

A MATLAB Interface to the GPU

Second Winter School
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2007-01-24

Outline

- 1 Motivation and previous work.
- 2 MATLAB interface.
 - Gauss-Jordan.
 - PLU factorization.
 - Tridiagonal Gaussian elimination.
- 3 Summary.

Why use the GPU from MATLAB?

- High-level, with mathematical syntax: $[U S V] = \text{svd}(A)$.
- A standard tool for scientists and engineers for prototyping.
- Extendible with user-defined MEX files.
- **Using the GPU can speed up computations.**

Previous work - GPU

- Matrix multiplication:
 - Fixed function (Larsen & McAllister, 2001),
 - Packing (Moravánszky, 2003; Hall et al., 2003),
 - Analysis (Fatahalian et al., 2004),
 - Automatic tuning (Jiang & Snir, 2005),
 - Analysis (Govindaraju et al., 2006).
- G-J/PLU factorization: Single component (Galoppo et al., 2005).
- Conjugate gradients: Sparse matrix-vector product (Bolz et al., 2003; Krüger & Westermann, 2003).
- CUBLAS.
- RapidMind, ~~PeakStream~~.

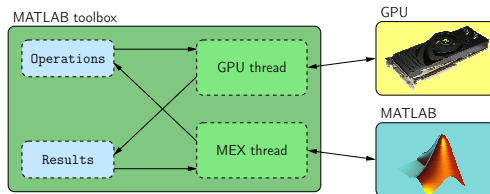
Previous work - MATLAB

- NVIDIA PhD fellow: CUFFT + MATLAB.
- NASA: GPU backend for MATLAB:
 - “**Virtually all software** requiring extensive numerical processing **could benefit** from the solvers developed in this project”,
 - “Most such software is already created using BLAS or similar libraries, thus **requiring very little modification** to be used with these accelerated solvers.”
- Parallel MATLAB survey: 8 message passing, 7 embarrassingly parallel, 6 backends, 7 compilers

What do we want?

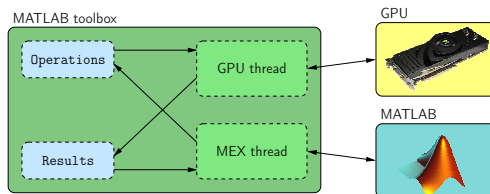
- Use the GPU as a coprocessor.
- An easy-to-use interface (tight integration with existing MATLAB syntax).
- BUT: (Almost) no thread-safe GPU APIs, and no thread-safe MEX API (MATLAB).

Two threads of execution



- A queue of operations, and a map of results.
- Similar to RapidMind and PeakStream ideas.
- Not a backend (as NASA's project), but a new class.

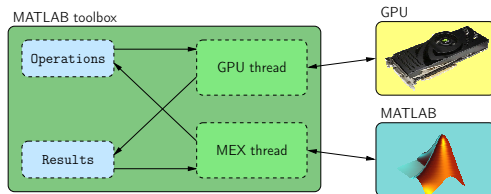
Two threads of execution



- 1** MATLAB user creates a gpuMatrix object.
- 2** Operation (+, -, rref, plu, etc) on the object.
- 3** MEX file is called using operator overloading.
- 4** The MATLAB thread enqueues the operation.

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Two threads of execution



...

- 5 The GPU thread computes the results, and moves to result map.
- 6 User requests result (e.g., `single(var)`).
- 7 Data is transparently moved from the GPU to MATLAB memory.

Syntax

Standard MATLAB

```
a = rand(n, n);  
b = a*a;  
[l u p] = lu(b);
```

GPU toolbox

```
a = gpuMatrix(rand(n, n));  
b = a*a;  
[l u p] = lu(b);
```

Background processing

```
a = gpuMatrix(rand(n, n));  
b = a*a;  
c = lu(b);  
read(c);
```

```
%CPU computations here
```

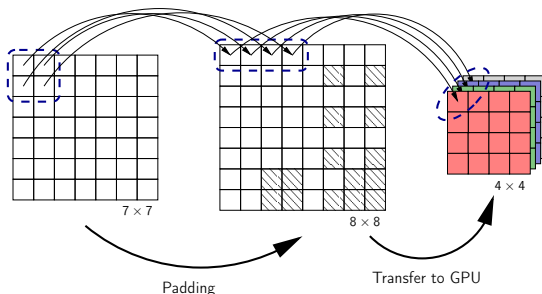
```
[l u p] = single(c);
```

Summary interface

- Simple interface with familiar MATLAB syntax.
- Background processing by splitting into two threads.
- Uses MATLAB objects, not a backend -> easier to reuse data?
- MATLAB has internal memory management.

Algorithms

- Four algorithms: (matrix multiplication), Gauss-Jordan, PLU factorization, tridiagonal Gaussian elimination (GE).
- All implemented using operator overloading to call the MEX file.
- All but tridiagonal GE are background (i.e. non-blocking) operators.
- Use of SIMD-vectorization (RGBA -> two-by-two packing)



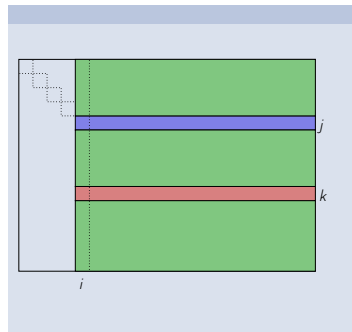
Gauss-Jordan factorization (i)

- Direct solver
- Slower than Gaussian elimination, but fewer passes needed.
- Need to employ a pivoting strategy for numerical stability.
 - Full - Overkill for most problems and not applicable for the chosen implementation (Doolittle)
 - Rook - Not applicable for the chosen implementation (Doolittle)
 - Partial - Works well for most cases
- Need to pivot two-by-two sub-matrices

Gauss-Jordan factorization (ii)

Algorithm

- 1** Find the pivoting element by reducing the pivot area to the largest element. Use quasi-harmonic norm $\left(\frac{a*b}{a+b}\right)$ as a measure.
- 2** Exchange two-by-two rows
- 3** Eliminate two-by-two column above and below pivot element



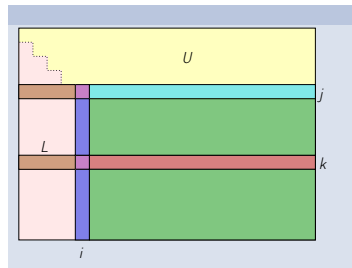
Items 2 and 3 are run in parallel!

PLU factorization (i)

- Direct solver.
- Can use a modification to the Doolittle algorithm.
- Can use same pivoting as for Gauss-Jordan elimination.
- Suitable for many right hand sides (Factorization $\mathcal{O}(n^3)$, while substitution is $\mathcal{O}(n^2)$).

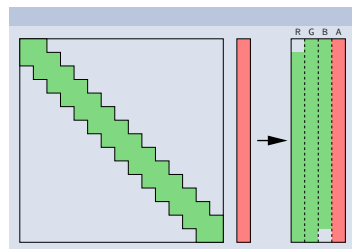
PLU factorization (ii)

- 1 Locate pivoting element
- 2 Exchange two-by-two rows, and calculate multipliers
- 3 Reduce below pivot element



Tridiagonal Gaussian elimination

- Tridiagonal storage - RGBA
- Can solve many systems in parallel
- Poor performance when solving only one system
- Stack many systems in columns.



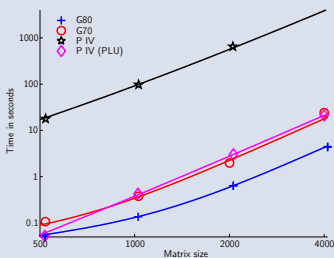
Summary operations

- Inherently parallel algorithms, but a lot of memory access per FLOP.
- Multi-pass algorithms are potentially expensive (reduction).
- Results will differ from CPU code (single precision, denormals).

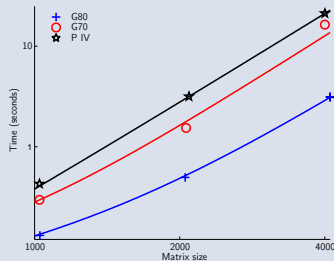
Accuracy

- Accurate storage and computation yields accurate results (e.g. most integral matrices).
- “A function of the condition number” (often matrix size).
- Experienced error is comparable to CPU single precision.

Runtime of different algorithms (loglog)



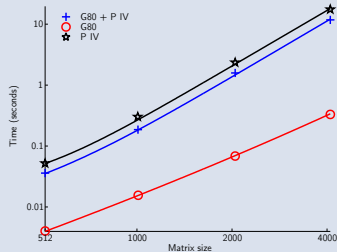
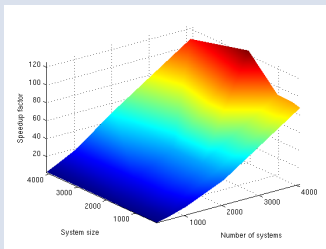
(a) Gauss-Jordan elimination (640x/4.3x)



(b) PLU factorization (7.55x)

- Fit $a + bx + cx^2 + dx^3$ to the runtimes.
- Compare the coefficient d , giving a speedupfactor valid for large n .
- Gauss-Jordan benchmarked against both rref (640x), and PLU (4.3x).

Runtime of different algorithms (loglog)



(c) Tridiagonal Gaussian elimination (a lot (d) Background computation PLU (1.7×)
(120×))

- Tridiagonal GE does not scale well wrt system size. Scales very well wrt number of systems solved.
- Background computation is free when enough CPU computation is computed simultaneously.

Summary

- Programmed using OpenGL -> difficult. CUDA remedies this.
- The GPU can be utilized as an efficient coprocessor, but MATLAB memory management “stinks”.
- As NASA says: A high-level mathematical interface to efficient algorithms is useful for prototyping

Contributions

- A high-level mathematical interface to the GPU.
- A new pivoting strategy for vectorized operations.
- The use of packing for Gauss-Jordan and PLU factorization.
- Paper accepted, soon to come:)
- Masters thesis available: <http://babrodtk.at.ifi.uio.no>

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