{\partial t} - \nabla \cdot (k\lambda_1 \nabla P_1) = q_1$$
$$-\phi \frac{\partial S}{\partial t} - \nabla \cdot (k\lambda_2 \nabla P_2) = q_2$$



Inverse problem

Inverse problem

Estimate the absolute permeability $k(\mathbf{x})$ based on pressure data in wells when the other model specifications are known.

Common characteristics

- Low spatial data density
- Limited prior information

$\Rightarrow$ But sufficient for identification of large-scale structures

Additional challenges

- Nonlinearity
- Resolving power of data is unknown

Minimization problem

Objective function

Minimize

$$J(k) = [d - m(k)]^T C^{-1} [d - m(k)]$$

with respect to k

Represent $k(x)$ with a function $p(x)$ with a finite number of degrees of freedom.

Representation of the parameter function

Tradeoff

finer spatial resolution $\Leftrightarrow$ increased parameter uncertainty

Relationship

small scale perturbations in $p(x)$ $\Leftrightarrow$ low sensitivity $\Leftrightarrow$ high model nonlinearity

Representation of the parameter function

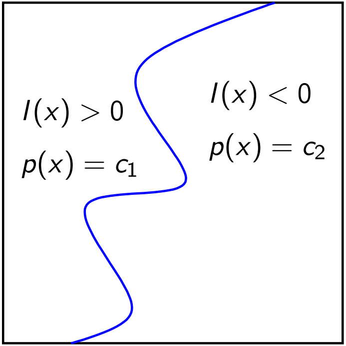
Linear representation (classical inverse problem)

$$p(x) = \Sigma_{j=1}^{N} c_j \Psi_j(x)$$

Composite representation (extended inverse problem)

$$p(x) = c_1 E_1\big(I(x)\big) + c_2 E_2\big(I(x)\big), \quad I(x) = \sum_{k=1}^{K} a_k \theta_k(x)$$

Special case: Level-set representation



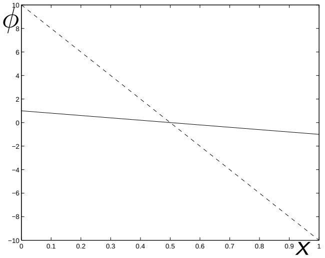
Level-set representation of permeability

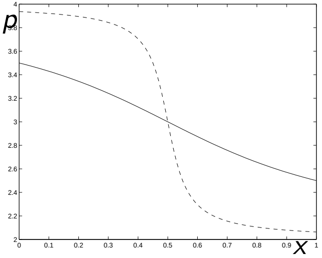
$$p(\mathbf{x}) = c_1 \underbrace{H(I(\mathbf{x}))}_{E_1(I(x))} + c_2 [\underbrace{1 - H(I(\mathbf{x}))}_{E_2(I(x))}]$$

Special case: Smooth Parameter Representation

Parameter function

$$p(x) = c_1 \Big[\underbrace{\frac{1}{2} + \frac{1}{\pi} \tan^{-1} \big(I(x) \big)}_{E_1(I(x))} \Big] + c_2 \Big[\underbrace{\frac{1}{2} - \frac{1}{\pi} \tan^{-1} \big(I(x) \big)}_{E_2(I(x)} \Big]$$





Parameter identification

Identifying parameterization structure and reducing nonlinearity

Composite representation

$$p(x) = c_1 E_1\big(I(x)\big) + c_2 E_2\big(I(x)\big), \quad I(x) = \sum_{k=1}^{K} a_k \theta_k(x)$$

Minimize

$$J(\eta) = [d - m(\eta)]^T C^{-1} [d - m(\eta)], \quad \eta = [c_1, c_2, a_1, \ldots, a_K]$$

Multi-level algorithm

Regular refinement of representation of $I(x)$

- lower sensitivity and lower nonlinearity for larger scales
- reduce the risk of getting trapped in local minima of the objective function

Optimization Method

Levenberg-Marquardt method

$\eta = [c_1, c_2, a_1, \ldots, a_K]$

$$\eta^{h+1} = \eta^h + \Delta\eta^h$$

$\Delta\eta^h$ is the solution of

$$\left(A^T A - \lambda^h I\right)\Delta\eta^h = A^T\left(d - m(\eta^h)\right),$$

Sensitivity matrix:

$$A = \left[\frac{dm_i}{d\eta_j}\right]_{i,j} = \frac{dm}{d\eta}$$

Sensitivity calculations

η	-	unknown coefficients in the representation
u	-	forward model solution vector from solving dynamic problem
m	-	model response (which is compared to data)

$$m = m(u; n), \quad u = u(\eta)$$

Sensitivity matrix

$$A = \left[\frac{dm_i}{d\eta_j}\right]_{i,j} = \frac{dm}{d\eta} = \frac{\partial m}{\partial u}\frac{du}{d\eta} + \frac{\partial m}{\partial \eta}$$

Direct method for sensitivity calculations

Discretized PDE's

$$F(u^{n+1}, u^n; \eta) = 0, \qquad (0)$$

Sensitivity equation

$$\underbrace{\frac{\partial F}{\partial u^{n+1}}}_{Jacobian} \frac{du^{n+1}}{d\eta} = -\frac{\partial F}{\partial u^n}\frac{du^n}{d\eta} - \frac{\partial F}{\partial \eta},$$

$\dim\eta = 2 + K$ linear systems (common coefficient matrix assembled when solving discretized PDE's)

Sensitivity calculations cont.

Chain rule

$$\frac{\partial F}{\partial \eta} = \frac{\partial F}{\partial p}\frac{dp}{d\eta}, \quad \frac{dp}{d\eta} = \left[\frac{\partial p}{\partial c_1}, \frac{\partial p}{\partial c_2}, \frac{\partial p}{\partial a_1}, \ldots, \frac{\partial p}{\partial a_K}\right]$$

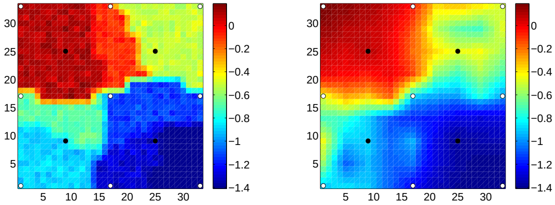
$$p(x) = c_1 E_1\big(I(x)\big) + c_2 E_2\big(I(x)\big)$$

$$I(x) = \sum_{k=1}^{K} a_k \theta_k(x)$$

$$\Rightarrow \quad \frac{\partial p}{\partial c_j} = E_j(I), \quad \frac{\partial p}{\partial a_k} = \frac{\partial p}{\partial I}\frac{\partial I}{\partial a_k}$$

Numerical Results

Smooth Representation



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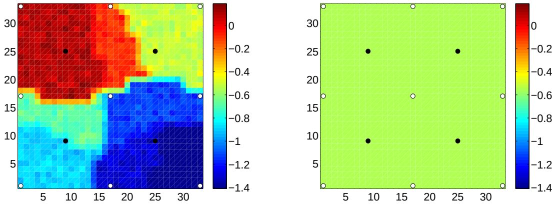

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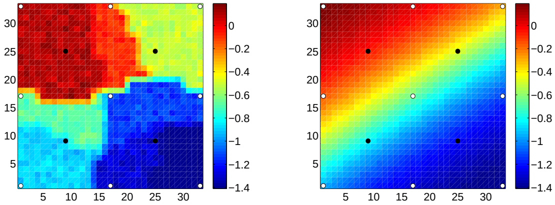

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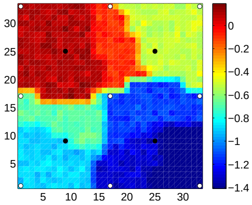

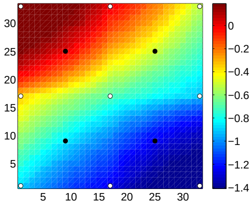

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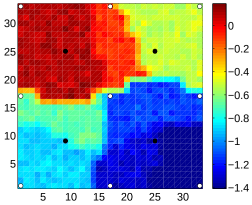

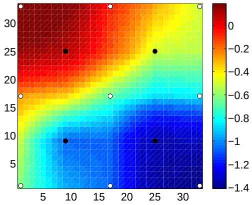

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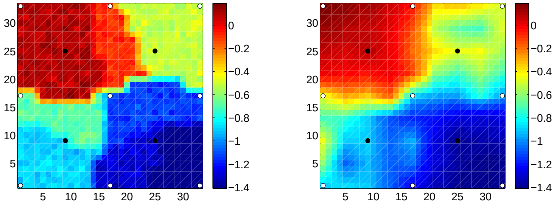

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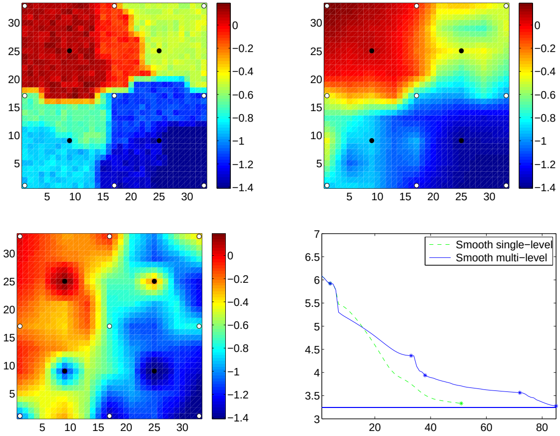

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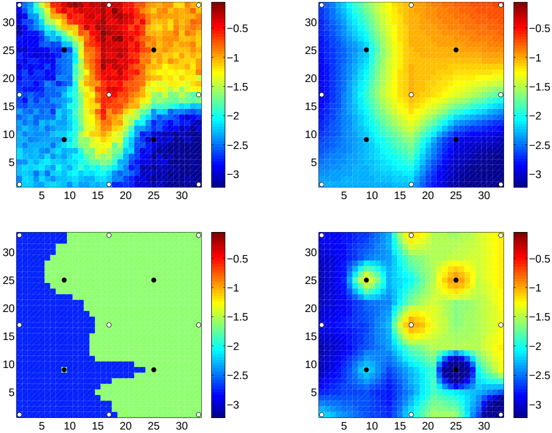
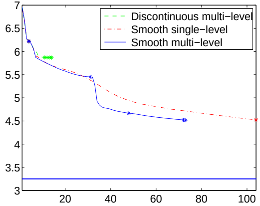
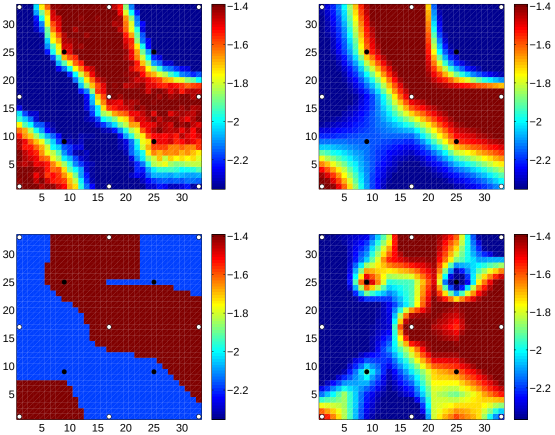
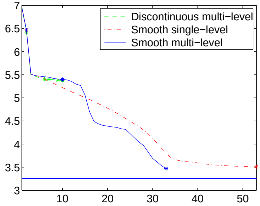




Numerical Results

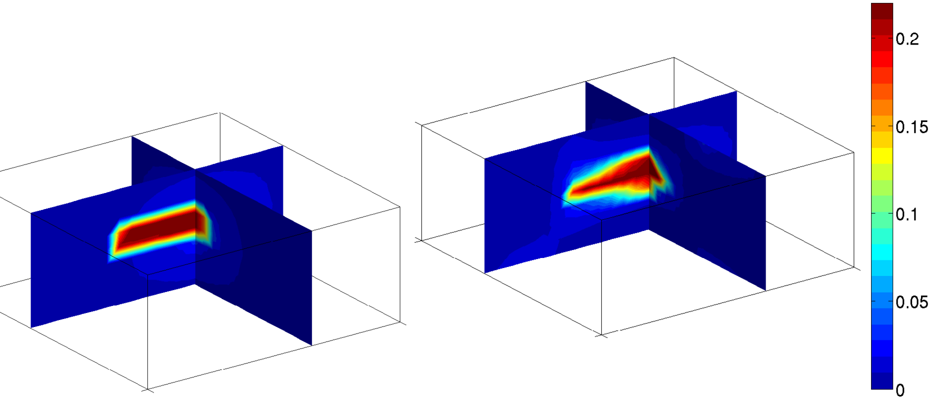
Smooth Representation





Numerical Results

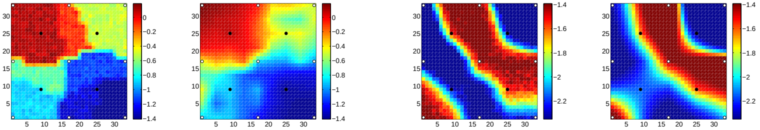
Estimation of electric conductivity based on low frequency electromagnetic data



25% noise

Conclusions

- The composite representation facilitates representation of smooth as well as sharp transitions between region of nearly constant parameter value with a low number of coefficients in the representation
- Gradual refinement stabilize the estimation and avoids local minima in parameter space
- Based on spatially sparsely distributed data, the method has the potential of recovering coarse permeability variations



Future work

Gradient/Sensitivity-based refinement

Objective function:

$$J(a,c) = [d - m(a,c)]^T C^{-1} [d - m(a,c)]$$

Compare

Gradient (magnitude of steepest descent)

$$\nabla J_a = \nabla_I J \frac{\partial I}{\partial a},$$

or approximation to objective function based on a linearized model response

$$J(a,c) \approx [d - m(a,c)]^T (C^{-1} - [C^{-1} A (A^T C A)^{-1} A^T C^{-1}])[d - m(a,c)]$$

for different choices of $\{\theta_k(x)\}_{k=1}^K$.

Testing of adaptive refinement

Linear expansion representation

Gaussian radial basis functions

$$p(x) = \Sigma_{j=1}^{N} c_j \Psi_j(x), \quad \Psi_j(x) = \exp(-\frac{1}{2\sigma_j^2}(x - x_j)^2)$$