Multiscale methods for reservoir simulation on a geological scale

Jørg Espen Aarnes
SINTEF Applied Mathematics

Multiscale Methods in Science and Engineering 2004

Motivation

- Modern reservoir simulators are not able to run routine simulations on the geological scale.
- Upscaling techniques are used to create coarsened grid models for day-to-day simulation.
  - The price to pay is less reliable results.
- Multiscale methods offer the possibility of bridging the gap between the geoscale and the simulation scale.

Model problem (heterogeneous porous media flow)

The Darcy law and the continuity equation for phase i reads

$$v_i = -k\lambda_i(\nabla p_i - \rho_i G) \tag{1}$$

$$\phi\partial_t S_i + \nabla \cdot v_i = q_i \ . \tag{2}$$

Combining (1) and (2) gives the elliptic equation for pressure

$$v = -k\lambda\nabla p + k\lambda_g G \ \text{ and } \ \nabla \cdot v = q \ .$$

Here k is bounded, symmetric and uniformly positive definite.

- The permeability k typically span over many length scales.
  - The solution may contain a multiple scale structure.
- Details at all scales have a strong impact on the true solution.
  - Conventional numerical methods which are not adaptive to the information at the subgrid scales may give poor accuracy.
  - A prohibitively large number of variables are often needed to resolve all the subgrid scales.
- Hierarchical or multiscale modeling approaches are needed.

The Multiscale Finite Element Methods

The multiscale finite element methods (MsFEMs) is a class of FEMs for (nearly) elliptic problems with multiple scale coefficients.

Multiscale methods: Methods that incorporate fine scale information into a set of coarse scale equations in a way which is consistent with the local property of the differential operator.

The MsFEMs are based upon the construction of appropriate "coarse-scale" approximation spaces that are adaptive to the local property of the elliptic differential operator.

Some of the features which make MsFEMs an attractive tool for reservoir simulation are:

- The elliptic "parallelization": subgrid resolution at a low cost.
- The flexibility: the natural ability to handle
  - heterogeneous and anisotropic materials,
  - irregular and unstructured grids.
- The ideal foundation for adaptive numerical schemes for the solution of advective transport equations.

The Mixed Multiscale Finite Element Method

Let the reservoir domain Ω be partitioned into a family of mutually disjoint elements $\mathcal{K} = \{K\}$ of arbitrary shape and size.

The original mixed MsFEM applies base functions ψ_{ij} which satisfy

$$\psi_{ij} = -k\lambda\nabla\phi_{ij} \ , \qquad \nabla\cdot\psi_{ij} = \pm|K|^{-1} \qquad \text{in} \qquad K_i \cup K_j \ ,$$

and are subject to prescribed boundary conditions $\psi_{ij}\cdot n_{ij} = \nu_{ij}$ on $\Gamma_{ij} = \partial K_i \cap \partial K_j$ and no-flow conditions on $\partial(K_i \cup K_j)\backslash\Gamma_{ij}$.

The boundary condition ν_{ij} is chosen such that $\psi_{ij} \in H^{1,div}(\Omega)$.

To formulate the corresponding mixed MsFEM we assume, for simplicity, homogeneous boundary conditions on $\partial\Omega$.

We then seek $v \in V = span\,\{\psi_{ij}\}$ and $p \in \mathcal{P}_0(\mathcal{K})$ such that

$$\begin{aligned} \int_\Omega (k\lambda)^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= \int_\Omega \frac{\lambda_g}{\lambda} G \cdot u \, dx \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx \end{aligned}$$

for all $u \in V$ and $l \in \mathcal{P}_0(\mathcal{K})$.

How to get subgrid resolution

- **Objective:** We want to be able to simulate the phase transport at the subgrid level without violating the mass balance.
- **Adjustments:** We need to alter the definition of the base functions and to add a correction term to the mixed FEM equations in order to account for the difference between the pressure at the wells and the average pressure in the well blocks.
- **Extensions:** For multi-phase flow we need to develop an efficient way to update the base functions throughout the simulation.

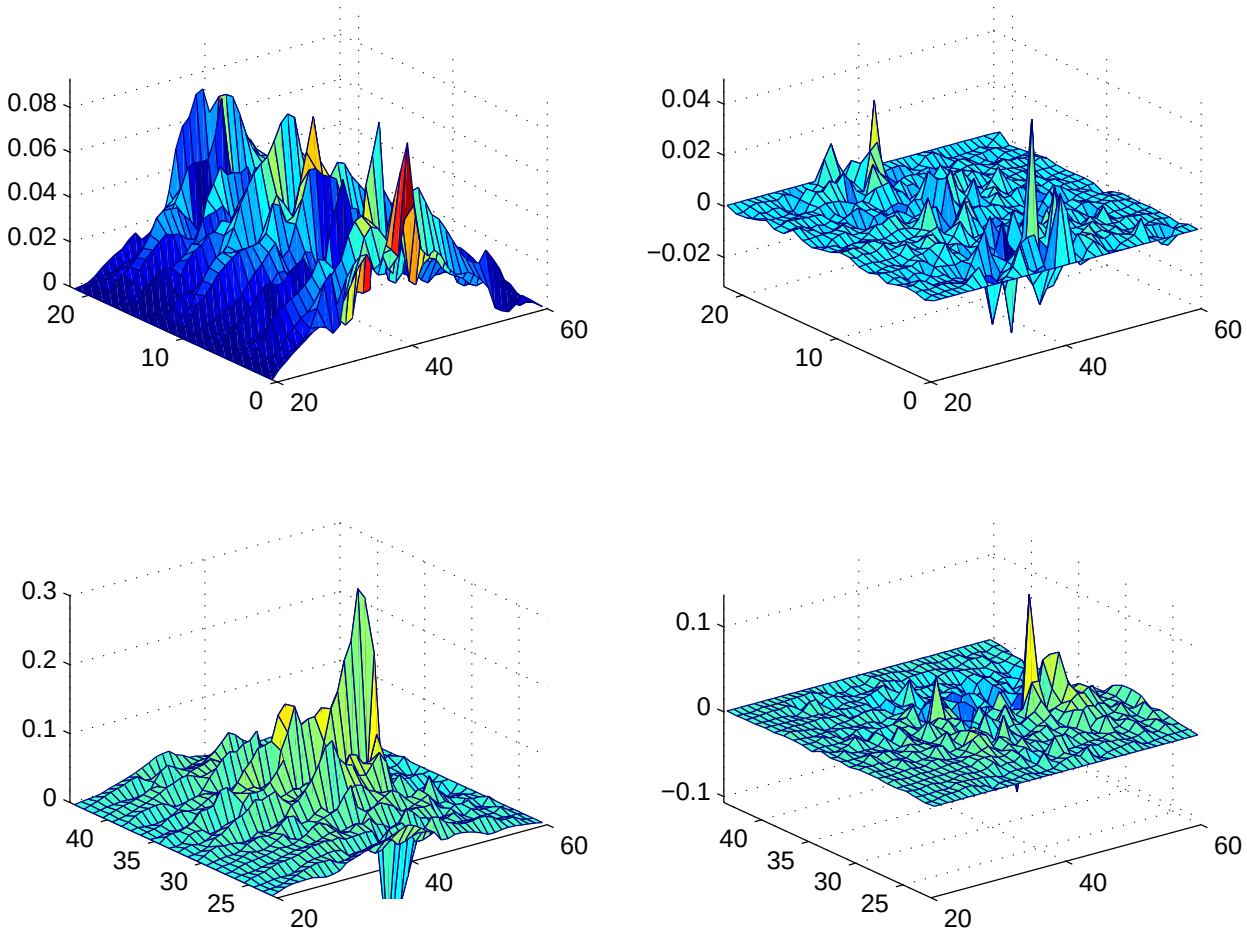
The modified base functions

The base functions ψ_{ij} for the modified mixed MsFEM satisfy

$$\nabla \cdot \psi_{ij} = \pm \begin{cases} \frac{1}{|K|} & \text{if } \int_K f \, dx = 0 \,, \\ \frac{f}{\int_K f \, dx} & \text{if } \int_K f \, dx \neq 0 \,, \end{cases}$$

in $K_i \cup K_j$ and prescribed boundary conditions on $\partial(K_i \cup K_j)$.

The boundary conditions ν_{ij} determine how well V incorporates the local property of the differential operator. They should therefore reflect the subgrid heterogeneity and radial flow in the well blocks.


0.08
0.06
0.04
0.02
0
20
10
0
20
40
60
0.04
0.02
0
−0.02
20
10
0
20
40
60
0.3
0.2
0.1
0
40
35
30
25
20
40
60
0.1
0
−0.1
40
35
30
25
20
40
60

The modified approximation space for the pressure

To modify the approximation space for the pressure, let $\hat{p}$ be a function that vanishes outside the well blocks and satisfies

$$\int_K \hat{p}\, dx = 0, \quad \hat{v} = -k\lambda\nabla\hat{p} + k\lambda_g G, \quad \nabla\cdot\hat{v} = q, \quad \hat{v}\cdot n = \nu_K$$

in the well blocks for some boundary condition $\nu_K \approx v\cdot n$ on ∂K.

If v solves the mixed formulation, $\nu_{ij} = \frac{v\cdot n}{\int_{\Gamma_{ij}} v\cdot n}$ on Γ_{ij} and $\nu_K = v\cdot n$ on ∂K, then there exist a $p \in U = \hat{p} + \mathcal{P}_0(\mathcal{K})$ such that v satisfies the mixed equations for all $u \in V$ and $l \in \mathcal{P}_0(\mathcal{K})$.

The modified mixed multiscale FEM

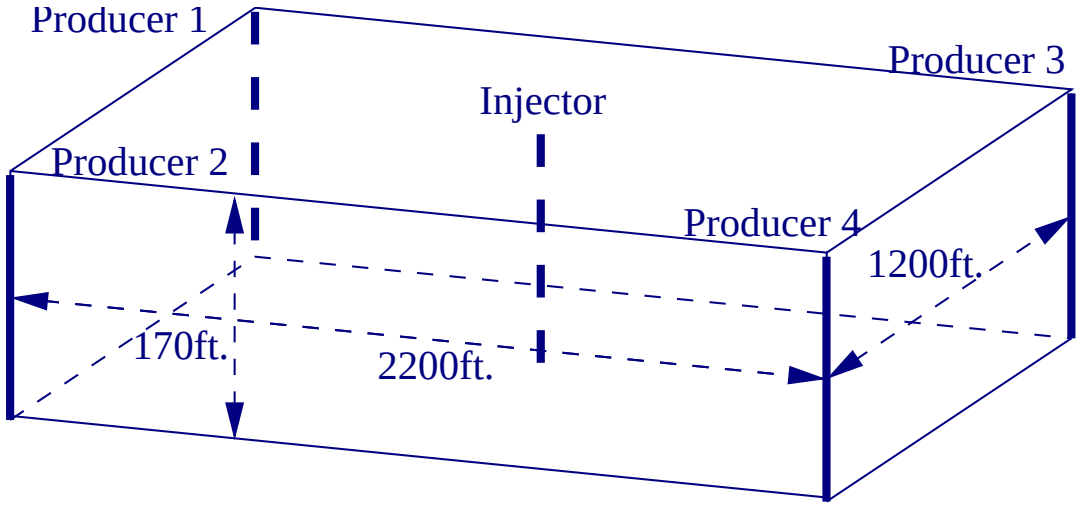
Find $v \in V = span\,\{\psi_{ij}\}$ and $p \in U$ such that

$$\begin{aligned} \int_\Omega (k\lambda)^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= \int_\Omega \frac{\lambda_g}{\lambda} G \cdot u \, dx \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx \end{aligned}$$

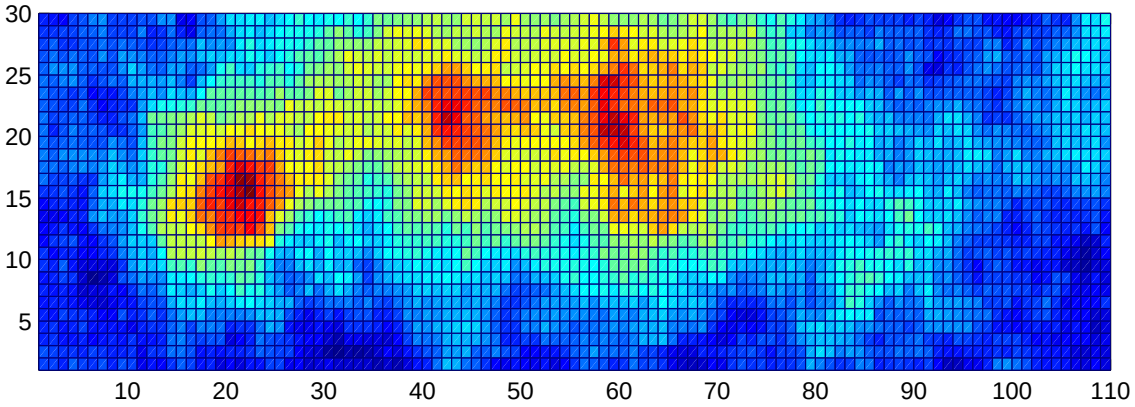
for all $u \in V$ and $l \in \mathcal{P}_0(\mathcal{K})$.

Numerical Test Case (two-phase flow)

The test case is an upscaled version of the 10th SPE comparative solution project, model 2. The model specifications are as follows.



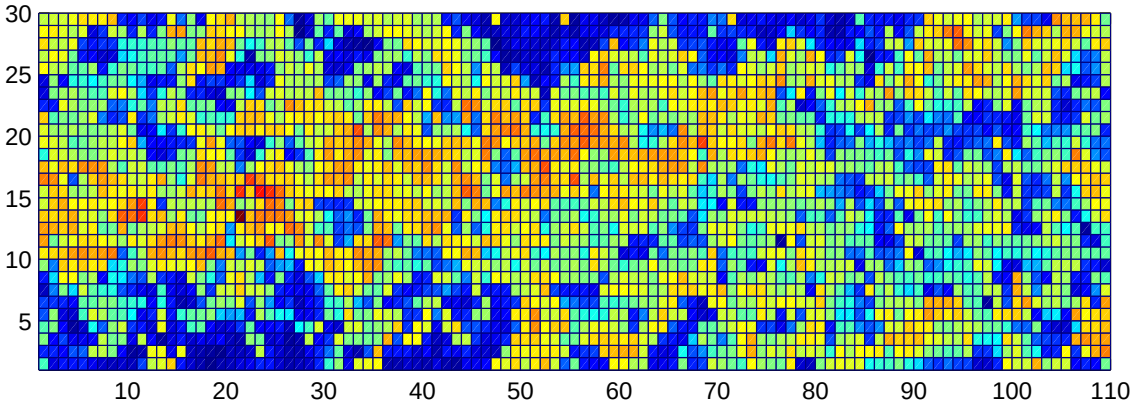
- $30 \times 110 \times 17$ cells.
- Sim. time: 2000 days.
- No-flow BC.



Tarbert formation

$30 \times 110 \times 7$ cells.

$5 \times 11 \times 1$ blocks.



Upper Ness formation

$30 \times 110 \times 10$ cells.

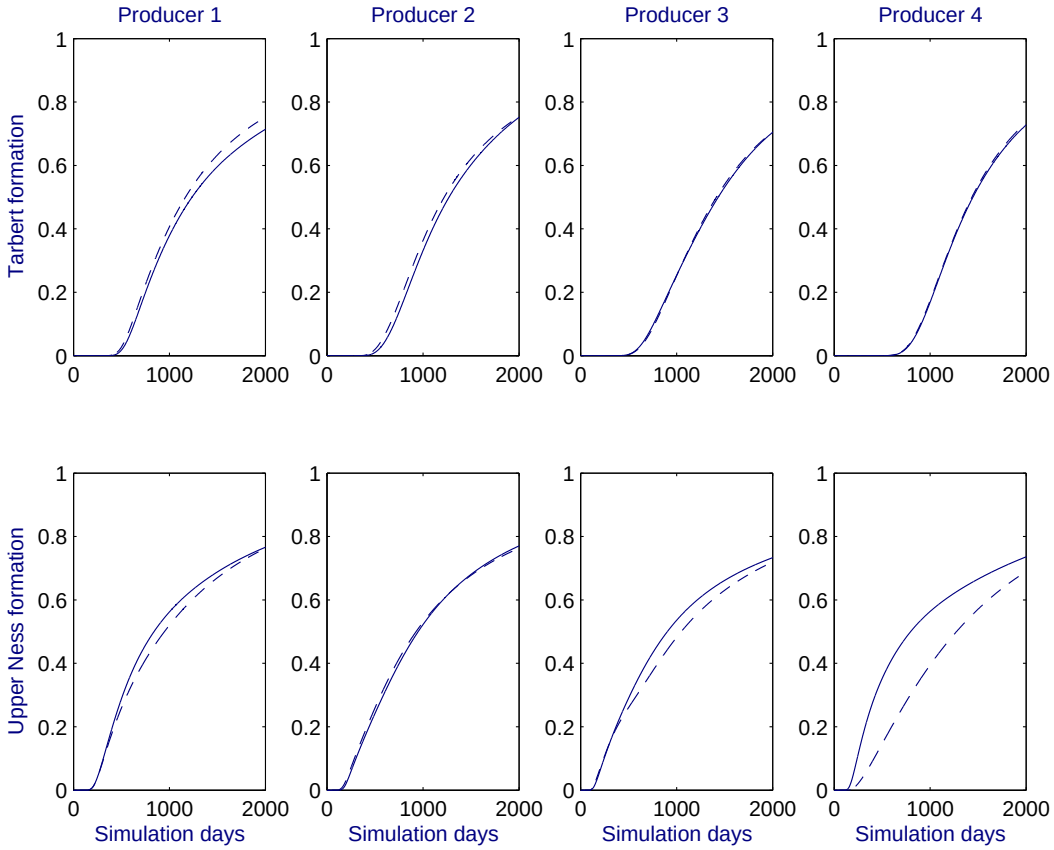
$5 \times 11 \times 2$ blocks.

To access the accuracy of the flow scenarios induced by the mixed MsFEM we compare the water-cut curve with the corresponding water-cut curve induced by a reference solution.

The water-cut curves plot the fraction of water in the produced fluid and is a more sensitive measure than the accumulated oil-production curves which are also frequently used.

We also quantify the discrepancy between the water-cut curves ω_i^{ref} and ω_i^{ms} for producer i with the norm measure

$$e_i^{ms} = \frac{\|\omega_i^{ref} - \omega_i^{ms}\|_{L^2}}{\|\omega_i^{ref}\|_{L^2}}.$$



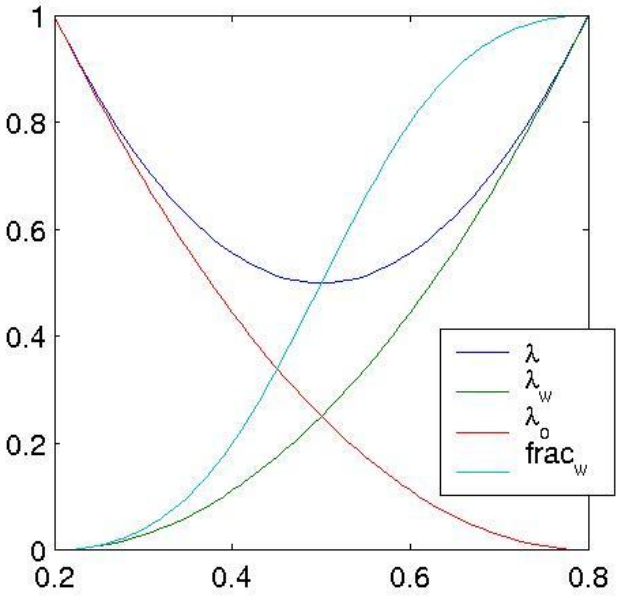
Steady-state two-phase flow: $\lambda \equiv 1$ and $\lambda_g = 0$.

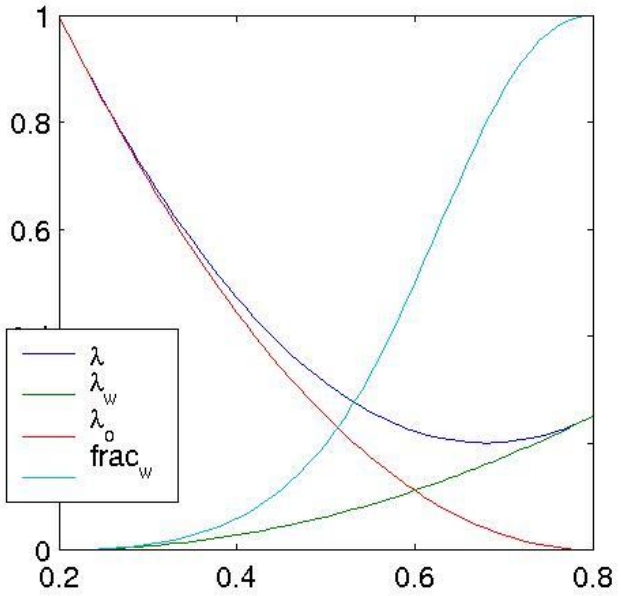
e_i^{ms}	Tarbert	U.Ness
e_1^{ms}	.044	.075
e_2^{ms}	.022	.024
e_3^{ms}	.008	.040
e_4^{ms}	.008	.174

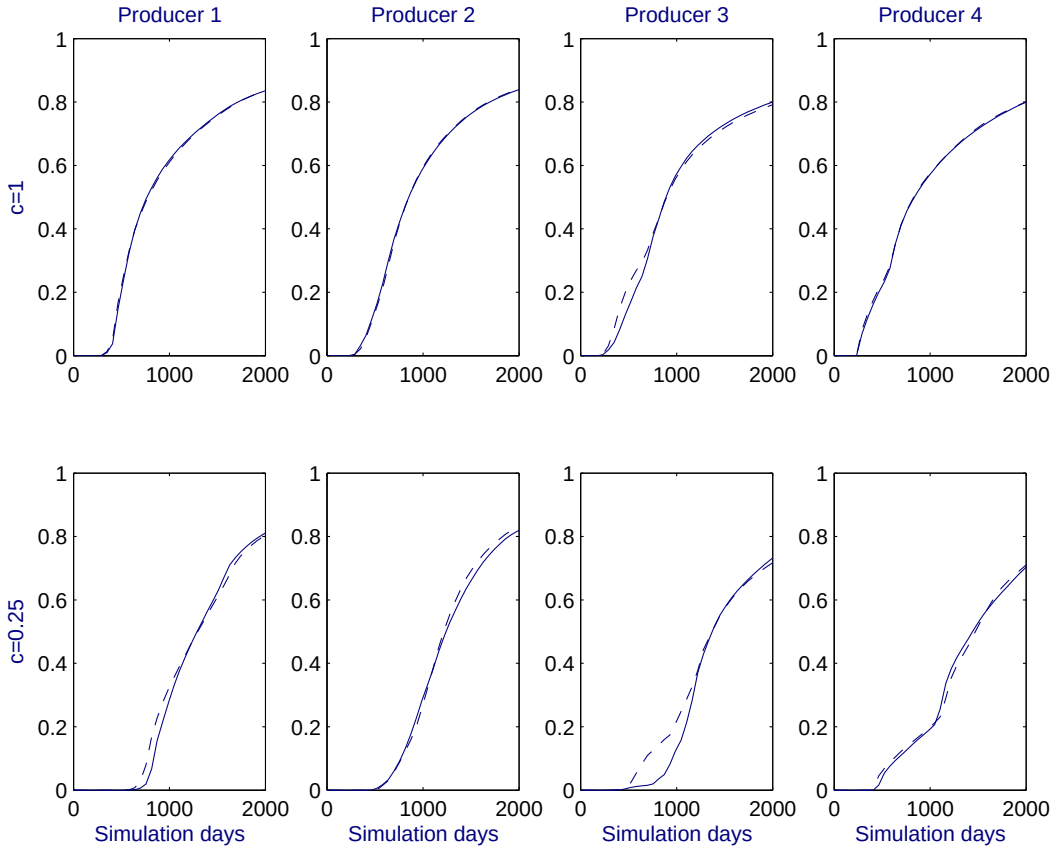
We now consider dynamic two-phase flow and define

$$\lambda_w(S) = c\left(\frac{S - S_{rw}}{1 - S_{rw} - S_{ro}}\right)^2 \quad \text{and} \quad \lambda_o(S) = \left(\frac{1 - S - S_{ro}}{1 - S_{rw} - S_{ro}}\right)^2$$

where $S_{rw} = S_{ro} = 0.2$ are the residual water and oil saturations.


1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
λ
λw
λo
fracw


1
0.8
0.6
0
0.2
0.4
0.6
0.8
λ
λw
λo
fracw



Dynamic two-phase flow: $\lambda = \lambda_w + \lambda_o$ and $\lambda_g = \rho_w \lambda_w + \rho_o \lambda_o$.

e_i^{ms}	$c = 1$	$c = .25$
e_1^{ms}	.007	.035
e_2^{ms}	.006	.020
e_3^{ms}	.031	.067
e_4^{ms}	.006	.022

Concluding Remarks

- The key to the success of the multiscale finite element methods (MsFEMs) is the selection of proper boundary conditions.
- MsFEMs may be used to accelerate simulations or give improved accuracy at a finer scale. Their flexibility also make them amenable for solving problems with irregular unstructured grids.
- Unexplored issues include "adaptivity without refinement" which can be used to model e.g., crack propagation in materials.