The GeoScale Project – Reservoir Simulation on a Geological Scale

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Outline

1. The GeoScale Project
   - Objectives
   - Facts
2. Multiscale Pressure Solution
   - Background: Upscaling
   - Multiscale Mixed Finite Elements
   - Accuracy and Robustness
   - Computational Complexity
   - Advantage: Flexibility
3. Future Work

The GeoScale Project (2004–08)

Primary objective:

Establish mathematical and numerical technology that facilitates direct simulation on high-resolution geomodels in 3D.

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Secondary objectives:

- Develop better simulation methods for industry-standard geomodels with small-scale heterogeneity, irregular grids and multiple wells.
  - Simulations should run within a few hours timeframe on standard desktop computers
  - Simulations should scale well with increasing computational resources
- Promote technology to industrial end-users
- Establish industrial funding

The GeoScale Project (2004–08)

Partners:

SINTEF and Univ. Bergen, Oslo, Trondheim

Funding:

1,8 million $ over 4 years from Research Council of Norway

+ 3-4 PhD grants (RCN, UoB, NTNU)

+ 2 postdoc grants (RCN, EU)

Collaboration:

Stanford, Texas A&M, ETH Zürich,

Schlumberger Moscow Research, Statoil Research Centre

Contact:

`http://www.math.sintef.no/geoscale/`

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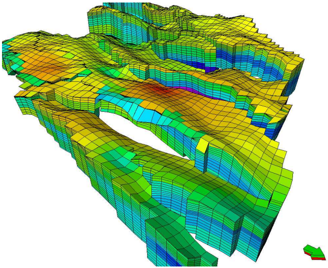
Simulation on Geological Models

For various reasons, there is a need for direct simulation on high-resolution geomodels. This is difficult:

- $\mathbf{K}$ spans many length scales and has multiscale structure

$$\max \mathbf{K} / \min \mathbf{K} \sim 10^3\text{–}10^{10}$$

- Details on all scales impact flow



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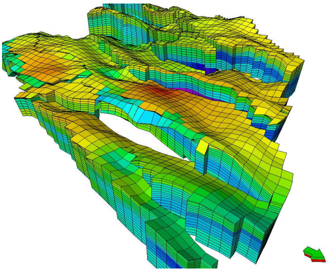
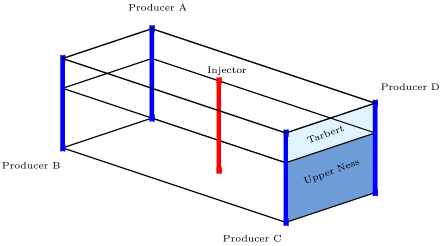
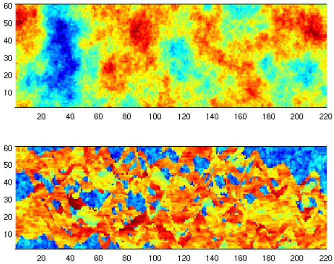


Gap between simulation models and geomodels:

- High-resolution geomodels may have $10^7 - 10^9$ cells
- Conventional simulators are capable of about $10^5 - 10^6$ cells

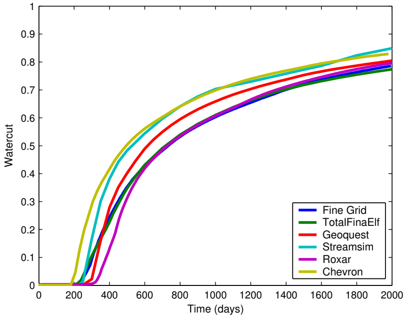


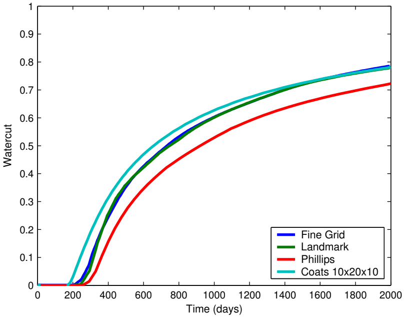


- Geomodel: $60 \times 220 \times 85 \approx 1,1$ million grid cells
- Simulation: 2000 days of production

10th SPE Comparative Solution Project

Upscaling results reported by industry



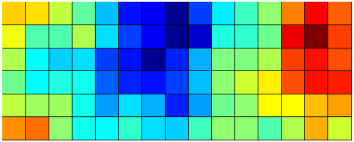


We seek a methodology that:

- gives a detailed image of the flow pattern on the fine scale, without having to solve the full fine-scale system
- is robust and flexible with respect to the coarse grid
- is robust and flexible with respect to the fine grid and the fine-grid solver
- is accurate and conservative
- is fast and easy to parallelise

Standard method

Upscaled model:



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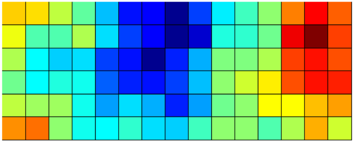

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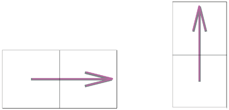
Building blocks:



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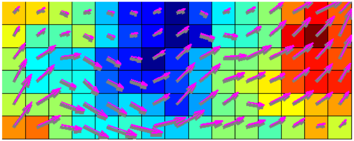

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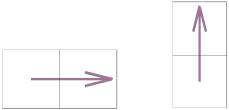

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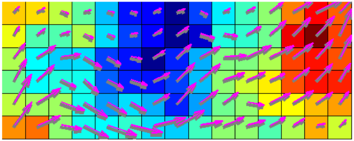

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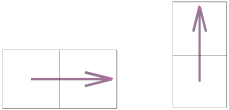
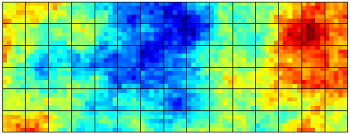


Two-scale method

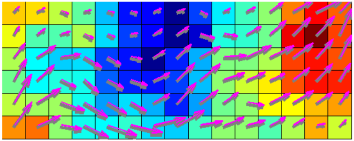
Geomodel:



From Upscaling to Multiscale Methods

Standard method

Upscaled model:



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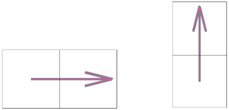

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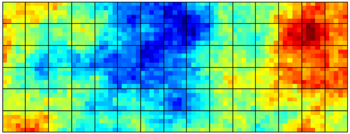

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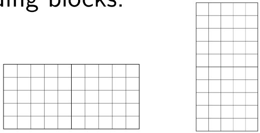

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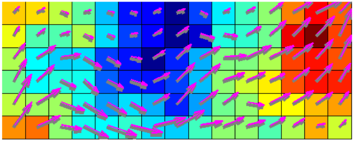

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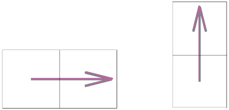

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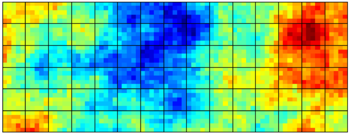

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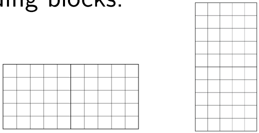

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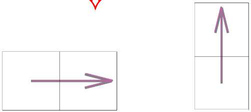

From Upscaling to Multiscale Methods

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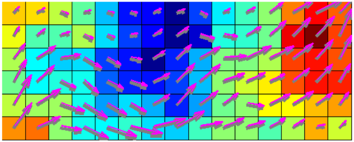

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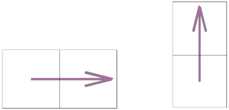

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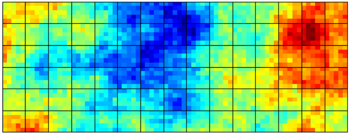

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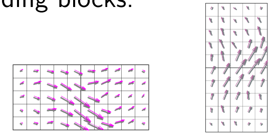

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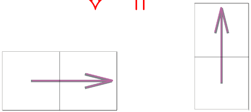

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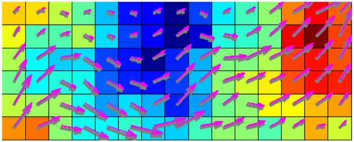

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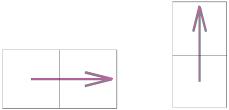

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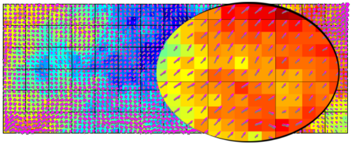

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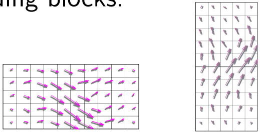

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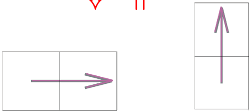


Mixed formulation:

Find $(v, p) \in H_0^{1,\text{div}} \times L^2$ such that

$$\int (\lambda K)^{-1} u \cdot v \, dx - \int p \nabla \cdot u \, dx = 0, \qquad \forall u \in H_0^{1,\text{div}},$$
$$\int \ell \nabla \cdot v \, dx = \int q \ell \, dx, \quad \forall \ell \in L^2.$$

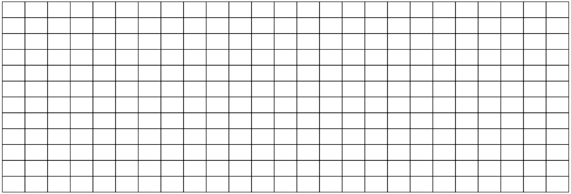
Multiscale discretisation:

Seek solutions in low-dimensional subspaces

$$U^{ms} \subset H_0^{1,\text{div}} \text{ and } V \in L^2,$$

where local fine-scale properties are incorporated into the basis functions.

We assume we are given a *fine* grid with permeability and porosity attached to each fine-grid block.



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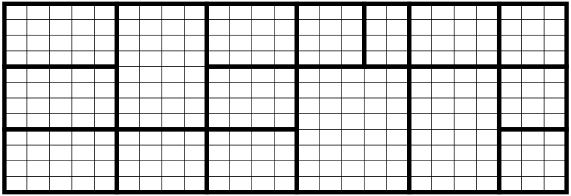

We construct a *coarse* grid, and choose the discretisation spaces V and U^{ms} such that:

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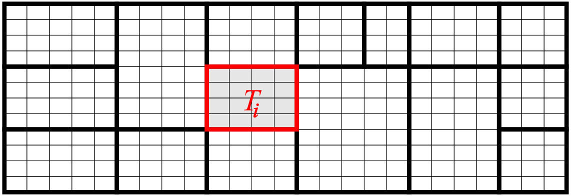

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- For each coarse block T_i, there is a basis function $\phi_i \in V$.

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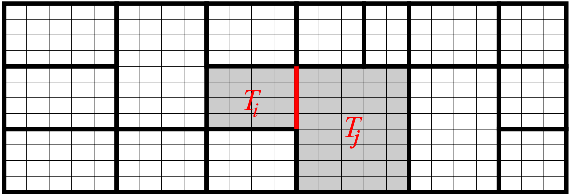

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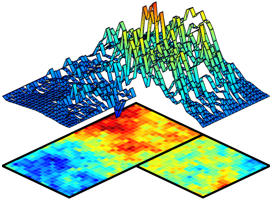
- For each coarse block T_i, there is a basis function $\phi_i \in V$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in U^{ms}$.

For each coarse edge Γ_{ij}, define a basis function

$$\psi_{ij} : T_i \cup T_j \to R^2$$

with unit flux through Γ_{ij} and no flow across $\partial(T_i \cup T_j)$.



We use $\psi_{ij} = -\lambda K \nabla \phi_{ij}$ with

$$\nabla \cdot \psi_{ij} = \begin{cases} w_i(x), & \text{for } x \in T_i, \\ -w_j(x), & \text{for } x \in T_j, \end{cases}$$

with boundary conditions $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$.

If $\int_{T_i} q dx \neq 0$ (T_i contains a source), then

$$w_i(x) = \frac{q(x)}{\int_{T_i} q(\xi)\, d\xi}.$$

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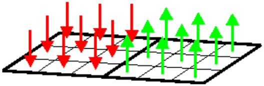

Multiscale Mixed Finite Elements
The Source Weights

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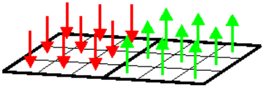
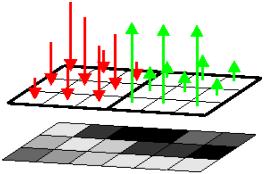


or to avoid high flow through low-perm regions

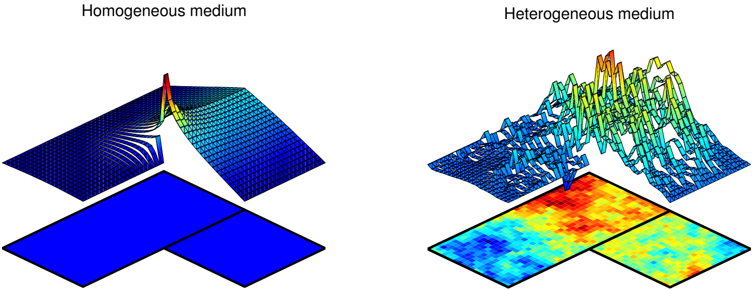
$$w_i(x) = \frac{\text{trace}(K(x))}{\int_{T_i} \text{trace}(K(\xi))\, d\xi}.$$



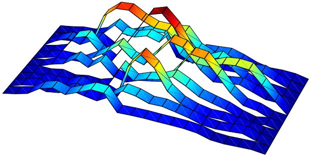
The latter is more accurate - even for strong anisotropy.

Homogeneous coefficients and rectangular support domain:
basis function = lowest order Raviart-Thomas basis

MsMFEM = extension to cases with subscale variation in coefficients and non-rectangular support domain



Velocity basis functions ψ_{ij} $\Longrightarrow$ Coarse-grid approximation space



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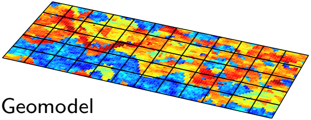
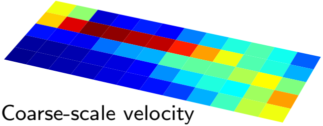


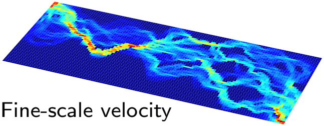
Geomodel

$\Downarrow$



Coarse-scale velocity

$\Downarrow$



Fine-scale velocity

For the MsMFEM the fine-scale velocity field is a linear superposition of basis functions: $v = \sum_{ij} v_{ij}^{*} \psi_{ij}$.

Multiscale:
Incorporates small-scale effects into coarse-scale solution

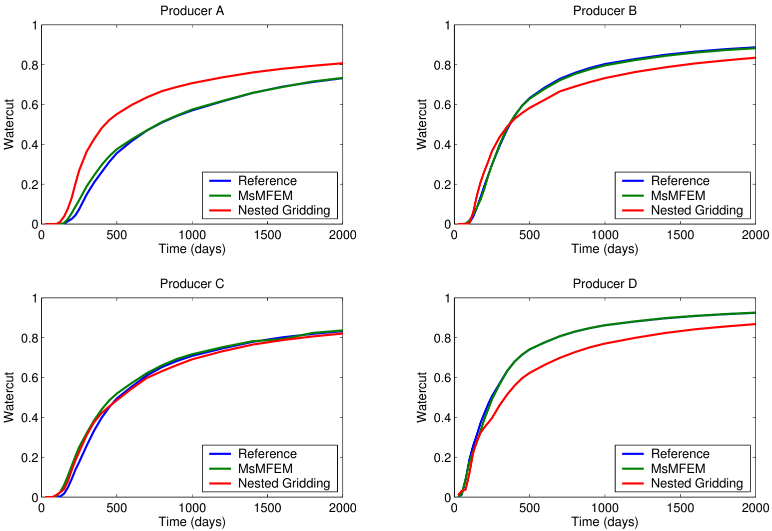
Conservative:
Mass conservative on coarse grid and on the subgrid scale

Scalable:
Well suited for parallel implementation since basis functions are processed independently

Flexible:
No restrictions on subgrids and subgrid numerical method. Few restrictions on the shape of the coarse blocks

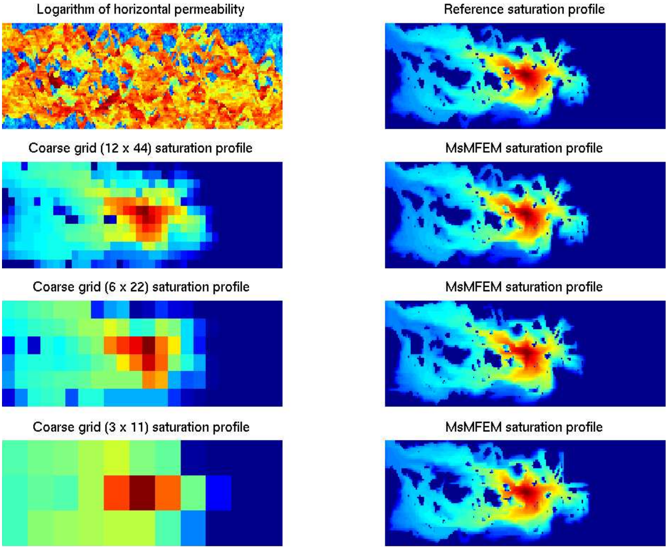
Fast:
The method is fast when avoiding regeneration of (most of) the basis functions at every time step

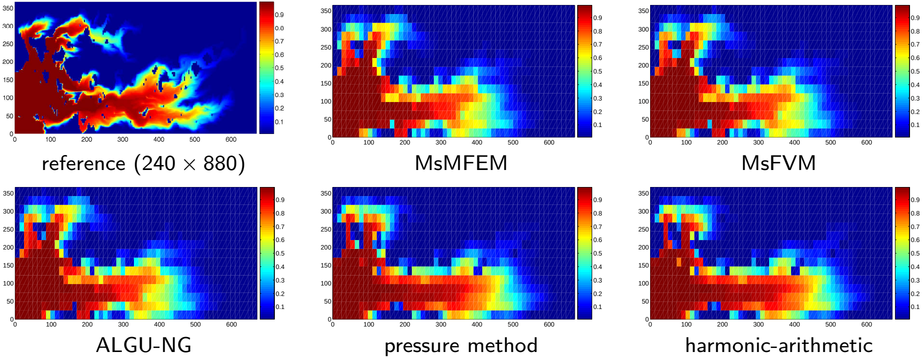
Advantages: Accuracy
SPE10 Benchmark ($5 \times 11 \times 17$ Coarse Grid)



Nested gridding: upscaling + downscaling

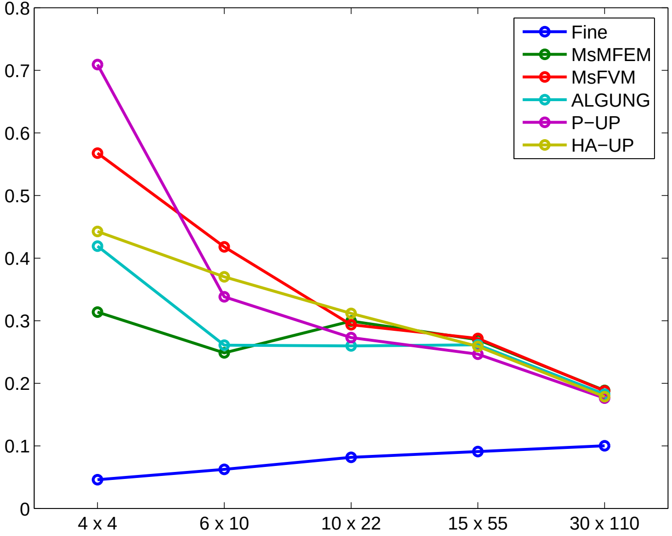
Advantage: Robustness
SPE10, Layer 85 (60×220 Grid)



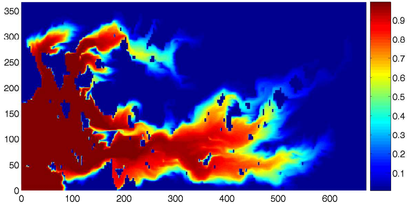


saturation computed on the coarse grid

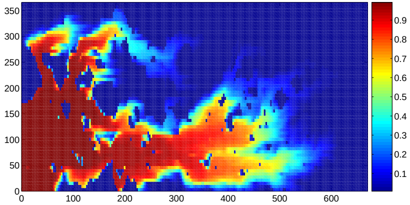
Multiscale vs. Upscaling
Saturation Errors on the Upscaled Grid



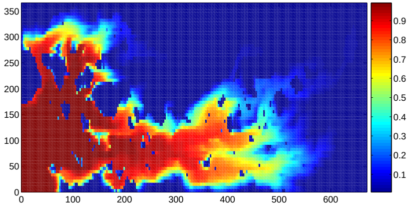
Multiscale vs. Upscaling/Downscaling



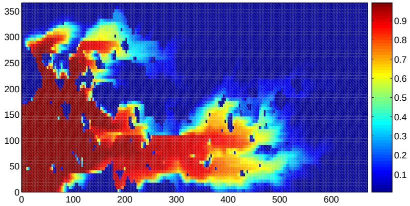
reference (240 × 880)



MsMFEM

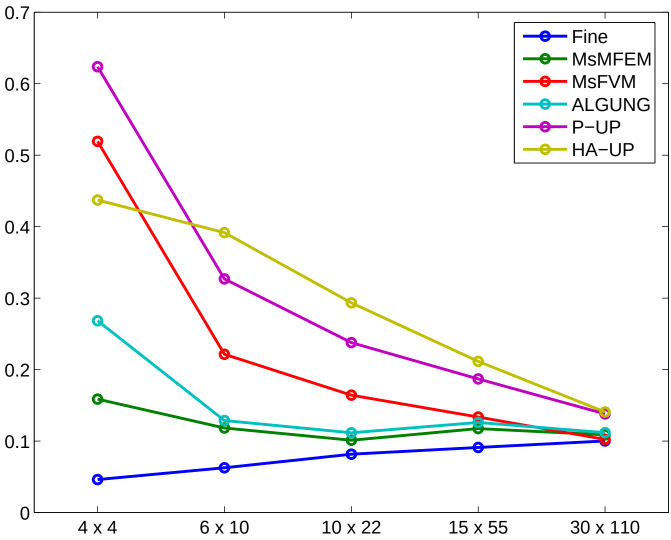


MsFVM



ALGU-NG

Multiscale vs. Upscaling
Saturation Errors on the Fine Grid



Assume:

- $N = n \cdot m$ grid blocks in geomodel
- n coarse blocks, each containing m fine blocks
- linear algebra with complexity N^α for N unknowns

Direct solution:

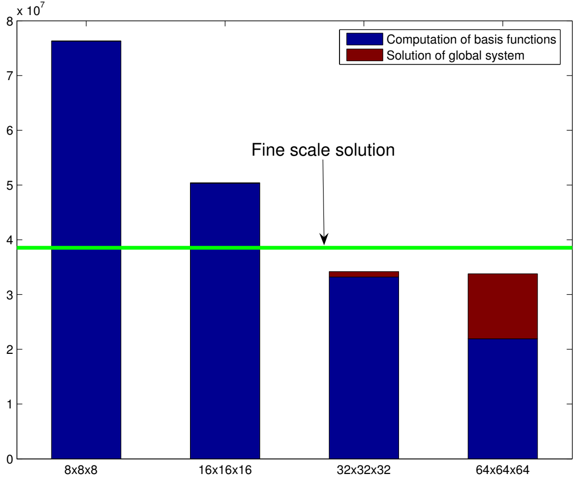
N^α operations for a two-point finite volume method

Multiscale solution:

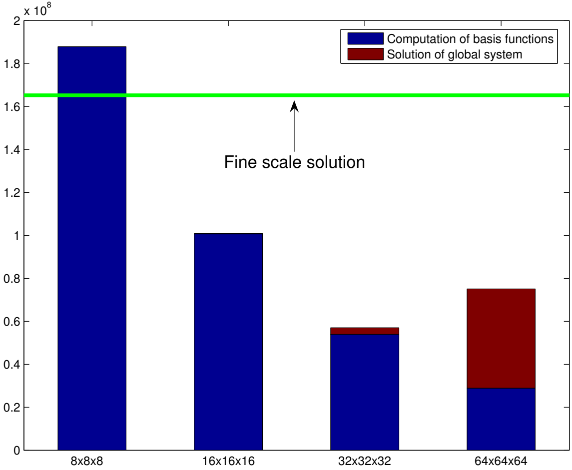
$dn \cdot (2m)^\alpha + (dn)^\alpha$ operations using a two-point FVM for fine-scale solution

Computational Complexity, cont'd

Example 3D (128x128x128), $\alpha = 1.2$



Example 3D (128x128x128), $\alpha = 1.3$



In practice:

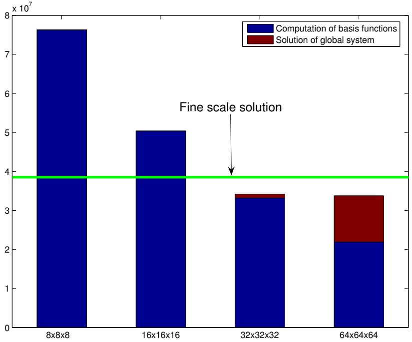
- Assembly time may become significant when solving many small problems since vectorization is harder.
- Efficient linear solvers typically require an initial setup phase, therefore the solution of many small systems may be more time-consuming than anticipated.

Direct solution may be more efficient, so why bother with multiscale?

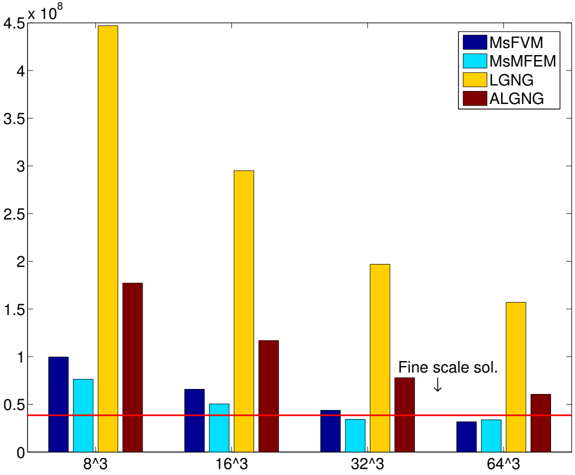
- Full simulation: $\mathcal{O}(10^2)$ time steps.
- Basis functions need not be recomputed

Also:

- Possible to solve very large problems
- Easy parallelization



Example: 3D (128x128x128), $\alpha = 1.2$ and $k = 3$

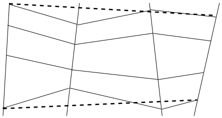


Flexibility

Multiscale mixed formulation:

coarse grid = union of cells in fine grid

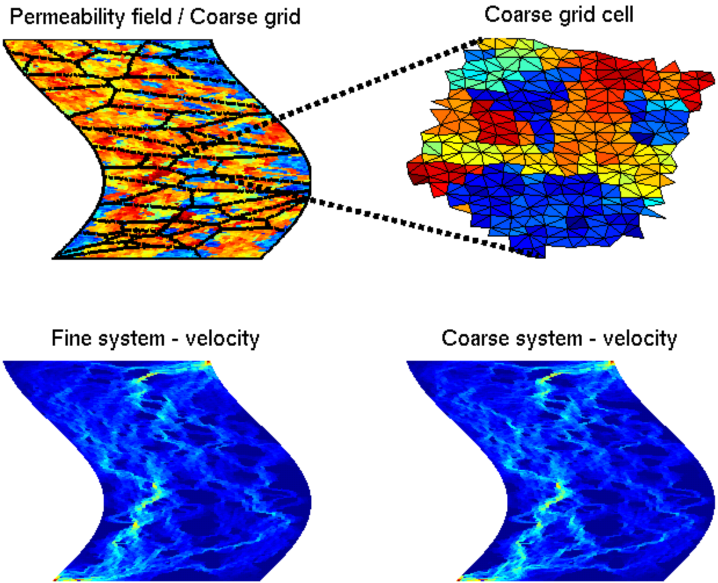
- Given a numerical method that works on the fine grid, the implementation is straightforward.
- One avoids resampling when going from fine to coarse grid, and vice versa



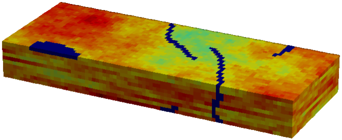
Other formulations:

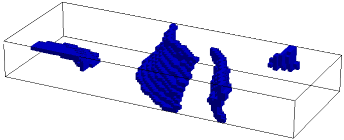
MsFVM and (A)LGU: based upon *dual grid* $\longrightarrow$ special cases that complicate the implementation

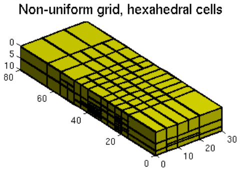
Flexibility wrt. Grids

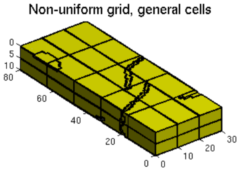


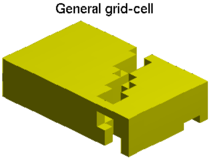
Flexibility wrt. Grids
Around Flow Barriers



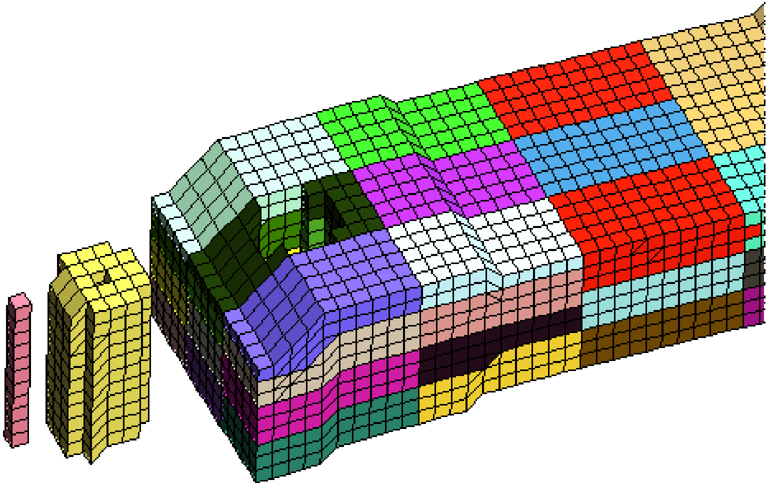






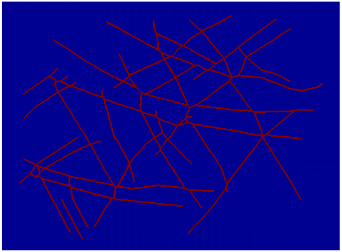


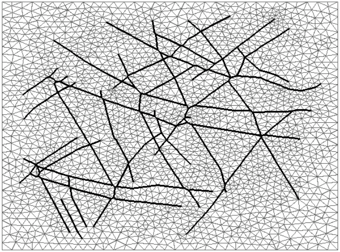
Flexibility wrt. Grids
Around Wells

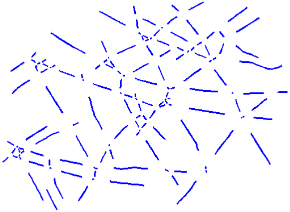


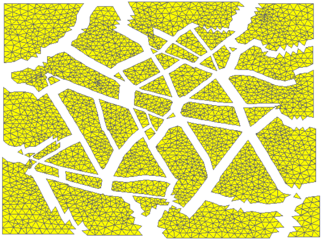
Flexibility wrt. Grids

Fracture Networks



[1]





[1] Courtesy of M. Karimi-Fard, Stanford

Future Work

Multiscale methods

- Well models (adaptive gridding, multilaterals)
- More general grids (block-structured, PEBI, ..)
- Compressibility, multiphase and multicomponent
- Adaptivity
- Fractures and faults

Applications:

- Multiscale history matching
- Carbonate reservoirs