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Models for proppant transport and deposition in hydraulic fracture simulation: A review of the state of the art

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ABSTRACT

Basics of proppant transport and deposition under hydraulic fracturing are presented. Models of proppant transport and deposition in hydraulic fracturing are described briefly starting with the earlier simplified models proposed in the 1970s and proceeding to the more recent and more sophisticated models based on the granular kinetic theory.

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1 Introduction

Hydraulic fracturing is one of the most popular stimulation techniques currently used in oil and gas industry. The role of this technique is expected only to increase in the years to come as the formations with increasingly lower permeability are involved in production.

The ultimate goal of hydraulic fracturing is to increase the productivity of the stimulated well. In order to maximize this increase, the fracture usually needs to be propped by a suitable agent, i.e. a granular material, such as sand. This agent, called *proppant*, prevents the fracture from closing after the injection has stopped, and creates a filling with relatively high permeability inside the fracture so as to facilitate the flow from the reservoir through the fracture and to the well. During injection, premature bridging of the fracture by proppant particles should be avoided since it leads to screen-outs. The deposited proppant must have sufficiently high permeability to provide low resistance to flow. High permeability can be achieved e.g. by injecting relatively coarse sand. The deposited proppant must also satisfy certain requirements with respect to its compressive strength that would allow it to withstand the in-situ stresses once the fracture has closed. Moreover, the proppant must sit firmly enough to remain in place at flowback.

In practical terms, thousands of litres of proppant-laden slurry can be injected during a stimulation job (Adachi *et al*., 2007). Mass concentration of proppant at perforations is about 2-3 lb/gal, which is equivalent to 0.24-0.36 kg/l, or a volume fraction of 0.1-0.15. As the slurry moves along the fracture, the concentration increases because of leak-off, and may reach 11 lb/gal, i.e. a volume fraction of above 0.5 (Novotny, 1977).

Since the placement process and the properties of the proppant bed strongly affect the eventual productivity of the well, proppant transport and deposition are an important part of hydraulic fracturing modelling. Proppant transport and deposition are even more important in hydrofrac treatments of low-permeability rocks, such as shale, because of the low viscosity of the carrying fluid (water) used in these applications. Early settling of proppant in water may significantly reduce the effective, i.e. propped, fracture length and thereby reduce the expected productivity of the stimulated well. Also, since in a low-permeability formation the leakoff is small, it will affect the solids concentration to much a lesser degree during proppant flow along the fracture. This is expected to have a manifold effect on proppant placement since (a) the hindered settling velocity of proppant particles in the carrying fluid will be increased by reduced concentration; (b) the proppant will tend to move away from the fracture faces and towards the fracture mid-plane more than it does with leakoff; (c) the latter effect may facilitate settling by convection (gravity-driven settling, or gravity current) of proppant.

Several processes may affect the final configuration of the proppant bed in the fracture. These are:

1. Leak-off of the carrying fluid through the fracture walls into the formation.
2. Settling of proppant particles due to gravity.
3. Variation in the rheology of the carrying (non-Newtonian) fluid as the temperature of the fluid gradually increases as it moves away from the well.

During injection and transport of proppant inside the fracture, and if the formation is sufficiently permeable, fluid leaks off through the fracture walls into the formation. As a result, the concentration of proppant in the fluid gradually increases. Increasing sand concentration reduces the hindered settling velocity thus slowing down the settling process.

The simplest model of leak-off widely used in stimulation modelling is the so-called Carter's law given by

$$q_L(t) = \frac{2C_L}{\sqrt{t-\tau}} \tag{1.1}$$

where q is the volume leak-off flow rate per unit area of the fracture surface; τ is the time at which the leak-off starts; C_L is an empirical coefficient. Alternatively, $C_L(p - p_\infty)$ can replace C_L in the nominator in the right-hand side of Eq. (1.1) (with a corresponding change in the dimensions), with p being the fluid pressure at the leak-off place in the fracture and p_∞ being the far-field formation fluid pressure (Clifton and Wang, 1988). Even though the Carter's law is most often used in the form given by Eq. (1.1), the equation with $C_L(p - p_\infty)$ replacing C_L seems more logic since the leak-off rate must be an increasing function of the difference between the fluid pressure in the fracture and the formation pore pressure.

Eq. (1.1) describes the fluid loss through the filter cake *after* the filter cake has built up. The leak-off *during* filter cake deposition is usually described as the so-called spurt loss, represented by a lump amount of fluid lost instantaneously through the wall.

In a vertical or inclined fracture, particles will start to settle as soon as they enter the fracture at the injection interval, and the results of this process will become apparent at some distance from the well: the granular material becomes segregated. The settling process will continue after the injection has stopped and during the closing of the fracture, and this later stage will largely affect the final proppant distribution (Novotny, 1977). The settling stops when (a) the proppant forms a bank at the bottom of the (vertical) fracture; (b) the proppant concentration in the slurry becomes so high that particles can no longer settle; or (c) the fracture closes on the slurry trapping the proppant (Novotny, 1977).

Injection is performed though a perforated interval of the well. When the fracture is sufficiently long and high, the injection source is often modelled as being distributed uniformly along the fracture height at wellbore. However, it can be conceived that for shorter fractures it might be necessary to take into account the actual distribution of the injection sources (perforations).

It is believed that, as injection proceeds, the sand bed in a vertical fracture builds up according to the following scenario (Daneshy, 1978), represented schematically in Figure 1.1 (in this very rough schematic representation it is assumed that all proppant particles have the same size):

1) The sand settles during transport, which results in a gradual increase of the sand holdup (solid lines in Figure 1.1; bed growth is indicated with an arrow). The growth of the bed continues up and forward along the flow until the first proppant particle introduced at the top of the injection interval reaches the bottom of the fracture (the lowermost dashed line in Figure 1.1).
2) Then, the sand bed grows in height until the angle of repose is reached at the bed front.
3) Simultaneously, since the flow velocity has increased due to a reduction of the clearance between the bed top and the fracture top, the sand can be transported longer before it is deposited. The sand bed has an equilibrium height now, h_{eq}. The front of the bed can propagate forward, while the angle of repose is maintained at the front and the bed height (sand holdup) is maintained at the equilibrium level, h_{eq}. This is illustrated by dotted lines in Figure 1.1.

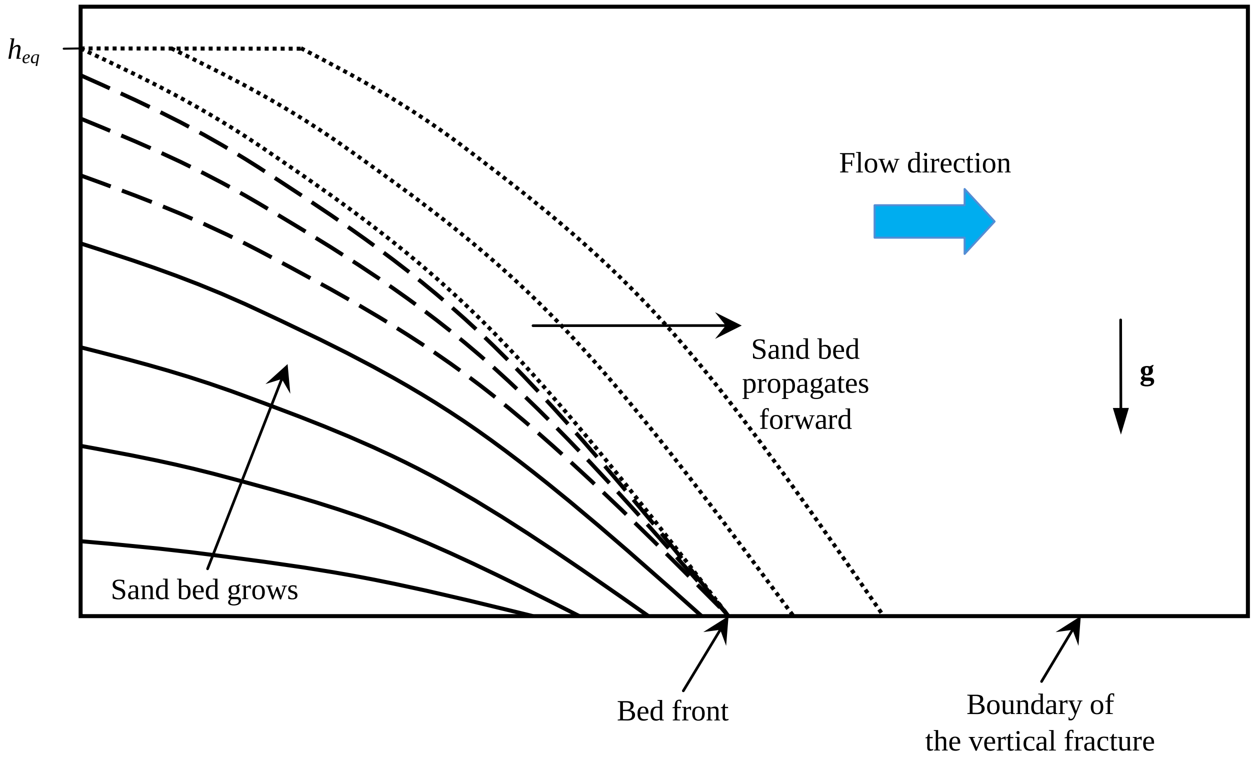

Figure 1.1 Growth and propagation of sand bed during proppant injection into a vertical hydraulic fracture.

In reality, materials having different grain size could be injected sequentially, which would complicate the simplified picture of Figure 1.1 resulting in a sand bed of a complex shape. This shape may be further complicated by settling of the suspended particles after injection stops. Moreover, the viscosity of the carrying fluid may be so high that no deposition occurs during transport.

In Sections 2 through 4, different models of proppant transport and deposition in hydraulic fracturing are outlined, in the order of increasing complexity and detail, from the simplified models proposed in the 1970s and 1980s to the models that make use of the granular kinetic theory and proposed just a few years ago.

2 Simplified proppant transport and settling models

In simplified models of proppant transport and deposition, such as those by e.g. Daneshy (1978) or Novotny (1977), the fracture is vertical and is discretised into slim vertical sections (columns) as shown in Figure 2.1.

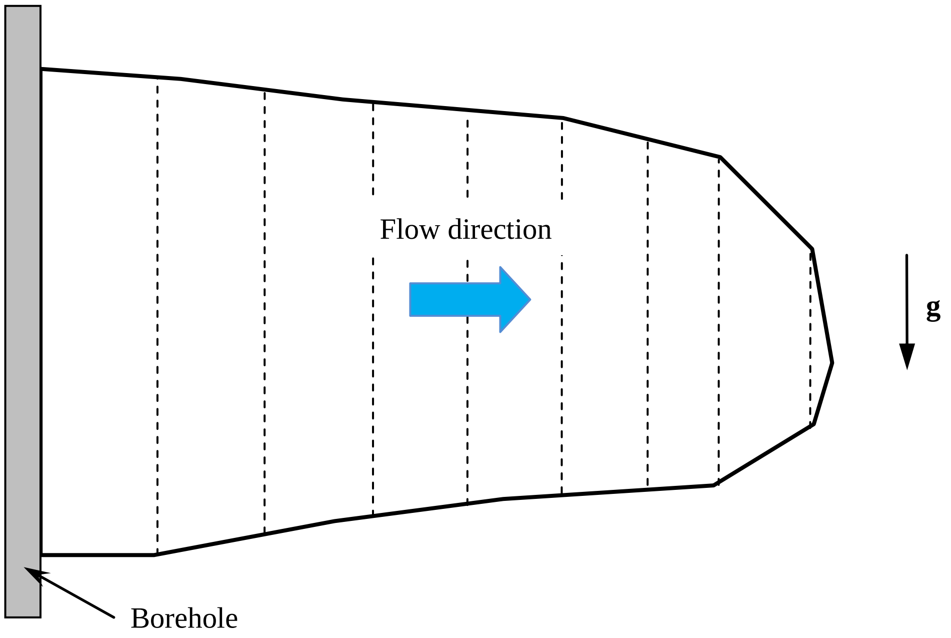

Figure 2.1 Fracture discretization employed in simplified proppant transport and settling models.

At each time step, the fluid loss (leak-off), the resulting increase in the solids volume fraction, and the settling velocity are computed for each element. Based thereupon, the increment in the deposited sand, the holdup and the suspended sand are computed for each element. The results of the computation are the bed profile and the sand in suspension at the end of injection, and the final bed shape at the end of settling after the injection has stopped.

In order to close the model, the settling law is needed. A number of empirical laws are available in the literature, developed for different Reynolds numbers, fluid rheologies, hindered settling conditions and accounting for wall effects (e.g. Daneshy, 1978; Novotny, 1977; Clark and Quadir, 1981; Valkó and Economides, 1995; Gadde *et al*., 2004). The equations are bulky and are omitted here. They can be found in standard texts on particle transport and deposition.

For water-fracs, inertial effects and the effects of turbulence on particle settling are important. In particular, three regimes with respect to the inertial effects are usually considered (Novotny, 1977): Re below 2 (Stokesian regime); Re between 2 and 500 (formation of a turbulent wake behind a particle, Bolster *et al*., 2011); Re above 500 (with the terminal settling velocity unaffected by the fluid viscosity). Inertial effects slow the settling down. $Re < 2$ is sometimes assumed for gel-fracs, while $Re = 2 \ldots 500$ for water fracs. The effects of turbulence on particle settling are poorly understood (Gadde *et al*., 2004).

The above-described simple model can be extended to incorporate the heat transfer between the slurry and the formation (Novotny, 1977). In any event, it is evident from the brief description above that the model is limited to planar vertical fractures. Finer details of flow in the vicinity of the injection interval are not resolved, which is not so critical for long fractures, but may become a serious limitation for shorter fractures. Also, in a non-Newtonian fluid, the effective viscosity seen by the particle and affecting the hindered settling velocity, is a function of the shear rate: in a shear-thinning gel, the effective viscosity decreases with shear rate. Such cross-coupling pointed out e.g. by Clark and Quadir (1981) will lead to an underestimation of the settling velocity in the flowing slurry, thus leading to overoptimistic predictions of proppant transport along the fracture.

3 Mixture models

A more general model of proppant transport in a hydraulic fracture is represented by a mixture-type model. In this type of model, the proppant velocity is assumed to be equal to the carrying fluid velocity. In other words, there is no slip and thus no momentum transfer between the carrying fluid and the granular phase. As a result, the concentration of proppant is constant across the fracture aperture in any cross-section. Also, no dispersion of proppant particles occurs, thus the front of the proppant concentration profile remains sharp (Adachi *et al.*, 2007).

The flow is assumed to be incompressible. The proppant particles are small compared to the fracture aperture; a relatively dilute suspension is considered. These assumptions break down near the fracture tip, where the aperture is small, and in the domains of high concentrations caused by leak-off.

Typically, the lubrication theory approximation (Reynolds equation) is used to describe the flow of the slurry. An essential element of this theory is an averaging of the field variables, such as the solids volume fraction and the fluid velocity, in the direction perpendicular to the fracture walls. For the lubrication theory to be a valid approximation, a collection of criteria for development lengths must be satisfied as follows (Pearson, 1994):

(1) The inertial effects must be negligible, i.e.:

$$\mathrm{Re}\left|\nabla w\right| << 1 \tag{3.1}$$

where Re is the fluid Reynolds number, $\mathrm{Re} = \rho Q / \mu$; w is the fracture aperture; ρ and μ are the fluid density and dynamic viscosity, respectively; Q is the flow rate (m^2/s); ∇ is applied in the plane of the fracture. Eq. (3.1) effectively compares the development length for inertial relaxation to the geometrical length scale along the flow direction.

(2) For a moving boundary, the boundary "must appear fixed for times over which the fluid moves many widths along the fracture":

$$w \frac{\partial w}{\partial t} Q^{-1} << 1 \tag{3.2}$$

(3) For a thixotropic or viscoelastic fluid with the longest of the relaxation/retardation times being τ_{max}, the relaxation effects, characterised by Deborah number, must be negligible:

$$\mathrm{De} = \tau_{\max} Q \left|\nabla w\right| / w^2 << 1 \tag{3.3}$$

Eq. (3.3) effectively provides a condition for the relaxation development length.

(4) For a non-isothermal flow, a more stringent requirement (here for Peclet number) is obtained based on the heat transfer from the fracture surfaces:

$$\mathrm{Pe} = Q/\kappa << \left|\nabla w\right|^{-1} \tag{3.4}$$

where κ is the thermal diffusivity of the fluid. Eq. (3.4) effectively compares the thermal relaxation length to the geometrical length scale.

Eqs. (3.1) through (3.4) are formulated based on the governing equations for a single-phase flow. For a particle-laden flow, additional development lengths arise in conjunction with particle settling and migration. Only if all development lengths are sufficiently small compared to the geometrical scale, $w\left|\nabla w\right|^{-1}$, will the lubrication theory approximation hold. A last requirement for its validity is that the boundary conditions involving the superficial leakoff velocity, v_{lo}, and the heat flux through the fracture face, q_{ht}, vary sufficiently slowly with time:

$$\frac{w^2}{v_{\mathrm{lo}} Q_m} \frac{\partial v_{\mathrm{lo}}}{\partial t} << 1 \tag{3.5}$$

$$\frac{w^2}{q_{\mathrm{ht}} Q_m} \frac{\partial q_{\mathrm{ht}}}{\partial t} << 1 \tag{3.6}$$

where Q_m is the mixture (suspension) flow rate.

The dimensional analysis reasoning presented above provides criteria for the validity of the lubrication theory approximation for a channel *with perfectly smooth walls*. Zimmerman *et al*. (1991) considered fluid flow in a fracture with varying (periodic) aperture and concluded that the lubrication approximation becomes invalidated by roughness only when the spatial wavelength of the dominant roughness component becomes on the order of or smaller than the amplitude of that roughness component.

In the case of a Newtonian carrying fluid, the dynamic viscosity of the slurry, μ_{SL}, is a function of the solids volume fraction, e.g. (Eskin and Miller, 2008):

$$\mu_{SL} = \mu_L \left[1 + 2.5c + 10c^2 + 0.0019\exp(20c)\right] \tag{3.7}$$

μ_L being the dynamic viscosity of the carrying fluid; c being the volume fraction of proppant;

or (Adachi *et al*., 2007; Barree and Conway, 1994):

$$\mu_{SL} = \mu_L \left(1 - \frac{c}{c_*}\right)^{\beta} \tag{3.8}$$

where c_* is the saturation concentration; -3 < β < -1.

In the case of a power-law fluid, Eqs. (3.7), (3.8) would apply to the effective dynamic viscosity of the slurry.

As the concentration increases, the slurry becomes packed and starts to behave more like a porous media through which the carrying fluid flows. The lubrication approximation for such a system is apparently invalid.

Since there is no slip between the particles and the fluid, there will be no momentum exchange terms in the momentum conservation equations, and the momentum conservation for this multiphase system reduces to the single momentum equation for the mixture.

Assuming a planar vertical fracture in the *xy* plane, with the *y* direction being vertical, the governing equations for the fluid flow and proppant transport are as follows:

(i) The mass conservation for the mixture given by (Clifton and Wang, 1988; Adachi *et al*., 2007):

$$\frac{\partial w}{\partial t} = \nabla \cdot \left[D(w,p)\left(\nabla p - \rho \mathbf{g}\right)\right] - \frac{2C_L}{\sqrt{t - t_0(x,y)}} - 2S_0 \delta\left[t - t_0(x,y)\right] \tag{3.9}$$

where ρ is the density of the slurry; t_0 is the initiation time for the leak-off, i.e. the time at which the fluid reaches the point (*x*, *y*); *w* is the aperture of the fracture; *p* is the slurry pressure; **g** is the gravity; S_0 is the spurt loss (see Section 1), and $D(w,p)$ is the solution of the momentum conservation equation under the assumptions of the lubrication theory, given, for a power-law fluid (e.g. gels used in hydraulic fracturing), by:

$$D(w,p) = \frac{n}{2n+1}\left(\frac{1}{2^{n+1}K}\right)^{1/n} w^{(2n+1)/n} \left|\nabla p - \rho \mathbf{g}\right|^{(1-n)/n} \tag{3.10}$$

where *n* and *K* are the power-law exponent and the consistency index of the power-law fluid, respectively.

(ii) The mass conservation equation for the proppant volume fraction *c* (Pearson, 1994):

$$\frac{\partial (cw)}{\partial t} + \nabla \cdot \left(cw\mathbf{v}_p\right) = 0 \tag{3.11}$$

where $\mathbf{v}_p$ is the proppant velocity given by (Adachi *et al*., 2007):

$$\mathbf{v}_p = \mathbf{v} - (1-c)\mathbf{v}_{slip} \tag{3.12}$$

where $\mathbf{v}$ is the slurry (mixture) velocity obtained from the fluid solver; $\mathbf{v}_{slip}$ is the slip velocity that can be found using the same empirical relations for the hindered settling velocity as those used in the simplified models of Section 2.
Combined with mass conservation for an incompressible slurry under the assumption of no leak-off, given by $\frac{\partial w}{\partial t} + \nabla \cdot \left(w\mathbf{v}_p\right) = 0$, Eq. (3.11) reduces to:

$$\frac{\partial c}{\partial t} + \mathbf{v}_p \cdot \nabla c = 0 \tag{3.13}$$

which is an incompressible advection equation for *c* and can be solved by e.g. the method of characteristics (Clifton and Wang, 1988).

A boundary condition for slurry in hydraulic fracturing applications is typically given by injection rate at perforations. Boundary condition for Eq. (3.13) is given by the solids volume fraction in the slurry at perforations.

The proppant solver then is coupled to the hydraulic fracture simulator to provide input (*c*) that affects the slurry rheology (Figure 3.1). According to Adachi *et al*. (2007), the proppant solver is used only to update the slurry properties, i.e. they imply that the coupled fluid solver / proppant solver need not be run iteratively to convergence with full explicit coupling. A similar approach was taken by Gadde *et al*. (2004). Pearson (1994) suggested that the proppant concentration equation be solved iteratively with a coupling to the fluid flow equations.

Since there is no slip between solids and fluid in the model described above, other than that due to settling, shear-induced proppant migration and collisions between proppant particles cannot be captured by this type of model (Adachi *et al*., 2007). Nevertheless, the model is widely used, also for water-fracs[1] where the collisions and shear-induced effects might be more pronounced than in gel-based suspensions. The mixture model was employed by Hammond (1995) to investigate two types of the gravity-driven vertical motion of proppant in vertical hydraulic fractures: slumping[2] and settling.

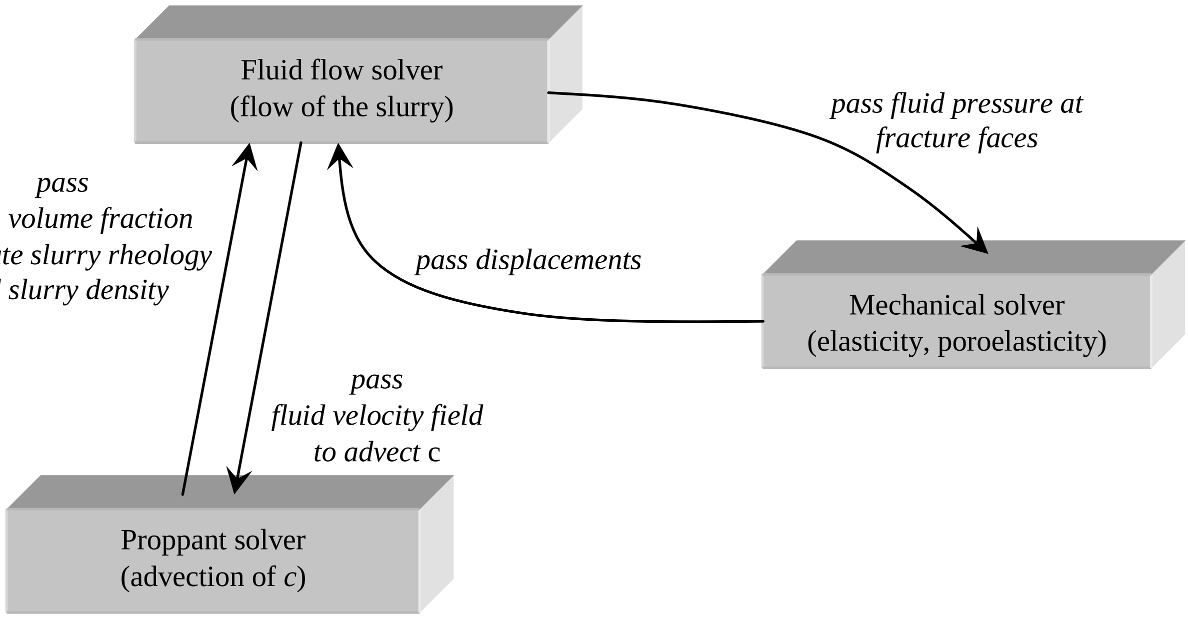

Figure 3.1 Coupling of different building blocks of a hydraulic fracture simulator. Thermal solver is omitted to avoid cluttering in the Figure.

[1] Cf. e.g. the finite-element-based proppant solver by Gadde *et al*. (2004).
[2] Also known as "convection" or "gravity current". The importance of gravity currents in proppant transport and deposition was emphasized and analyzed through experiments and modeling by Barree and Conway (1994).

4 Models using granular kinetic theory

A model of proppant transport in a fracture based on the concepts of granular kinetic theory was proposed by Eskin and Miller (2008). The model is briefly described in this Section.

Steady-state flow of a slurry in a fracture with flat faces without leak-off is considered (Figure 4.1).

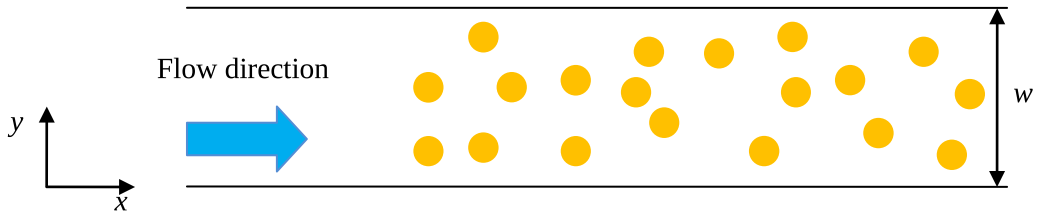

Figure 4.1 Solids-liquid flow in a vertical fracture. Yellow circles represent sand particles. The vector of gravity points in the direction perpendicular to the plane of the Figure.

It is assumed in the granular kinetic theory that sand particles collide with each other which leads to a Maxwellian distribution of the particle fluctuation velocities. Therefore, as in kinetic gas theory, it is possible to introduce a quantity called *granular temperature* given by

$$\Theta = \frac{\langle v_s^2 \rangle}{3} \tag{4.1}$$

where v_s is the fluctuation velocity of the particle, i.e. the difference between the particle velocity and the average velocity of the particles at this location.

One of the applicability criteria of the granular kinetic theory is that the ratio of the velocity relaxation time for a particle fluctuating in a liquid to the time of the particle free path between particle-particle collisions be large enough (at least larger than 2). From their numerical simulations, Eskin and Miller (2008) obtained that, with the power-law carrying fluid, this condition was satisfied only near the fracture walls where the granular temperature was relatively high. Thus, even though the particle migration from the fracture surfaces towards the fracture mid-plane might take place, it is not necessarily caused by the granular temperature gradient, but might be driven by other mechanisms (see e.g. a discussion by Hammond, 1995). For the sake of completeness of this review, however, the approach based on the granular kinetic theory is described in what follows. Also, in fluids with lower viscosity, such as the ones used in water-frac jobs, the role of particle collisions in particle migration might be more significant than in gels, thus making the granular kinetic approach more relevant for those problems.

Four governing equations describe the solids-liquid flow in granular kinetic theory (Eskin and Miller, 2008):

(i) Granular energy conservation:

$$\tau \frac{\mathrm{d}u}{\mathrm{d}y} = \frac{\mathrm{d}q}{\mathrm{d}y} + \varepsilon_{\text{visc}} + \varepsilon_{\text{coll}} + \varepsilon_{\text{heat}} \tag{4.2}$$

The meaning of Eq. (4.2) is as follows: the energy spent on friction between adjacent slurry layers is transformed into heat ($\varepsilon_{\text{heat}}$) and the energy of particle fluctuations. The latter dissipates through particle-liquid viscous interactions ($\varepsilon_{\text{visc}}$) and partially inelastic particle-particle collisions ($\varepsilon_{\text{coll}}$) and also is transported across the fracture by the granular conduction mechanism (q).

In Eq. (4.2):
u is the slurry velocity in the x direction (Figure 4.1);
q is the granular energy flux along the y direction in Figure 4.1 and is a function of particle size, density, concentration and granular temperature given by:

$$q = -k_{\Theta} \frac{\mathrm{d}\Theta}{\mathrm{d}y} \tag{4.3}$$

k_{Θ} is the granular conductivity given by

$$k_{\Theta} = \frac{150 \rho_s d_s c \sqrt{\pi \Theta}}{384 (1+k_n) g_0} \left[1 + \frac{6}{5} c g_0 (1+k_n) \right]^2 + \rho_s c^2 d_s (1+k_n) g_0 \sqrt{\frac{\Theta}{\pi}} \tag{4.4}$$

ρ_s is the particle density;
d_s is the particle diameter;
k_n is the restitution coefficient for particle-particle collisions[3];
c is the solids volume fraction;
g_0 is the radial distribution function taking into account the increase in the probability of particle-particle collisions in a dense system compared to a dilute one due to non-zero solids volume and given by

$$g_0 = \frac{1 - 0.5c}{(1-c)^3} \tag{4.5}$$

τ is the shear stress in the slurry given by

$$\tau = \mu_{SL} \frac{\mathrm{d}u}{\mathrm{d}y} \tag{4.6}$$

with the slurry viscosity according to Eq. (3.7);

$\varepsilon_{\text{visc}}$ is the energy dissipation rate caused by a viscous interaction of the fluctuating particles with the fluid and given by

[3] The coefficient of restitution is known to be an increasing function of the Stokes number (e.g. Joseph *et al.*, 2001; Lavrov and Laux, 2007). It is not clear from the description by Eskin and Miller (2008) if they assume k_n to be a function of St or a constant.

$$\varepsilon_{\text{visc}} = 9\pi\mu_L d_s n_s \Theta R_{\text{diss}} \tag{4.7}$$

where μ_L is the effective viscosity of the power-law fluid given by:

$$\mu_L = K\left\langle \bar{\dot{\gamma}}_{m\Theta} \right\rangle^{n-1} \tag{4.8}$$

n_s is the particle concentration by number given by

$$n_s = \frac{6c}{\pi d_s^3} \tag{4.9}$$

R_{diss} is the dissipation coefficient (effective drag coefficient) given by

$$R_{\text{diss}} = R_{\text{diss0}} + \text{Re}_\Theta \, k(c) \tag{4.10}$$

with R_{diss0} taking into account the effect of the lubrication force and $\text{Re}_\Theta \, k(c)$ taking into account the dissipation due to particle-liquid viscous friction.

$$\text{Re}_\Theta = \frac{d_s \rho_L \Theta^{1/2}}{\mu_L} \tag{4.11}$$

$$R_{\text{diss0}} = k_1(c) - k_2(c)\ln \varepsilon_m \tag{4.12}$$

$$k_1(c) = 1 + 3\sqrt{\frac{c}{2}} + \frac{135}{64} c \ln c + 11.26c\left(1 - 5.1c + 16.57c^2 - 21.77c^3\right) \tag{4.13}$$

$$k_2(c) = c g_0 \tag{4.14}$$

where ε_m is the dimensionless gap between two particles at which the lubrication force is assumed to stop increasing (cut-off distance for the lubrication force).

The total shear rate $\bar{\dot{\gamma}}_{m\Theta} = \bar{\dot{\gamma}}_m + \bar{\dot{\gamma}}_\Theta$ is produced by the shear rate of the slurry given by $\dot{\gamma}_m = \frac{\mathrm{d}u}{\mathrm{d}y}$ and the shear rate caused by particle fluctuations and given by $\dot{\gamma}_\Theta \approx \frac{\sqrt{\left\langle v_s^2 \right\rangle}}{d_s}$. It is shown by Eskin and Miller (2008) that the average length of the share rate vector is given by

$$\left\langle \bar{\dot{\gamma}}_{m\Theta} \right\rangle = \left|\bar{\dot{\gamma}}_m\right| \left[1 + \frac{1}{3} \left(\frac{\left|\bar{\dot{\gamma}}_\Theta\right|}{\left|\bar{\dot{\gamma}}_m\right|} \right)^2 \right] \tag{4.15}$$

$$k(c) = \frac{0.096 + 0.142c^{0.212}}{(1-c)^{4.454}} \tag{4.16}$$

Returning to Eq. (4.2), $\varepsilon_{\text{coll}}$ is the energy dissipation rate due to the kinetic energy loss caused by partially inelastic particle-particle collisions ($k_n < 1$). It is given by

$$\varepsilon_{\text{coll}} = \frac{12}{d_s\sqrt{\pi}} \left(1 - k_n^2\right) c^2 \rho_s g_0 \Theta^{3/2} \tag{4.17}$$

$\varepsilon_{\text{heat}}$ is the power directly turned into heat and is given by:

$$\varepsilon_{\text{heat}} = \mu_L (1-c) \left(\frac{\mathrm{d}u}{\mathrm{d}y} \right)^2 \tag{4.18}$$

The boundary conditions for Eq. (4.2) are as follows:

$q = 0$ at $y = 0$ and $y = w$.

$\frac{\mathrm{d}\Theta}{\mathrm{d}y} = 0$ at $y = w/2$.

(ii) The solids diffusion equation:

$$\frac{\mathrm{d}c}{\mathrm{d}y} = -\frac{1}{\Theta}\frac{\mathrm{d}\Theta}{\mathrm{d}y} \frac{c + 2(1+k_n)c^2 g_0}{1 + 2(1+k_n)\frac{c}{(1-c)^4(2-0.5c)}} \tag{4.19}$$

The boundary condition for Eq. (4.19) is formulated in an implicit form as:

$$J_s - \int_0^w \rho_s c u \,\mathrm{d}y = 0 \tag{4.20}$$

where J_s is the solids mass flow rate per unit of the fracture height.

(iii) Momentum conservation for the slurry in a steady state flow:

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mathrm{d}\tau}{\mathrm{d}y} \tag{4.21}$$

where $\mathrm{d}p/\mathrm{d}x$ is the pressure gradient, constant across the fracture within the lubrication theory approximation.

The boundary conditions for Eq. (4.21) are as follows:

$u = 0$ at $y = 0$ and $y = w$.

$\frac{\mathrm{d}u}{\mathrm{d}y} = 0$ at $y = w/2$.

(iv) The mass conservation equation for the liquid phase in an integral form:

$$J_L - \int_0^w \rho_L (1-c) u \mathrm{d}y = 0 \tag{4.22}$$

where J_L is the liquid phase mass flow rate per unit of the fracture height.

This completes the model.

5 Challenges

With regard to the simplified proppant models outlined in Section 2, the challenges are:

1) Settling velocities for non-Newtonian fluids are not readily available.
2) Particle clustering effects in non-Newtonian fluids are not captured by the simple empirical models.
3) Settling of non-spherical particles is poorly understood.

The model by Eskin and Miller (2008) is for steady-state only. Extending it to transient flows may make it computationally prohibitive for any practical use in a hydraulic fracturing coupled simulator. As the authors of the model point out, application of the granular kinetic theory, though mature for gas-solids flows, is still in its infancy for liquid-solids flows. Many of the equations used in the model of Eskin and Miller (2008), such as e.g. Eq. (4.13) or Eq. (4.16), make use of empirical laws and correlations and are therefore valid only within certain ranges. Leak-off was neglected in the numerical computations (its introduction should be straightforward, if needed, though). Also, there is no uniqueness in the choice of the closures such as e.g. Eq. (4.10). The significance of the collision effects for suspensions with power-law carrying fluids was questioned by the authors themselves, even though the necessary conditions seems to be present at least near the walls. Also, in the presence of leak-off, the migration away from the walls would be reduced. Additional complications with the model would arise in non-vertical fractures. Two such effects were pointed out by Eskin (2008): (i) the Boycott effect on an inclined fracture wall (a granular layer moving down the wall); (ii) the effect of surface roughness on particle transport.

In a less viscous carrying fluid, however, such as those used in water-fracs, the granular kinetic effects might be quite relevant.

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