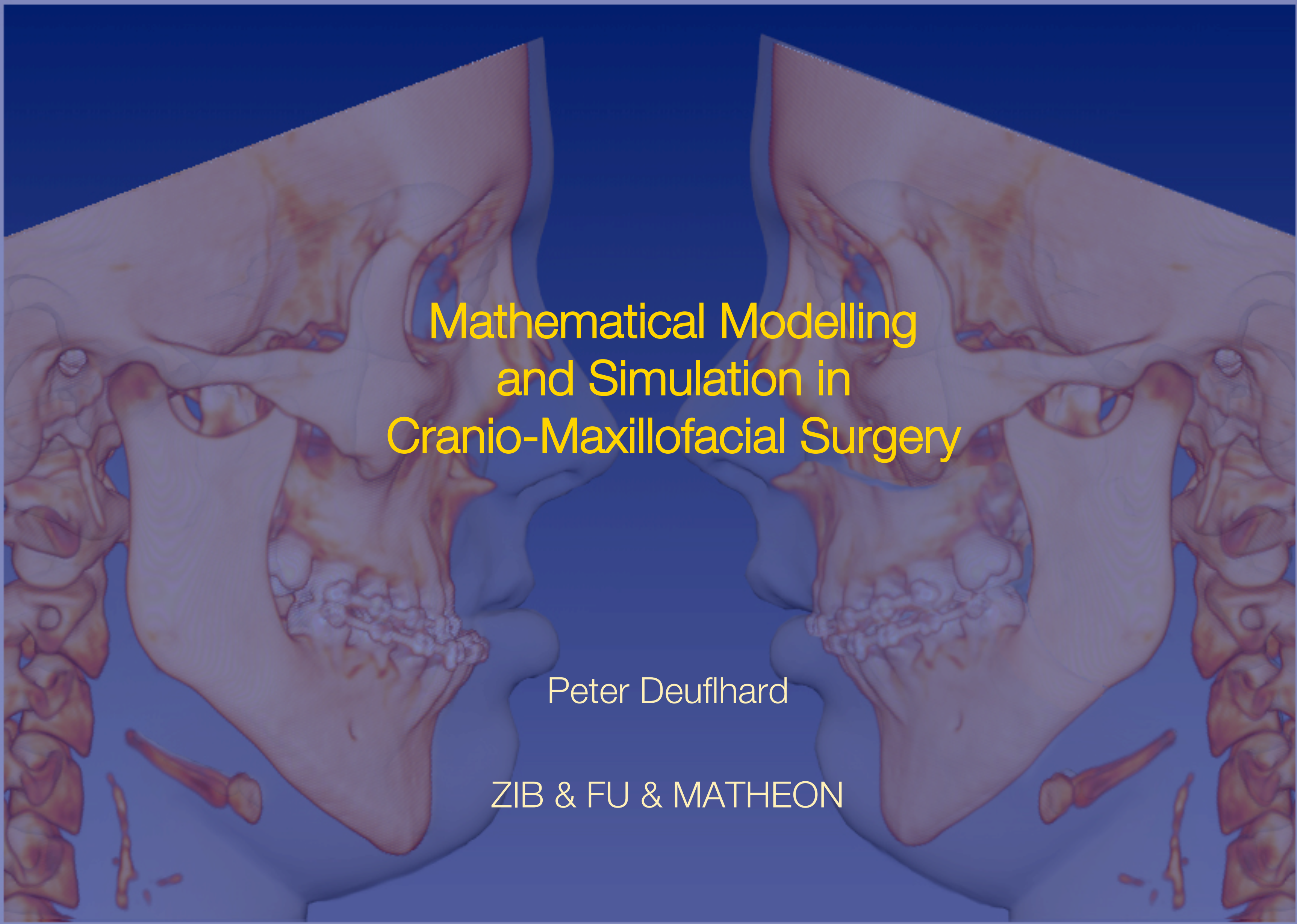
Mathematical Modelling and Simulation in Cranio-Maxillofacial Surgery

Peter Deuflhard

ZIB & FU & MATHEON



Imago animi vultus

Cicero, 106 - 43 v. Chr.

Mathematical Modelling and Simulation in Cranio-Maxillofacial Surgery

Peter Deuflhard

Zuse Institute Berlin (ZIB)

and

Freie Universität Berlin
Dept. Mathematics / Computer Science

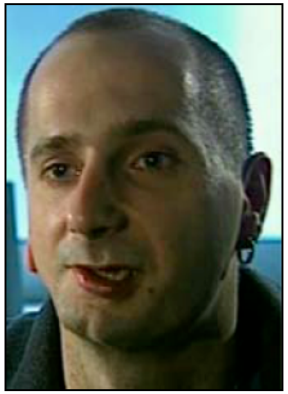
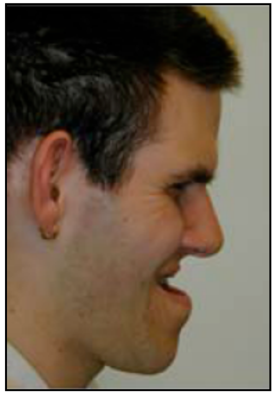
and

DFG Research Center MATHEON

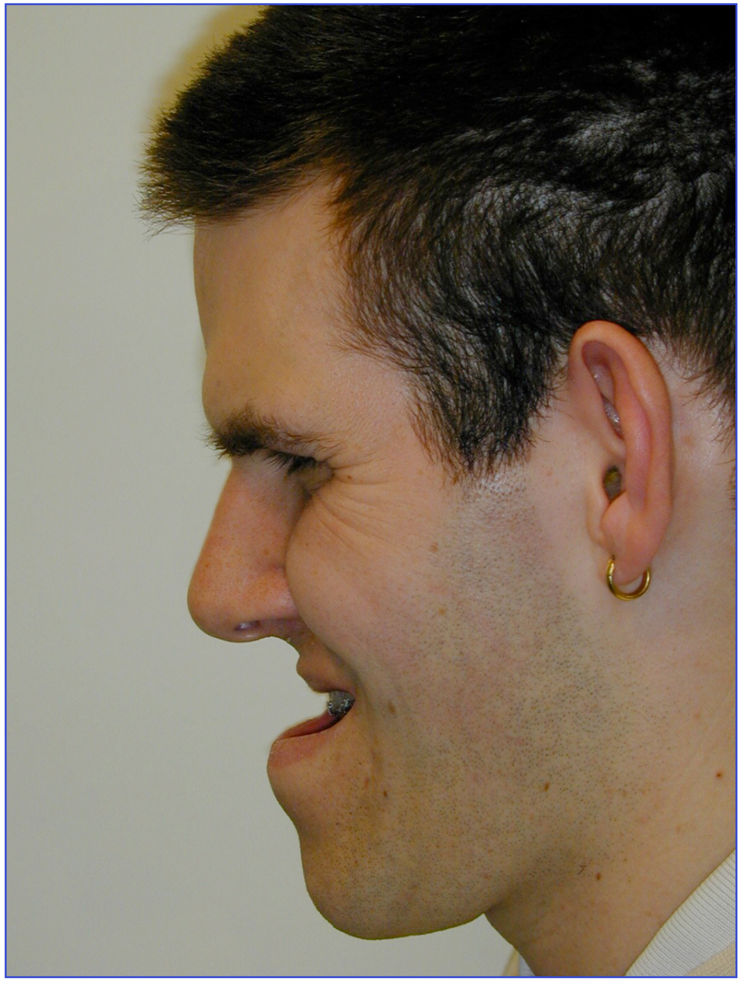
Cranio-maxillofacial surgery

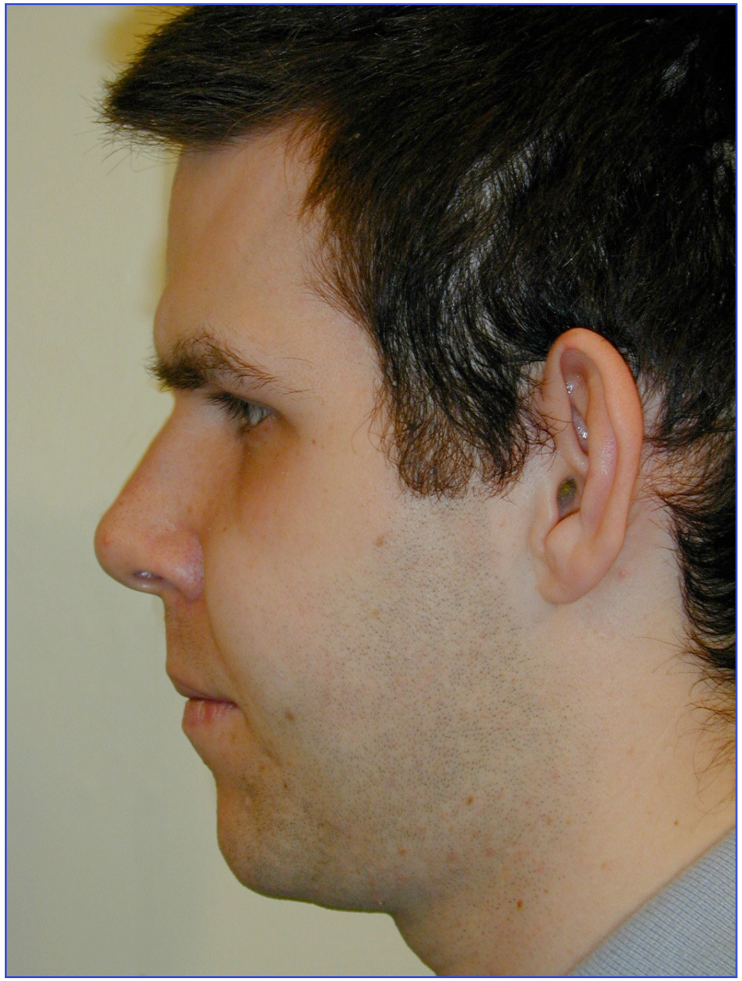
Facial distortions

Patient Bogumil (27)



Patient *before* operation



Patient *after* operation

Segmentation

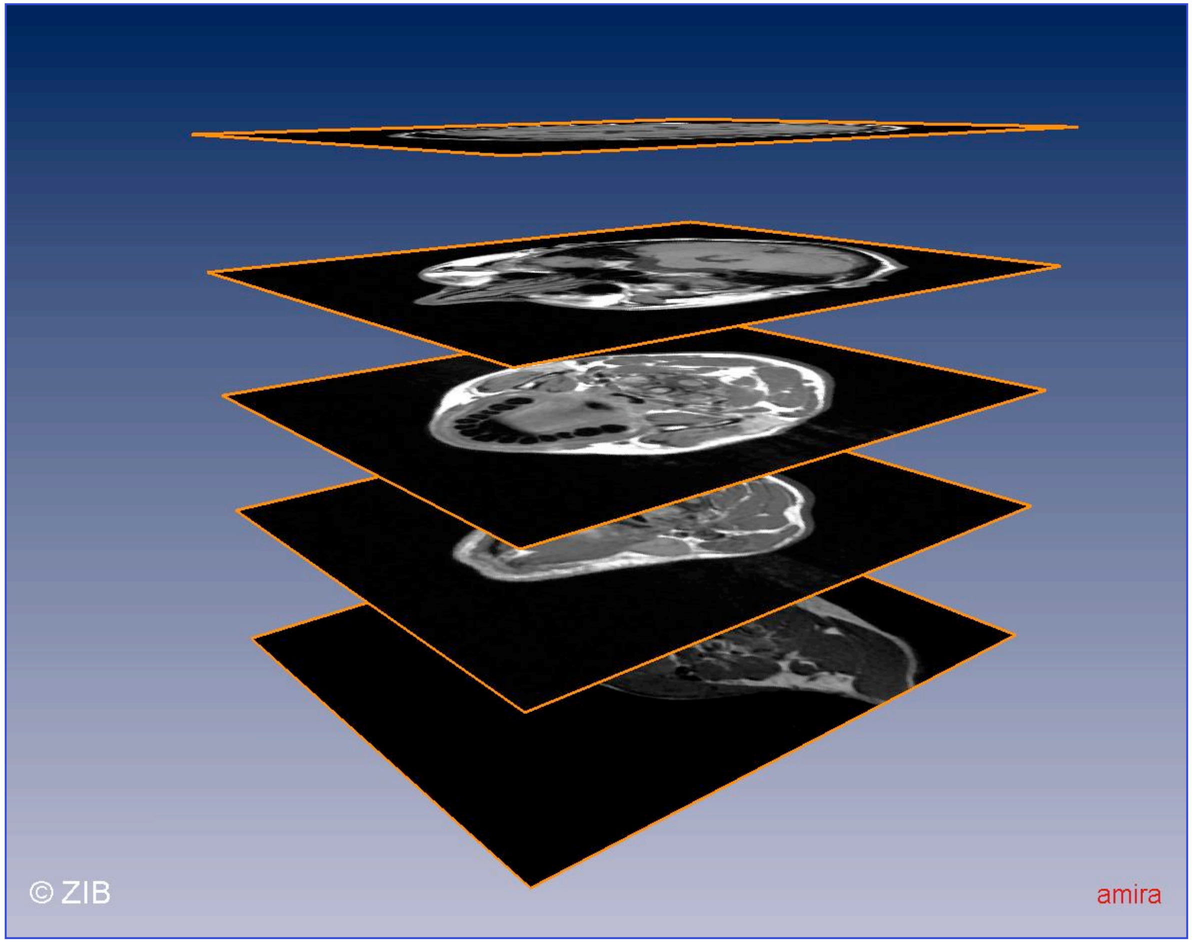
Operation planning

Soft tissue modelling

Affine conjugate Newton methods

Postoperative facial appearance

Input: 2D image stack (CT, MRT, ...)



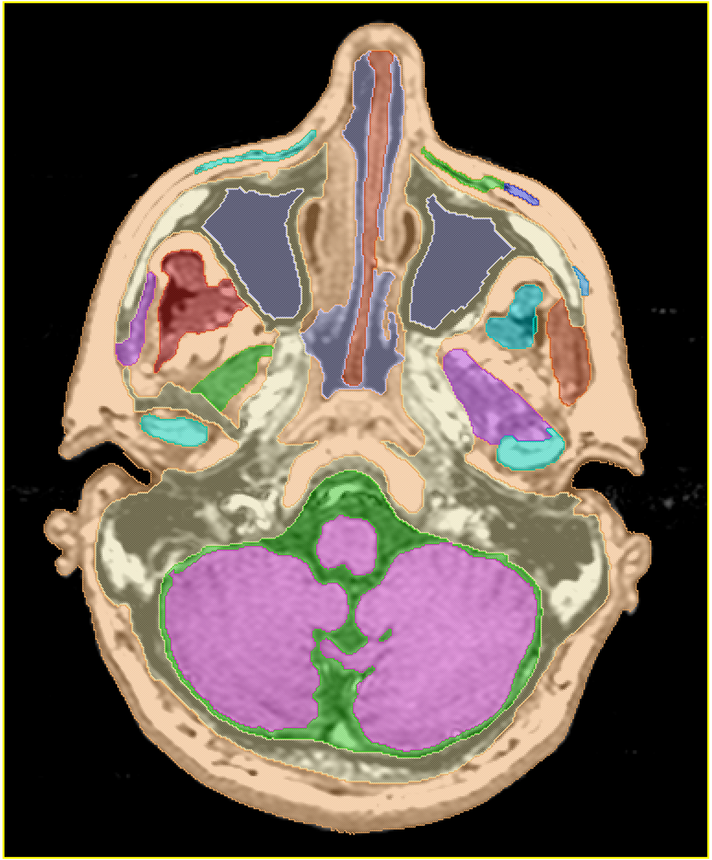
Density Image from CT, MRT, ...

Needed:

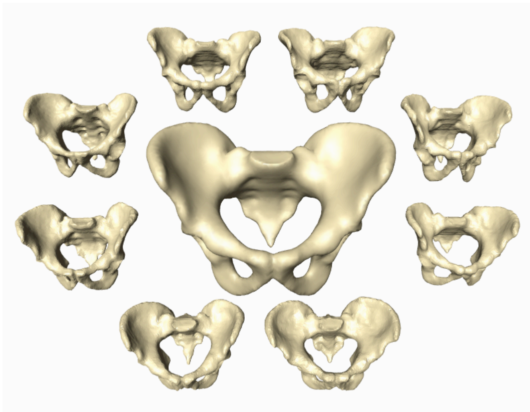
further material properties

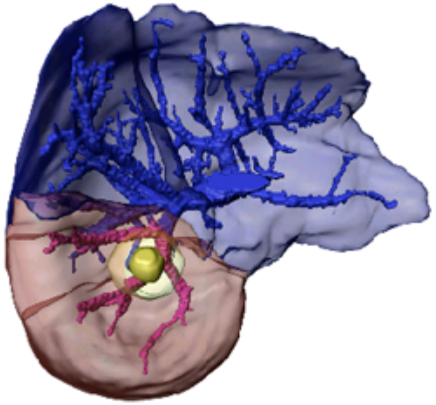
for virtual patient

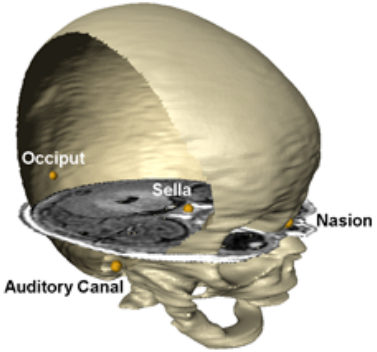
2D Segmentation

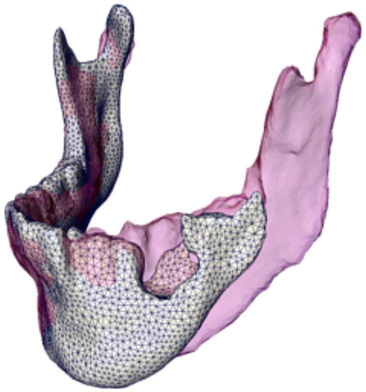


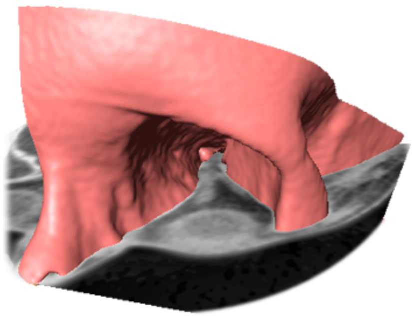
Statistical Shape Modelling

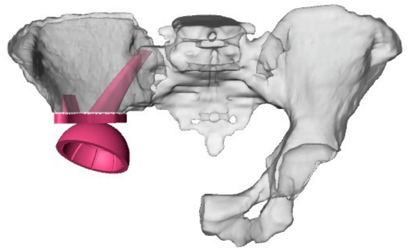






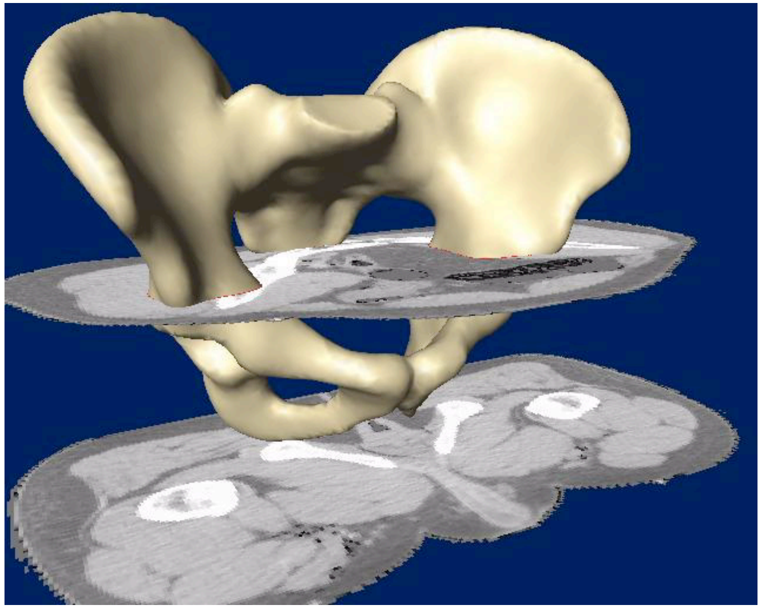


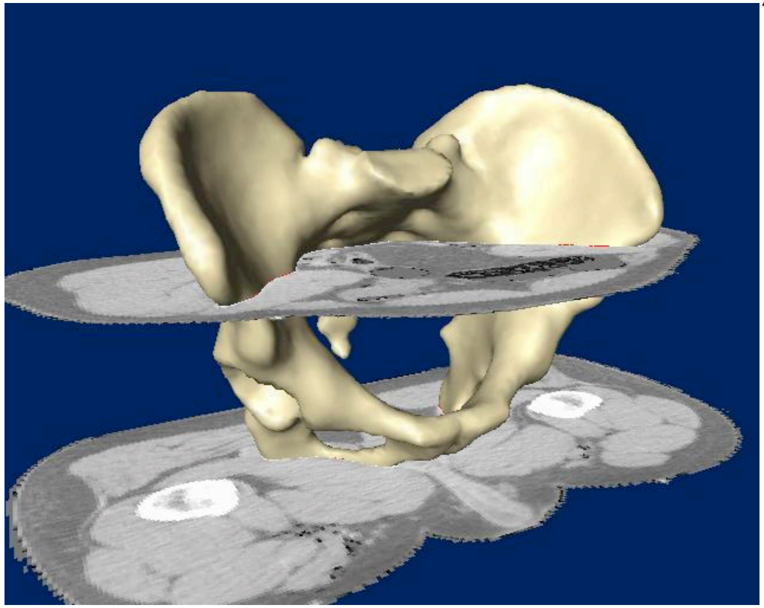




Lamecker, Hege, Deuflhard 2006

2D Segmentation via 3D





adaptation of new CT stack

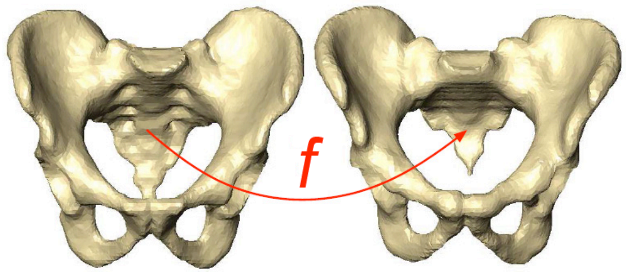
on the basis of known patient data

Variational Shape Matching

$$E[f] = \frac{1}{|\mathcal{M}|} \int_{\mathcal{M}} (\mu\rho(\lambda) + (1-\mu)\chi) \, d\mathcal{M}$$

$$\rho(\lambda) = \lambda \frac{(\Gamma - \gamma)^2}{\det(C)} + (1-\lambda) \frac{(\Gamma\gamma - 1)^2}{\det(C)}$$

$$\chi = \int_0^{2\pi} |II_1(X_\varphi) - II_2(df(X_\varphi))| \, d\varphi$$



Deuflhard, Lamecker, Wardetzky, Polthier 2006

Segmentation

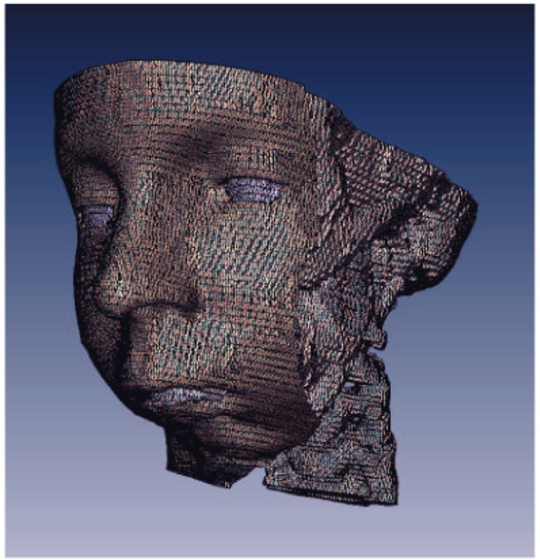
Operation planning

Soft tissue modelling

Affine conjugate Newton methods

Postoperative facial appearance

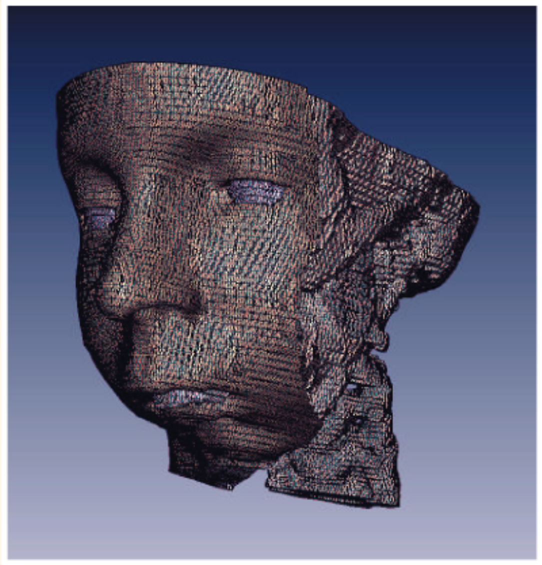
3D Grid generation



Interface grid (triangles)

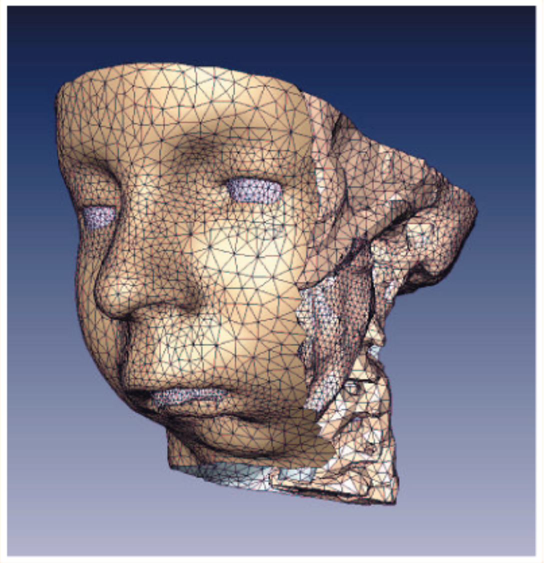
GMC method, ZIB

3D Grid generation



Interface grid (triangles)

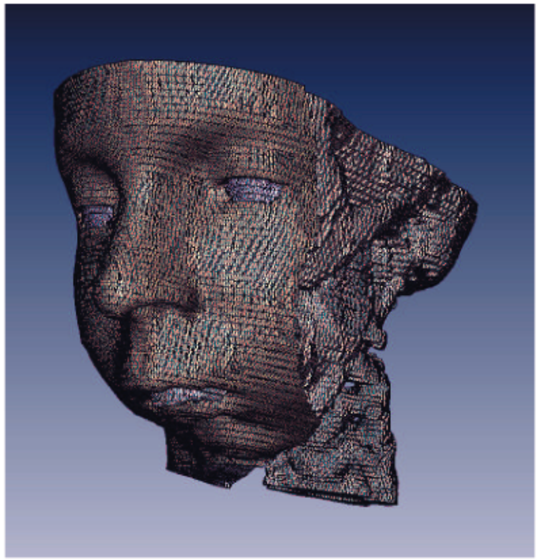
GMC method, ZIB



Grid coarsening (triangles)

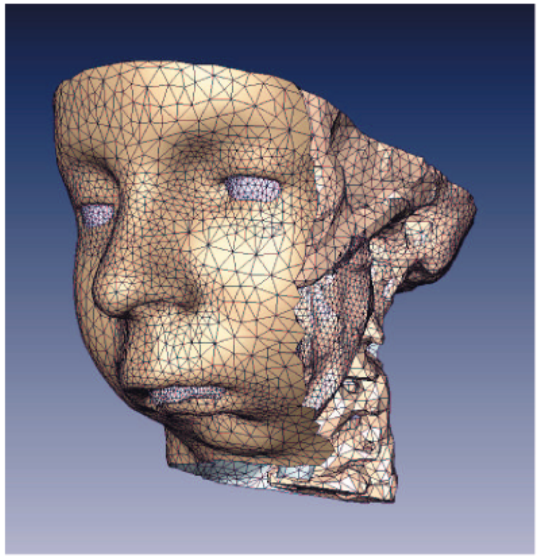
curvature dependent method, ZIB

3D Grid generation



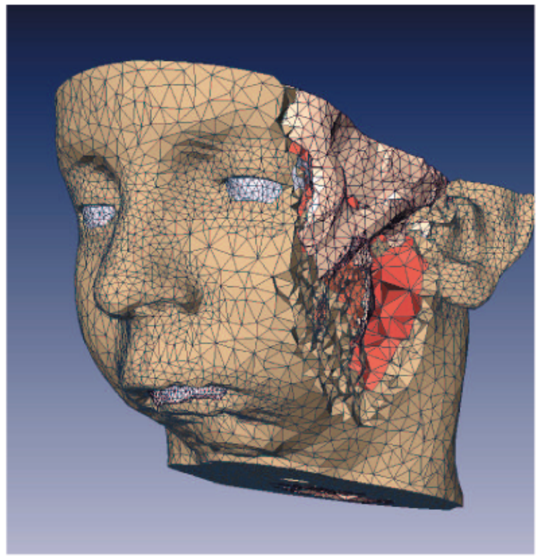
Interface grid (triangles)

GMC method, ZIB



Grid coarsening (triangles)

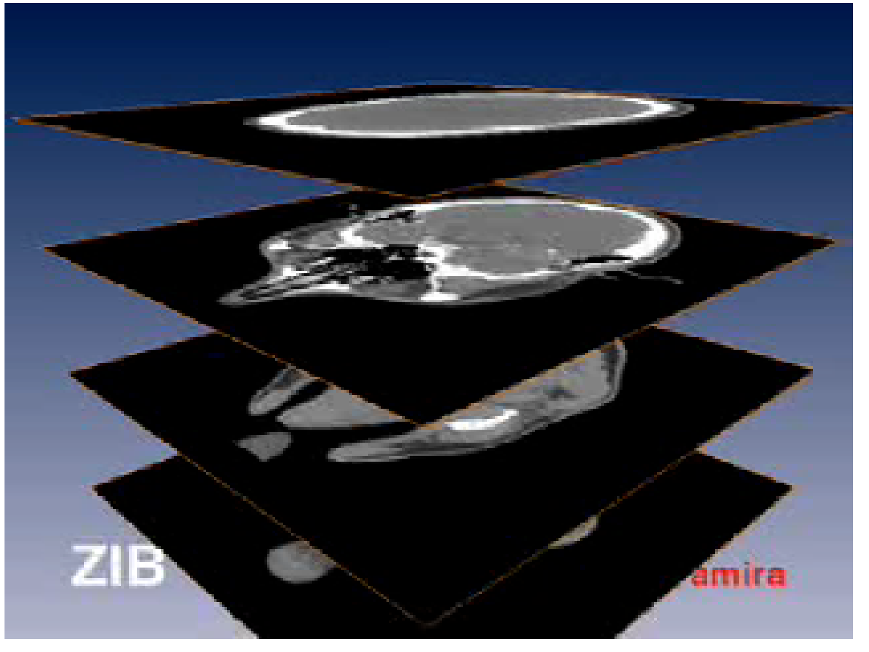
curvature dependent method, ZIB



Volume grid (tetrahedrals)

advancing front method, ZIB

Virtual head (Bogumil)



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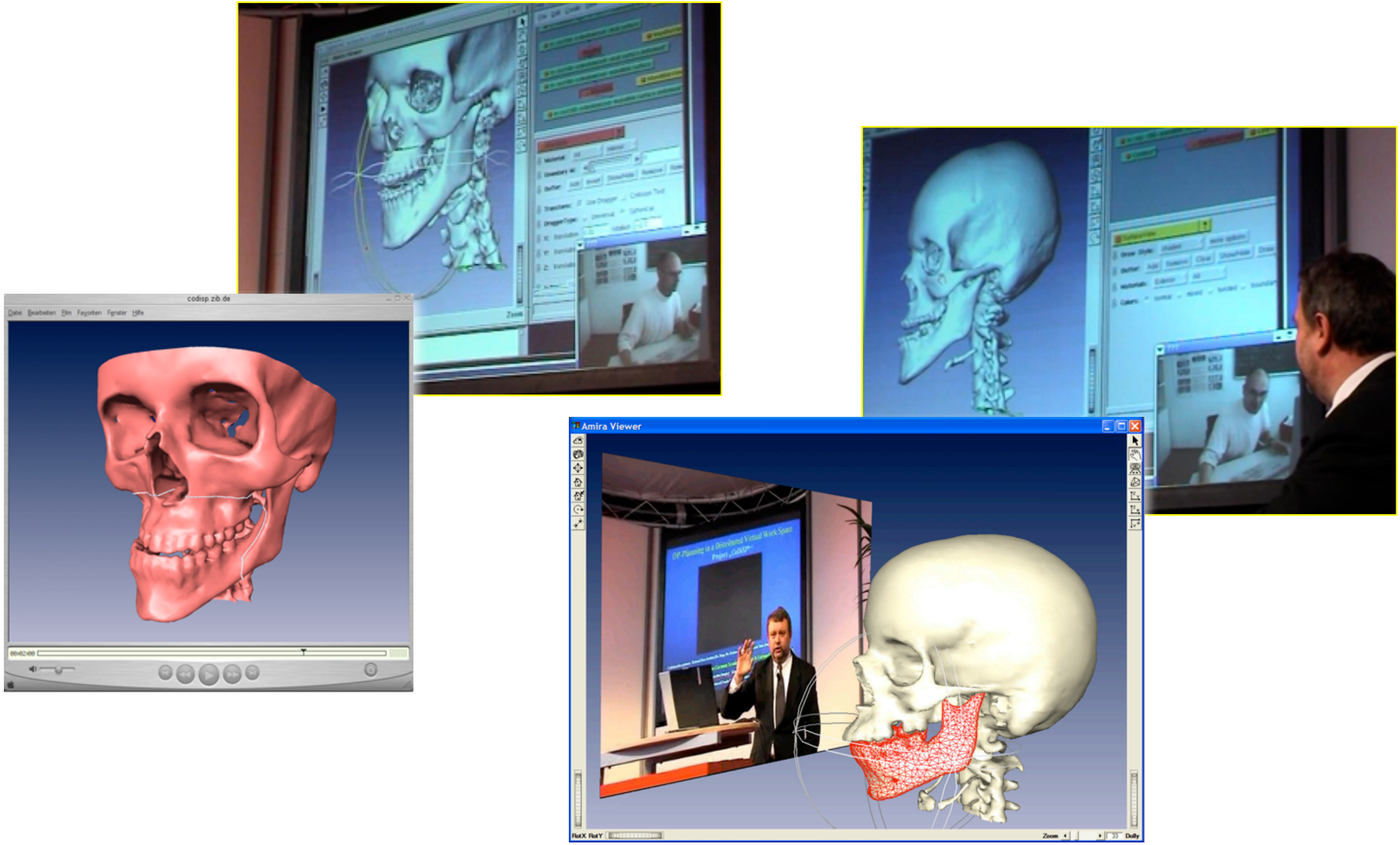
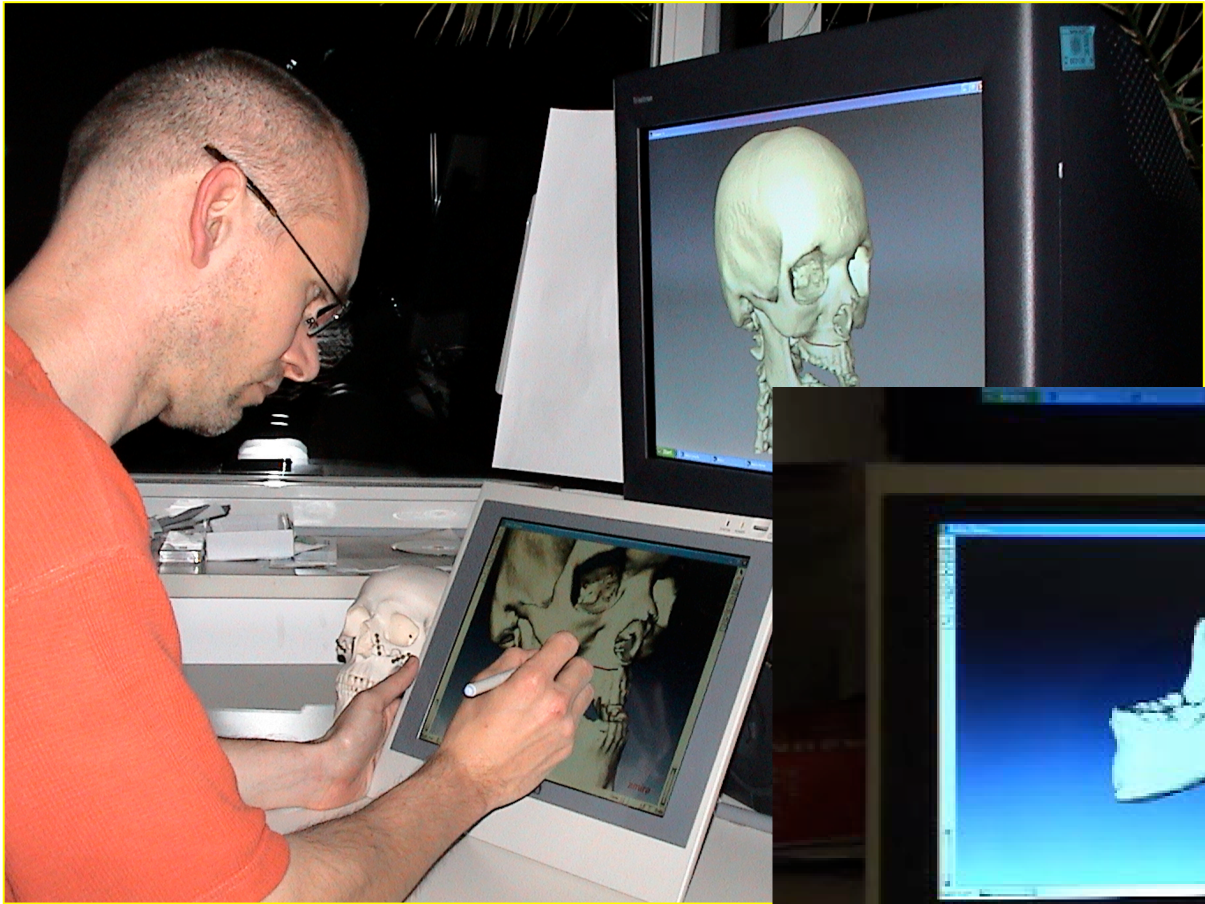
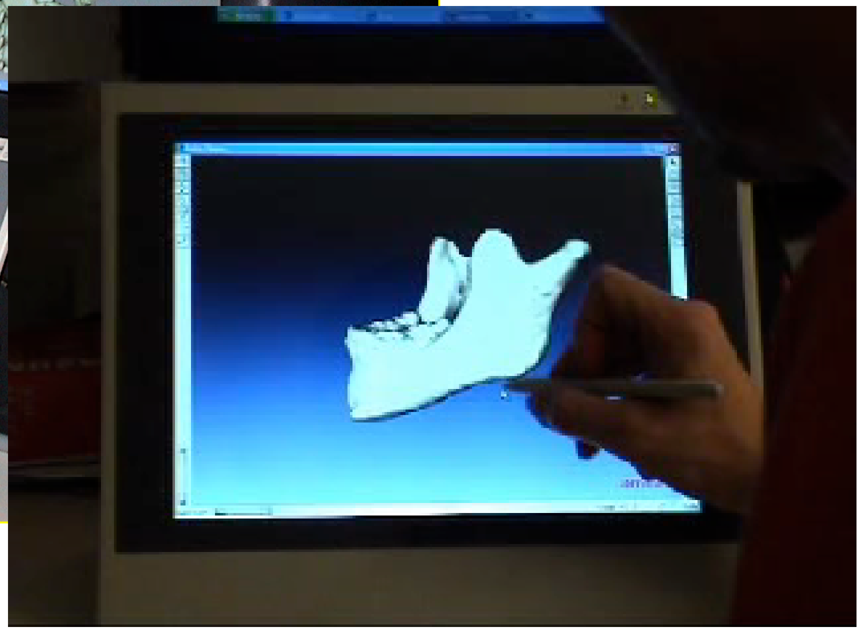
Tele-conference clinics / ZIB



Osteotomy planning at ZIB





Cranio-maxillofacial surgery

Segmentation

Operation planning

Soft tissue modelling

Affine conjugate Newton methods

Postoperative facial appearance

Biomechanical model

Tissue deformation

$$\begin{aligned} \Phi : \Omega &\longrightarrow \mathbf{R}^3 \\ \Omega &\subset \mathbf{R}^3 \\ \Omega' &\subset \mathbf{R}^3 \end{aligned}$$

Dirichlet interface

$$\Gamma_D \subset \partial\Omega$$

Stored energy to be minimized:

$$f(\phi) = \int_\Omega W(\nabla\phi)dx$$

Biomechanical linear model

Linear St. Venant-Kirchhoff material

$$W(\nabla\phi) = \frac{\lambda}{2}(\operatorname{tr}\varepsilon)^2 + \mu \operatorname{tr}\varepsilon^2$$

Linearized Green-Lagrange strain tensor

$$\varepsilon = \frac{1}{2}(\nabla u^T + \nabla u)$$

(not rotation invariant)

Biomechanical linear model – PDE

Lamé-Navier equation

$$
\begin{aligned}
-2\mu \operatorname{div} \varepsilon - \lambda \nabla \operatorname{div} u &= 0 && \text{in } \Omega \\
u &= u_0 && \text{on } \Gamma_D \\
2\mu\varepsilon + \lambda \operatorname{tr} \varepsilon I &= 0 && \text{on } \partial\Omega \backslash \Gamma_D
\end{aligned}
$$

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Biomechanical nonlinear model

Nonconvex material law of Ogden type

$$\begin{aligned} W(\nabla\phi) &= W(I+\nabla u) \\ &= a\,\mathrm{tr}\,E + b(\mathrm{tr}\,E)^2 + c\,\mathrm{tr}\,E^2 + d\Gamma(\det(I+\nabla u)) \end{aligned}$$

Full Green-Lagrange strain tensor

$$E = \tfrac{1}{2}(\nabla u^T + \nabla u + \nabla u^T \nabla u)$$

$$\begin{aligned} &A = -d\Gamma'(1), \qquad b = \tfrac{1}{2}(\lambda - d(\Gamma'(1) + \Gamma''(1))), \\ &c = \mu + d\Gamma'(1), \quad \Gamma(s) = s^2 - \ln s \end{aligned}$$

$d \to 0^+$: linear model

Biomechanical model – material characterization

Muscle $\quad \mathrm{E}_m \geq 300\mathrm{kPa}, \quad \nu_m \approx 0.44$

Soft tissue $\quad \mathrm{E}_s \leq 50\mathrm{kPa}, \quad \nu_s \approx 0.46$

For $\quad \nu < 0.5:$

$$\lambda = \frac{\mathrm{E}\nu}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{\mathrm{E}}{2(1-\nu)}$$

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Postoperative facial appearance

Convex minimization

Convex optimization

$$f(x) = \min, \qquad f : D \subset X \to \mathbb{R}$$
$$F(x) = \operatorname{grad} f(x) = f'(x)^* = 0, \quad x \in D$$

Affine transformation

$$g(y) = f(By) = \min$$
$$G(y) = B^* F(By) = 0$$
$$G'(y) = B^* F'(x) B$$

Affine conjugate Lipschitz condition

$$\|F'(x)^{-1/2}(F'(\bar{x}) - F'(x))(\bar{x} - x)\| \leq \omega \|F'(x)^{1/2}(\bar{x} - x)\|^2$$

Deuflhard, Weiser 1997/98   Deuflhard 2004/06

Affine conjugate exact Newton algorithm

$$f(x^{k+1}) \leq f(x^k)$$

Convergence analysis

$$f(x^k + \lambda \Delta x^k) \leq f(x^k) + \lambda \langle F(x^k), \Delta x^k \rangle + \frac{\lambda^2}{2}\epsilon_k + \frac{\lambda^3}{6}\omega \epsilon_k^{3/2} =: t(\lambda;\omega)$$

$$\epsilon_k = \langle F'(x^k)\Delta x^k, \Delta x^k \rangle$$

$$\lambda_{\text{opt}} = \arg\min_{\lambda>0} t(\lambda;\omega) = \frac{1}{1+\sqrt{1+2\omega\sqrt{\epsilon_k}}} < 1$$

Adaptive trust region strategy

$$[\lambda_{\text{opt}}] = \arg\min_{\lambda>0} t(\lambda;\omega) = \frac{1}{1+\sqrt{1+2[\omega]\sqrt{\epsilon_k}}} \leq 1$$

$X = \mathbb{R}^N$: NLEQ1-OPT

Affine conjugate inexact Newton algorithm

Inner iteration PCG $\|F'(x)^{1/2}(\delta x_i - \Delta x)\| = \min$

Threshold condition

$$\delta_k = \frac{\|F'(x^k)^{1/2}(\delta x^k - \Delta x^k)\|}{\|F'(x^k)^{1/2}\delta x^k\|} \leq \delta < 1$$

Convergence analysis

$$f(x^k + \lambda \delta x^k) \leq f(x^k) + \lambda \langle F(x^k), \delta x^k \rangle + \frac{\lambda^2}{2}\epsilon_k^\delta + \frac{\lambda^3}{6}\omega \epsilon_k^{\delta^{3/2}}$$

$$\epsilon_k^\delta = \langle F'(x^k)\delta x^k, \delta x^k \rangle = \frac{\epsilon_k}{1+\delta_k^2}$$

$X = \mathbb{R}^N, N$: "large": Code GIANT-PCG

Affine conjugate function space Newton algorithm

Function space -----> adaptive multilevel approach

infinite dimension -----> variable dimension

Inexact Newton frame:

inner iteration errors -----> discretization errors

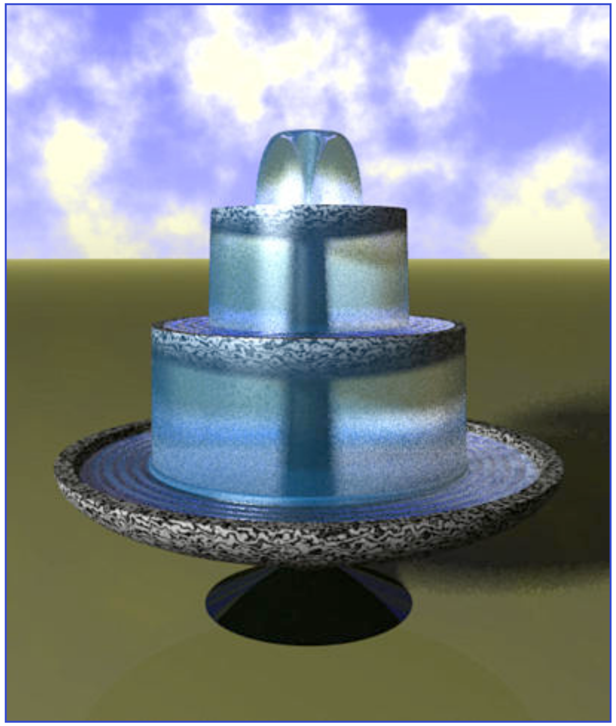
Affine conjugate adaptive multilevel FEM for nonlinear elliptic PDEs:

$X = W_p^1, \quad 1 < p < \infty$: Code NEWTON-KASKADE

Newton-KASKADE

Inner loop:

Cascadic multigrid method
with CCG as inner iteration.



Face:

90.000 - 300.00 unknowns

geometrically nonlinear model

Newton method for nonlinear mechanical model

$$F'(x_k) = M + N(x_k), \quad M = F'(0)$$

$$N(x) = \mathcal{O}(\|\nabla x\|) \text{ for } \|\nabla x\| \to 0$$

Global energy norm : $\|\cdot\|_M = \|M^{1/2} \cdot \|$

Affine conjugate Lipschitz condition :

$$\|M^{-1/2}(F'(y) - F'(x))(y - x)\| \leq \omega \|M^{1/2}(y - x)\|^2$$

Weiser, Deuflhard, Erdmann 2004

Nonconvex (polyconvex) exact minimization

$$f(x^{k+1}) \leq f(x^k)$$

Convergence analysis

$$\begin{aligned} f(x^k + \Delta x^k) &\leq f(x^k) + \langle F(x^k), \Delta x^k \rangle + \tfrac{1}{2}\epsilon_k + \tfrac{\omega}{6} \|\Delta x^k\|_M^3 =: t(\Delta x^k; \omega) \\ \epsilon_k &= \langle F'(x^k)\Delta x^k, \Delta x^k \rangle \\ \Delta x_{\text{opt}}^k &= \arg\min_{\Delta x^k \in X} t(\Delta x^k; \omega) \end{aligned}$$

Adaptive trust region strategy

$$[\Delta x_{\text{opt}}^k] = \arg\min_{\Delta x^k \in X^k} t(\Delta x^k; [\omega])$$

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Nonconvex (polyconvex) inexact minimization

Inner iteration truncated PCG will produce wrong direction

$$\delta_k = \frac{\|F'(x^k)^{1/2}(\delta x^k - \Delta x^k)\|}{\|F'(x^k)^{1/2}\delta x^k\|} \le \delta < 1 \quad \text{or} \quad \langle F'(x^k)p_i^k, p_i^k\rangle \le 0$$

Convergence analysis

$$f(x^k + \lambda\delta x^k) \le f(x^k) + \langle F(x^k), \delta x^k\rangle + \tfrac{1}{2}\epsilon_k^\delta + \tfrac{1}{6}\omega\|\delta x^k\|_M^3$$
$$\epsilon_k^\delta = \langle F'(x^k)\delta x^k, \delta x^k\rangle$$
$$\delta x_{\text{opt}}^k = \arg\min_{\delta x^k \in X_k} t(\delta x^k;\omega)$$
$$X_k \subset X, \ \dim X_k \ll \dim X$$

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Nonconvex (polyconvex) function space minimization

Function space -----> adaptive multilevel approach

infinite dimension -----> variable dimension

Inexact Newton frame:

inner iteration errors -----> discretization errors

Affine conjugacy via global energy norm (M)

$$X = W_p^1,\ 1 < p < \infty : \text{ Variant of code NEWTON-KASKADE}$$

Segmentation

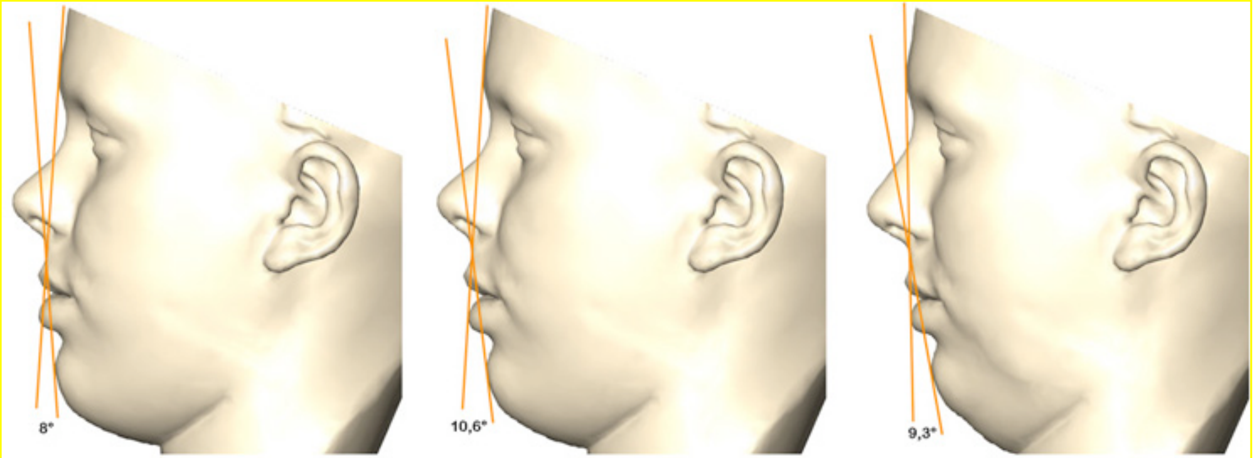
Operation planning

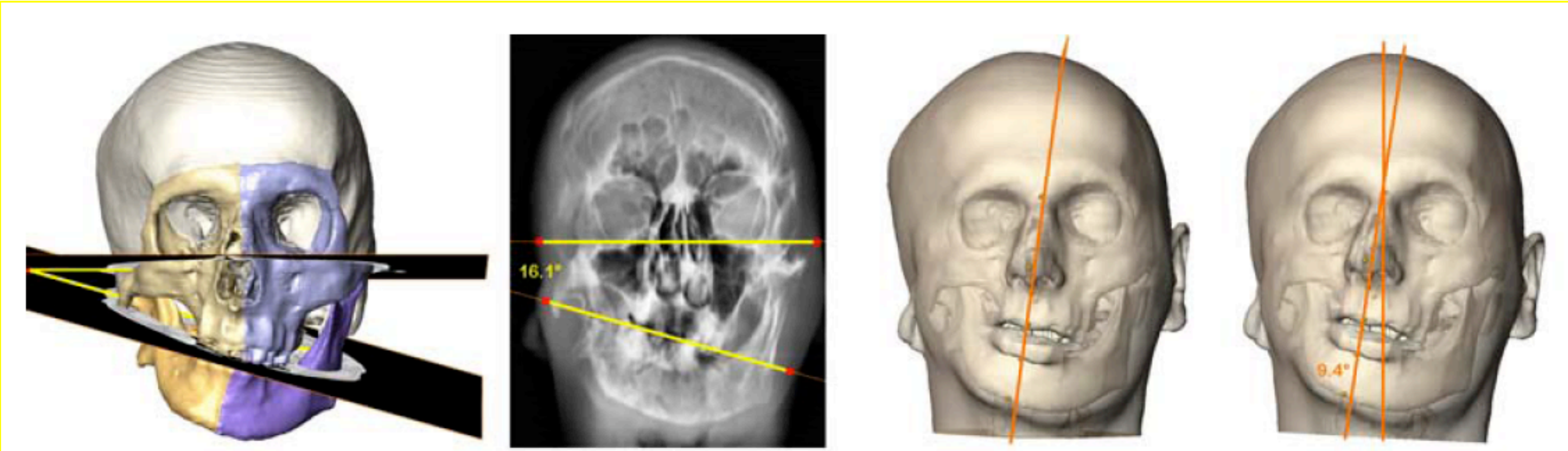
Soft tissue modelling

Affine conjugate Newton methods

Postoperative facial appearance

Patient specific proportion analysis





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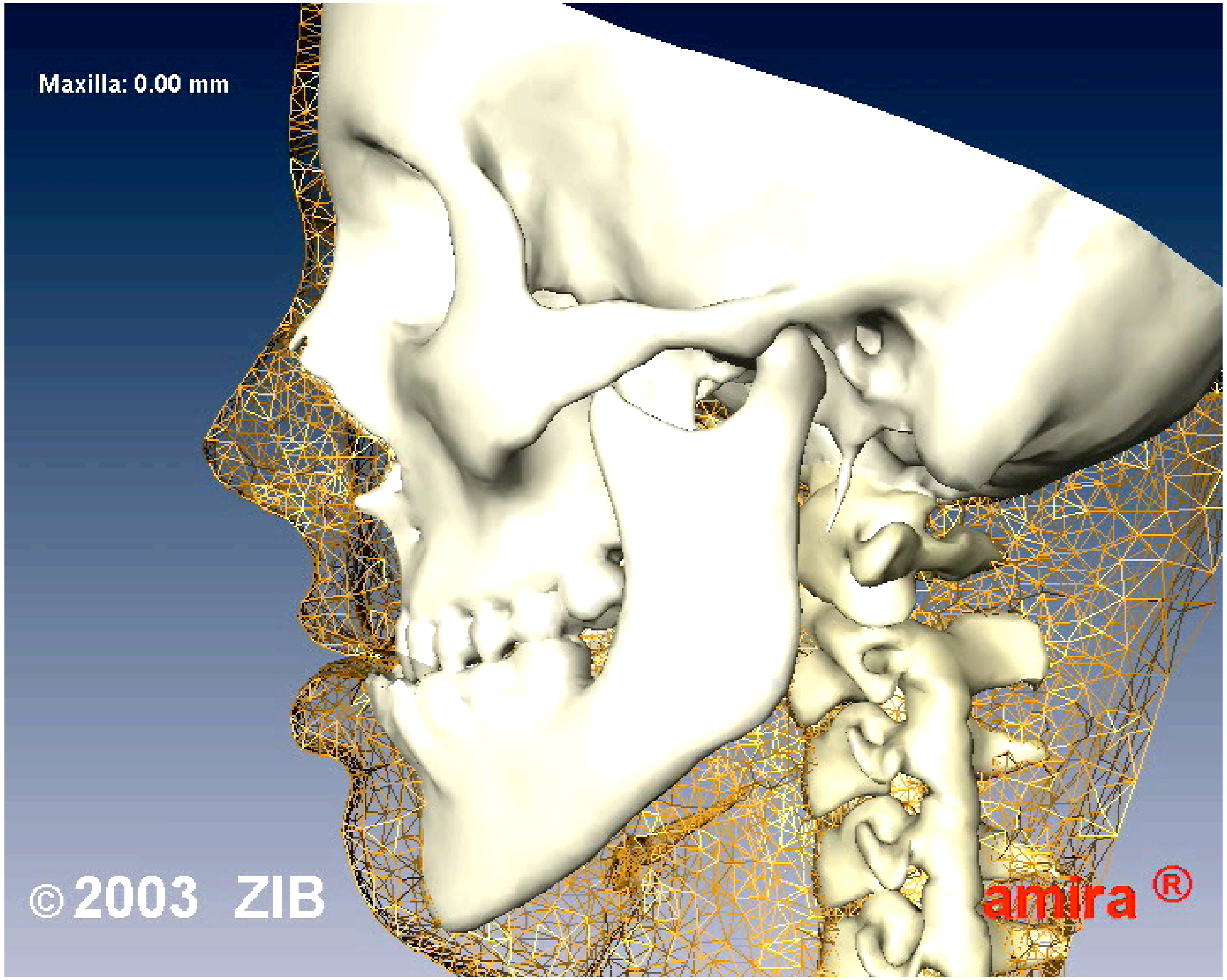
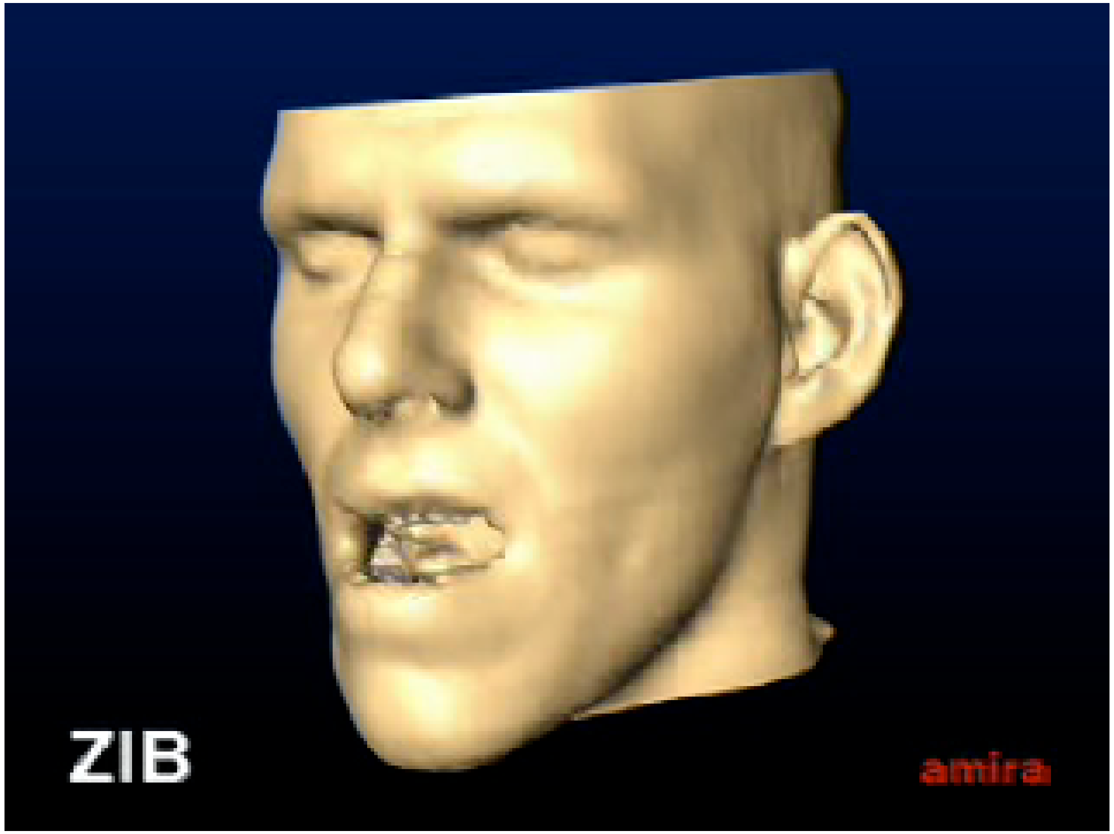
Operation planning: bone and grid movement



Zachow 2003

Operation planning: Bogumil (27)

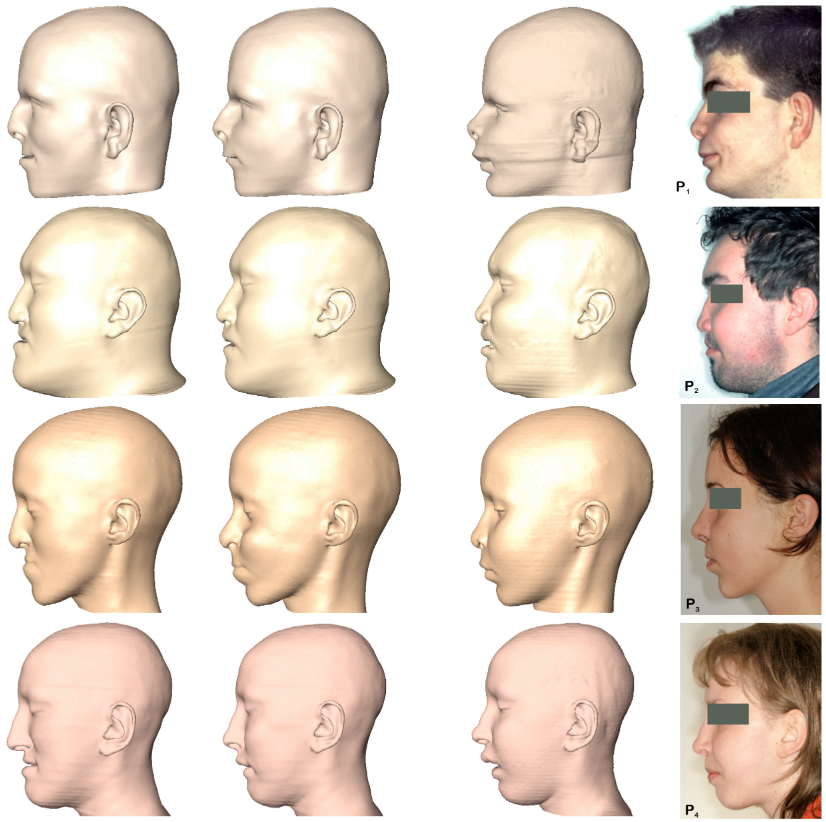


Zachow, Hege 2002

Bogumil: comparison before / after / prognosis

Karin: comparison before / after / prognosis

Four out of many patients



preoperative CT / planning / postoperative CT / photograph

Collaboration

ZIB

Computational medicine

Martin Weiser, Bodo Erdmann, Rainer Roitzsch

Medical planning

Stefan Zachow, Hans Lamecker

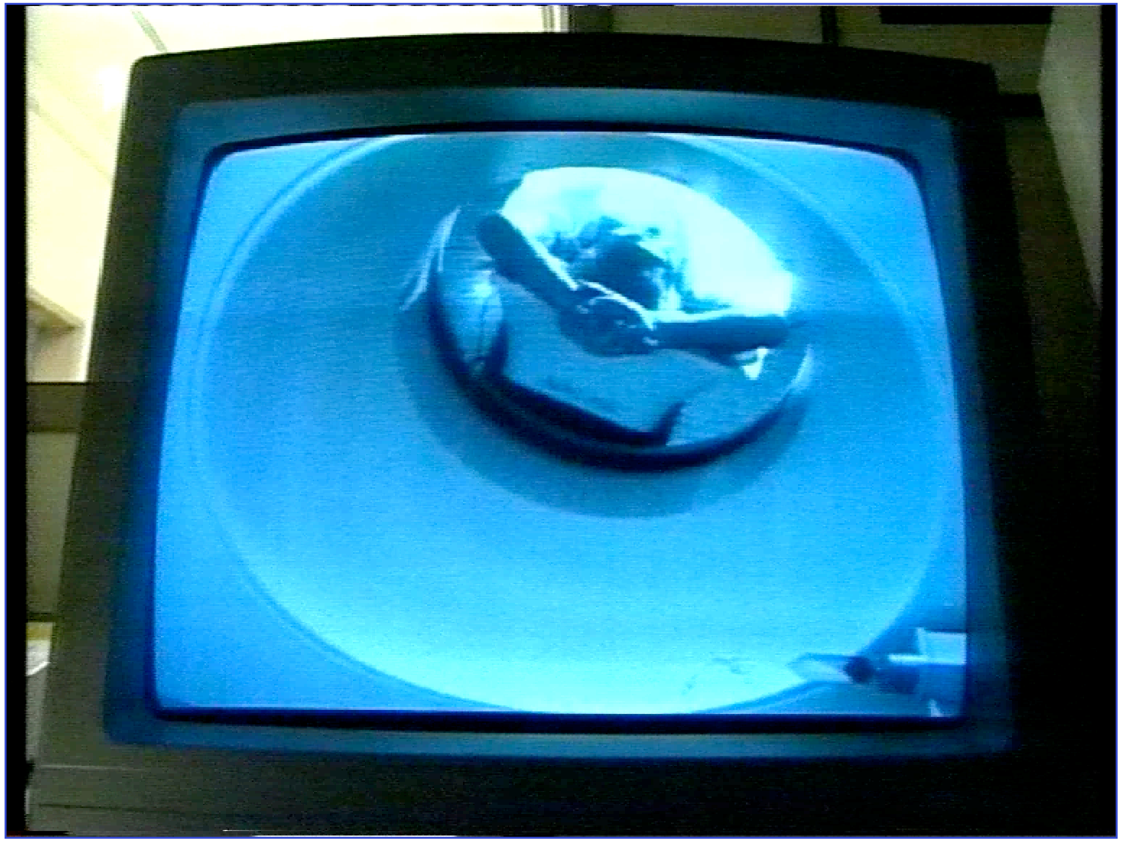
Clinical cooperations

U Basel, Kantonsspital, Hans-Florian Zeilhofer (formerly TU Munich)

U Frankfurt/M, Klinikum, Robert Sader (formerly TU Munich)

TU Dresden, KTH Stockholm, U Vienna, ….

Computer tomograph in hospital



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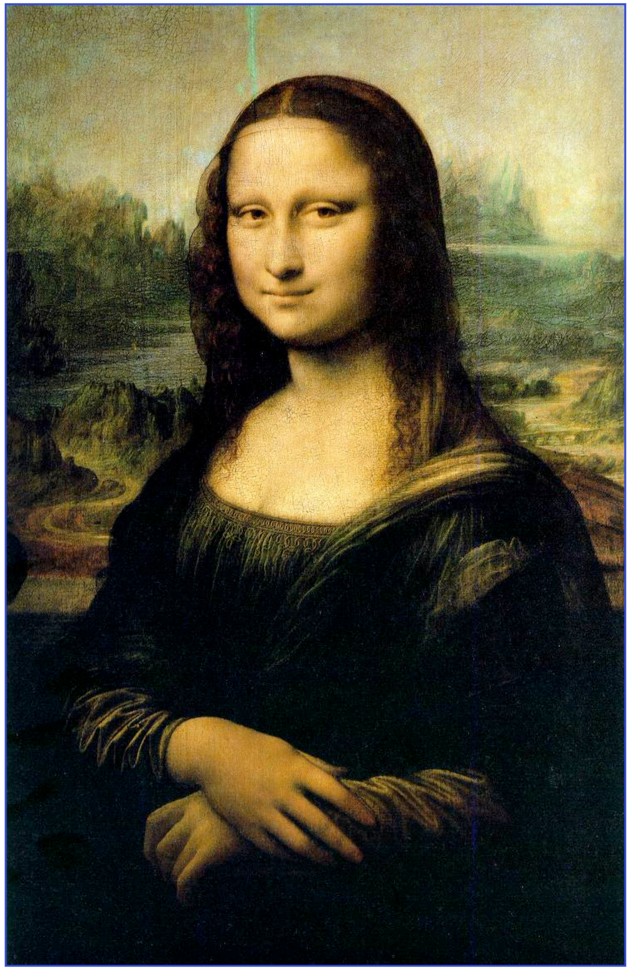
La Gioconda (1507)



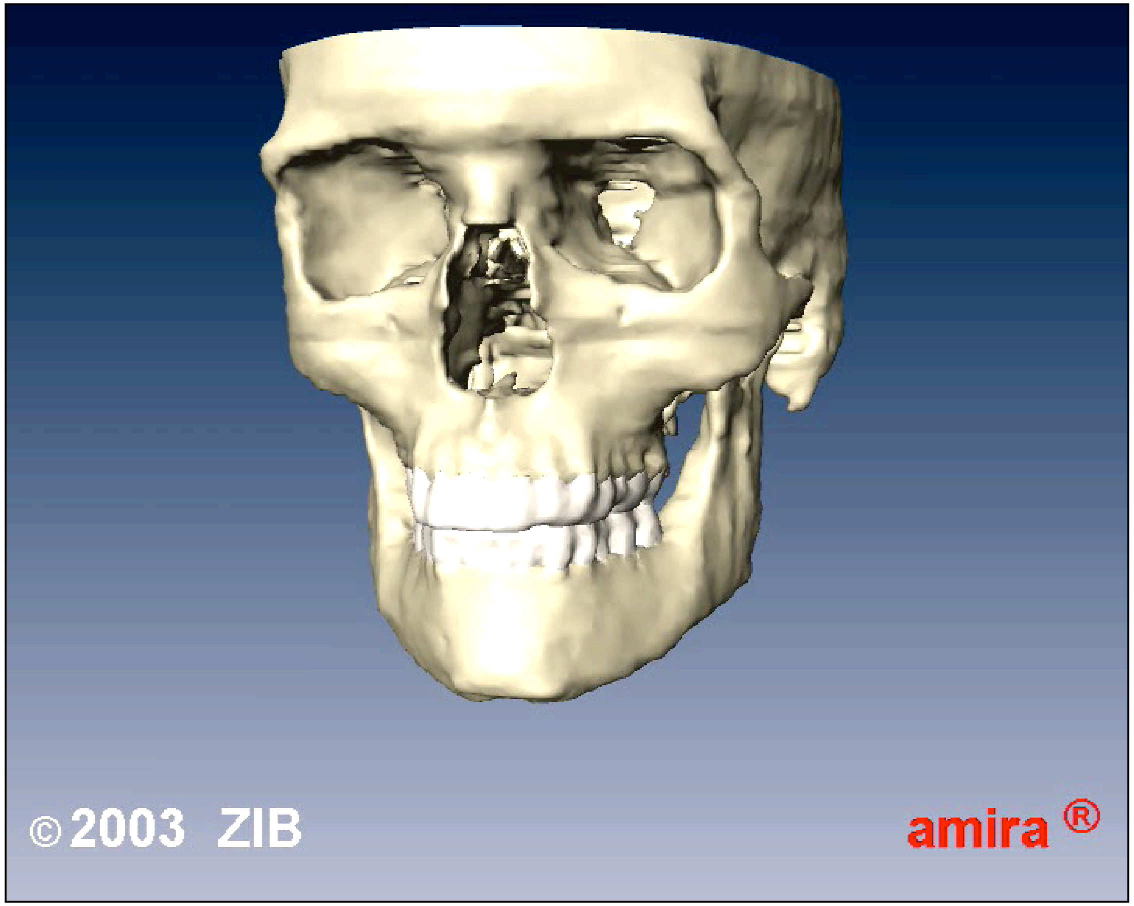
Leonardo da Vinci (1452 - 1519)

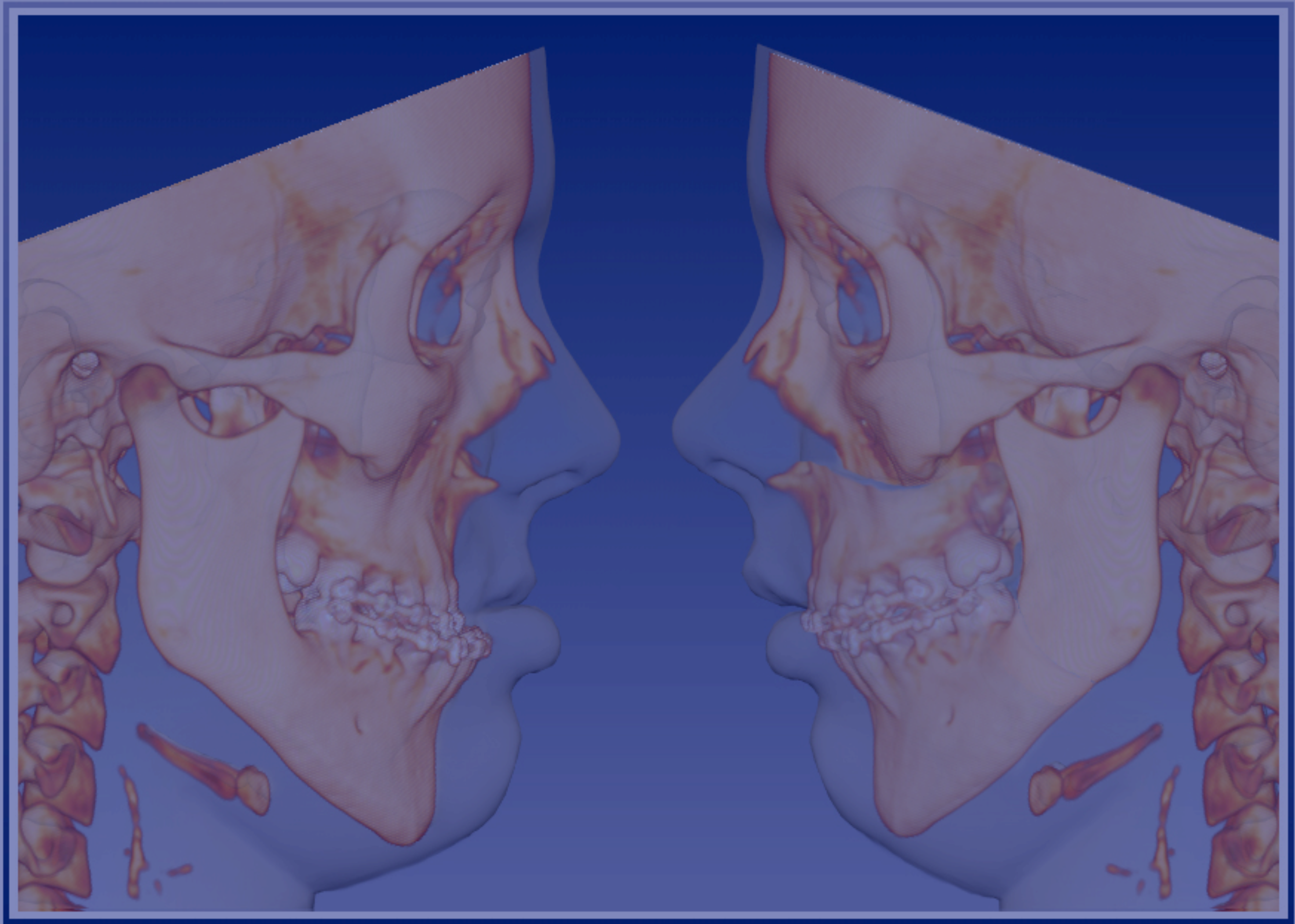
painter

mathematician

engineer

Smile with 10 out of 30 muscles





Imago animi vultus

Cicero, 106 - 43 v. Chr.