

Robust Passivity Enforcement of Frequency Dependent Transmission Line Models

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Abstract

Passivity must be enforced on rational macromodels in order to ensure a stable time domain simulation. This paper investigates procedures for enforcing passivity for transmission line models based on the Method of Characteristics. Adding a conductive correction term externally to the line terminals is shown to be a robust and simple procedure for ensuring a passive model. The perturbation of the line model behavior is reduced by shaping the correction term using a low order rational function.

Introduction

The modeling of linear devices and systems by rational macromodels has reached considerable interest, due to the potential high accuracy of the resulting model. The modeling can be done from tabulated data in either the frequency domain or the time domain, resulting in a model on state-space form or pole-residue form that can be easily included in most simulation programs. Although the modeling process is fairly straightforward by usage of robust fitting techniques such as vector fitting [1], passivity should also be enforced in order to ensure a stable simulation. This is usually achieved by a postprocessing step where the model parameters are perturbed [2],[3],[4]. Techniques are also available for passivity checking via algebraic tests [4].

This paper focuses on passivity enforcement for transmission line models based on the Method of Characteristics (MoC). The previous techniques for passivity enforcement are not directly applicable to MoC based models since the latter makes use of delay terms. For that purpose, an algorithm is introduced that adds a rational correction term externally to the line terminals (ports). The approach is demonstrated for use with the so-called Universal Line Model (ULM) [6] as implemented in the PSCAD simulation environment [7].

The Universal Line Model

In the simulation of electromagnetic transients in power systems, the Universal Line Model (ULM) is frequently applied. This is a direct formulation of the MoC in phase coordinates, without assumption of a real transformation matrix. The main strength of this approach is the ability to accurately simulate situations where the propagation function has small but important off-diagonal elements. This is sometimes the situation in EMC problems such as undesired coupling between a power line and telecommunication line.

Consider a line with ends k and m . In the frequency domain, the voltage-current relation at end k can be written

$$\mathbf{i}_k = \mathbf{Y}_c \mathbf{v}_k - 2\mathbf{i}_{ki} \quad (1)$$

$$\mathbf{i}_{ki} = \mathbf{H}^T \mathbf{i}_{mr} \quad (2)$$

where i and r denote incident and reflected wave.

Matrices \mathbf{H} and \mathbf{Y}_c are respectively the matrices of propagation characteristic admittance. They are obtained directly from the p.u.l. series impedance matrix \mathbf{Z} and shunt admittance matrix \mathbf{Y} , and the line length l

$$\mathbf{Y}_c = \mathbf{Z}^{-1} \sqrt{\mathbf{Z}\mathbf{Y}} \quad (3)$$

$$\mathbf{H} = e^{-\sqrt{\mathbf{Z}\mathbf{Y}}l} \quad (4)$$

In the ULM [6], the rational fitting for \mathbf{Y}_c and \mathbf{H} are on the form

$$\mathbf{Y}_c \cong \sum_{m=1}^N \frac{\mathbf{R}_m}{s - a_m} + \mathbf{D} \quad (5)$$

$$\mathbf{H} \cong \sum_{g=1}^G \left(\sum_{m=1}^{N_g} \frac{\mathbf{R}_{m,g}}{s - a_{m,g}} \right) e^{-s\tau_g} \quad (6)$$

Poles and delays for the fitting of \mathbf{H} in (6) are precalculated by fitting the modes h_i of \mathbf{H} . (Nearly equal modes are lumped before doing the fitting, thereby reducing the order of the final model).

$$h_i \cong \sum_{m=1}^N \frac{r_m}{s - a_{m,i}} e^{-s\tau_i}, \quad i = 1, \dots, n \quad (7)$$

Nodal Admittance Matrix And Passivity

The admittance matrix with respect to the line terminals can be calculated directly from the rational approximations for \mathbf{H} and \mathbf{Y}_c as follows [8]

$$\mathbf{Y}_n = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \quad (8)$$

where

$$\mathbf{A} = \mathbf{Y}_c (\mathbf{I} + \mathbf{H}^2) (\mathbf{I} - \mathbf{H}^2)^{-1} \quad (9)$$

$$\mathbf{B} = -2\mathbf{Y}_c \mathbf{H} (\mathbf{I} - \mathbf{H}^2)^{-1} \quad (10)$$

The model is passive if the nodal admittance stamp (8) is positive for all frequencies, i.e. [2],[9]

$$\text{eig}(\text{Re}\{\mathbf{Y}_n(s)\}) > 0 \quad \forall s, \quad s = j\omega \quad (11)$$

An approach for passivity checking based on algebraic tests is shown in [10].

Passivity Enforcement by External Correction Term

Large passivity violations can occur outside the fitting band. Most EMTP-type programs add a small shunt

conductance to \mathbf{Y} in order to ensure a physical (passive) behavior at low frequencies [11], see [12] for a recent discussion. With ULM, passivity violations can with short lines also occur at high frequencies. This problem is overcome by multiplying \mathbf{H} with a low-pass-filter [13].

One way of removing any remaining passivity violations is to add losses by connecting an external model to the line terminals. Using the Simplistic Approach in [2, Section VII], a correction matrix \mathbf{D}_{corr} is established incrementally in a frequency sweep over the interval(s) containing passivity violations. At the first frequency, \mathbf{D}_{corr} is initialized to zero. At each frequency, $\text{Re}\{\mathbf{Y}_n\} + \mathbf{D}_{corr}$ is separated into two modal decompositions that respectively contain the positive and negative eigenvalues.

$$\text{Re}\{\mathbf{Y}_n\} + \mathbf{D}_{corr} = \mathbf{T}\mathbf{\Lambda}\mathbf{T}^{-1} = \mathbf{T}\mathbf{\Lambda}_{pos}\mathbf{T}^{-1} + \mathbf{T}\mathbf{\Lambda}_{neg}\mathbf{T}^{-1} \quad (12)$$

Positive eigenvalues are ensured for $\text{Re}\{\mathbf{Y}_n\} + \mathbf{D}_{corr}$ at a given frequency by adding the negated value of the second term in (12) to \mathbf{D}_{corr} . This procedure ensures that all eigenvalues of the combined model are non-negative at all frequencies contained in the sweep. No iterations are needed since each update of \mathbf{D}_{corr} leads to higher losses at all frequencies. The procedure is illustrated in Fig. 1.

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Dcorr = 0
for k=1:Ns
  Re{Y(sk)} + Dcorr → TΛposT-1 + TΛnegT-1
  Dcorr := Dcorr - TΛnegT-1
end

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Fig. 1 Calculating conductive correction term

The resulting correction term is a nodal conductance matrix that gives an instantaneous coupling between all line terminals, see Fig. 2. The correction can be limited in bandwidth by multiplying with a suitable band-pass filter. If the passivity violations are located between frequencies $\{\omega_1, \omega_2\}$, it is proposed to use a correction

$$\mathbf{P}(s) = \mathbf{D}_{corr} \frac{-1.01 \cdot (-10\omega_2) \cdot s}{(s - (-0.1\omega_1))(s - (-10\omega_2))} \quad (13)$$

The two poles in (13) are respectively placed at one decade below and above ω_1 and ω_2 to ensure that the imaginary part of the band-pass filter is negligible between ω_1 and ω_2 . The factor 1.01 ensures that the real part of the band-pass filter is greater than unity at the boundaries ω_1 and ω_2 . For practical implementation in a time domain simulation program it may be advantageous to expand (13) into a sum of pole-residue terms

$$\mathbf{P}(s) = \frac{\mathbf{R}_1}{s - a_1} + \frac{\mathbf{R}_2}{s - a_2} \quad (14)$$

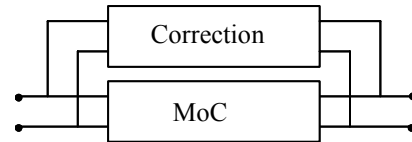


Fig. 2 Rational correction term (two-conductor line)

Example: Coaxial cable

As an example is used a homogenous two-conductor coaxial cable of 10 km length, buried 1 m below the surface of a lossy ground, see Fig. 3. The series impedance \mathbf{Z} is calculated including skin effects in conductors and earth [13].

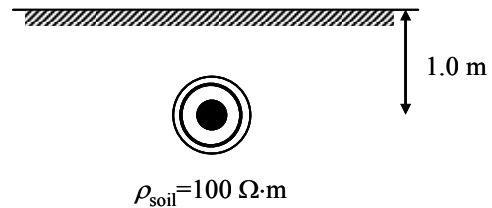


Fig. 3 132 kV cable

CABLE DATA	
Item	Property
Core	OD=39 mm, $\rho = 3.365E-8 \Omega\cdot\text{m}$
Insulation	$t=18.25$ mm, $\epsilon_r=2.85$
Sheath	$t=0.22$ mm, $\rho = 1.718E-8 \Omega\cdot\text{m}$
Jacket	$t=4.53$ mm, $\epsilon_r=2.51$

In the fitting process, eight poles are used for the fitting of \mathbf{Y}_c (5) and 12 poles are used for fitting each of the modes of \mathbf{H} (7). Out-of-band passivity violations at low frequencies are avoided by introducing an artificial shunt conductance at low frequencies such that a trapped voltage will discharge with a one second time constant. Out-of-band passivity violations at high frequencies are avoided by multiplying \mathbf{H} with a low-pass filter (10 MHz cut-off frequency) and asymptotic passivity is enforced for \mathbf{Y}_c [13].

Fig. 4 shows the rational fitting of \mathbf{Y}_c using an 8th order common pole set with weighting equal to the inverse magnitude of the matrix trace. The weighting is seen to ensure high relative accuracy at low frequencies where the matrix elements are small.

Fig. 5 shows the rational fitting of \mathbf{H} using a 12th order approximation for each of the two modes. It is seen that all elements are well fitted, including the small element that represents the induced sheath voltage.

Figs 6 and 7 (expanded view) show the resulting eigenvalues of $\text{Re}\{\mathbf{Y}_n\}$, calculated from the fitted model by (8)-(10). Some of the eigenvalues are negative in the frequency range 15.3 Hz–575 Hz, thereby making the line model non-passive.

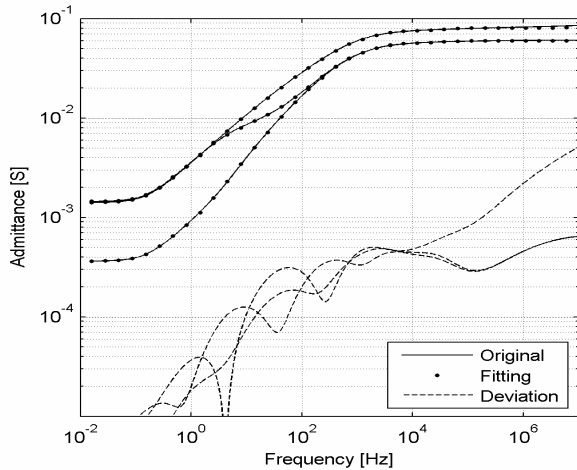


Fig. 4 Rational fitting of Y_c (8th order)

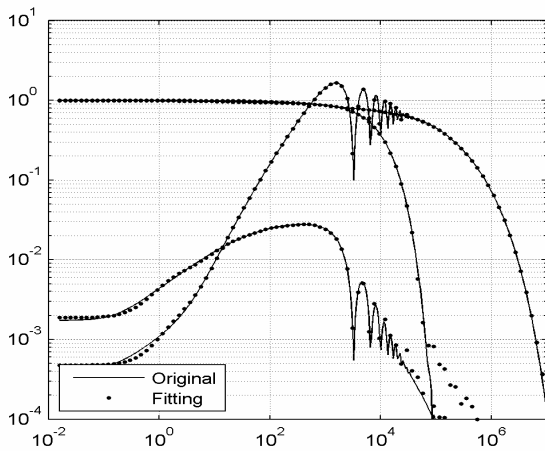


Fig. 5 Rational fitting of H (12th order per mode)

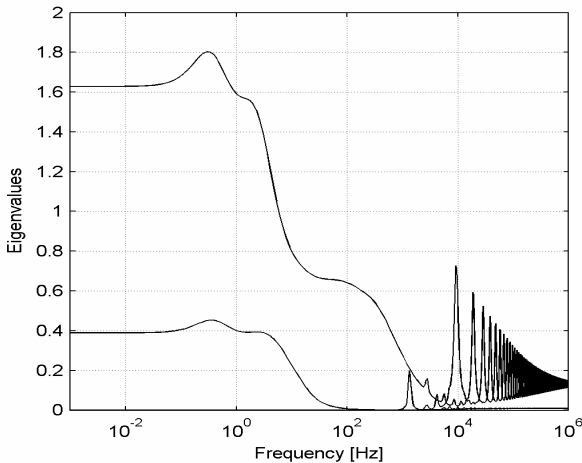


Fig. 6 Eigenvalues of $\text{Re}\{Y_n\}$.

A correction term (13) is calculated and added to the four line terminals. The result in Fig. 8 shows that the eigenvalues are modified to become positive at all frequencies, thereby giving a passive model. It is also seen that the eigenvalues are unaffected at very low frequencies due to the band pass function in (13).

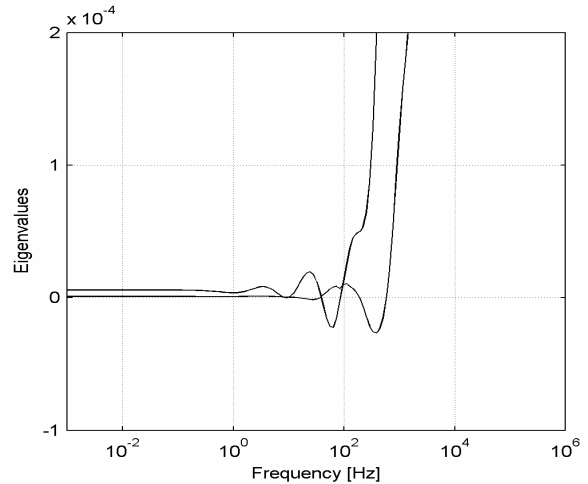


Fig. 7 Eigenvalues of $\text{Re}\{Y_n\}$. Expanded view

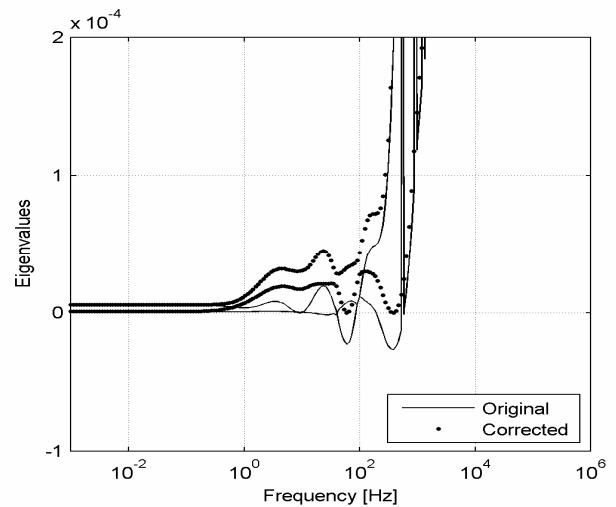


Fig. 8 Effect of external correction term on $\text{eig}(\text{Re}\{Y_n\})$

Time Domain Simulation

In the following example is used the obtained model in a transient simulation, see Fig. 9. The simulation is with the PSCAD software [7] with the correction term (14) included via a user-defined subroutine [14].

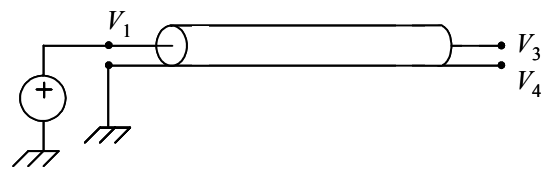


Fig. 9 Step voltage excitation

Figs 10 and 11 respectively show the simulated voltage at the far end core conductor and sheath conductor when the cable core conductor is subjected to a step voltage excitation. The sheath conductor is grounded at the sending end but is open at the far end. The simulated voltage is shown before and after adding the external correction term (13). In addition is shown the deviation between the two responses. It is seen that the effect of the correction term is insignificant. The relative impact is higher for the response in Fig. 11 since it is nearly two magnitudes smaller than the one in Fig. 10.

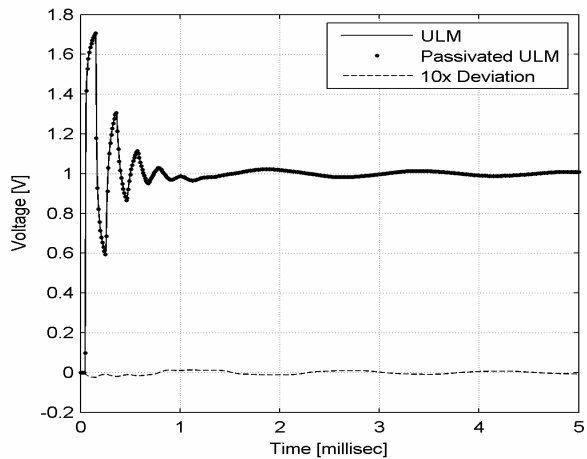


Fig. 10 Far end core voltage (V_3)

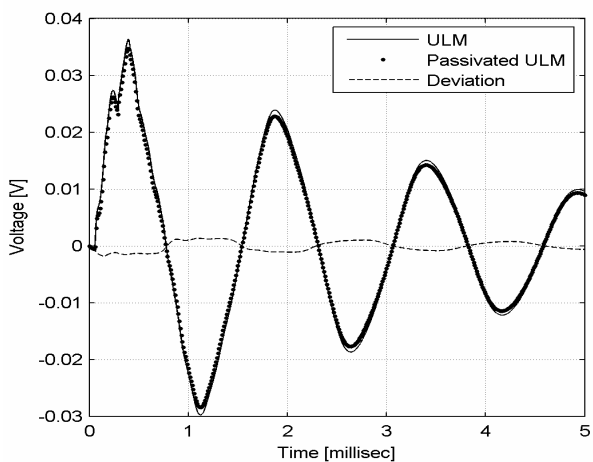


Fig. 11 Far end sheath voltage (V_4)

Discussion

The proposed approach for passivity enforcement is highly robust and has been successfully applied to a number of line cases, including a six-conductor cable system [13]. This method is fail-proof in the sense that it cannot diverge; a single sweep will guarantee passivity at the frequency samples contained in the sweep. A potential drawback is that the size of the perturbation is not minimal, but this should not be a problem as long as the needed correction is small, which is usually the case of in-band passivity violations. (In-band passivity violations generally decrease in size when increasing the fitting order.) An undesired side effect is that causality is violated due to the instantaneous coupling between the line ends. This coupling is however insignificant when the needed correction is small, see for instance Figs 6 and 7.

The passivity enforcement approach was in this work used together with the Universal Line Model, but it is equally well applicable to other MoC based models.

Conclusions

Passivity enforcement for MOC based transmission line models can be achieved by adding a rational correction term to the line terminals. The correction term is derived directly from the model nodal admittance matrix via a frequency sweep, giving a conductance matrix. The bandwidth of the

model perturbation is reduced by appropriate shaping of the correction term by a band pass function. Application to one case taken from power systems modeling showed that the passivity was achieved with a only a insignificant change to a transient response.

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