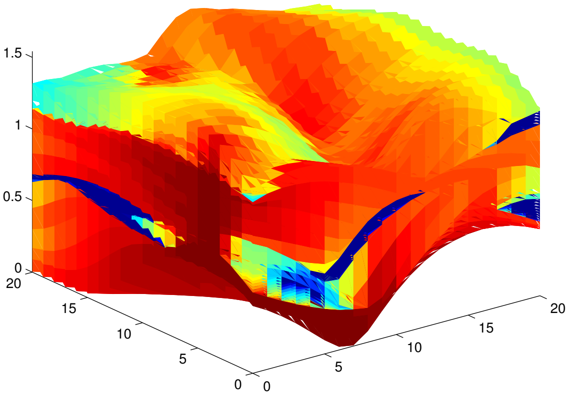
Multiscale methodology for flow in porous media

A versatile tool for handling unstructured corner-point grids

Jørg E. Aarnes and Yalchin Efendiev



A multiscale methodology for porous media flow

A multiscale mixed finite-element method (MsMFEM) for elliptic and slightly parabolic problems (pressure equation)

$$-\nabla \cdot (k(x)\nabla p) + c(x)\frac{\partial p}{dt} = q.$$

A two-scale "upscaling/downscaling" approach for hyperbolic problems (saturation equations)

$$\phi\frac{\partial S}{dt} + \nabla \cdot (f(S)v) = q_w.$$

Mixed finite element methods: $-\nabla \cdot (k(x)\nabla p) = q$

In mixed FEMs one seeks $v \in V$ and $p \in U$ such that

$$\begin{aligned} \int_\Omega k^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= 0 && \forall u \in V, \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx && \forall l \in U. \end{aligned}$$

Here $V \subset \{v \in (L^2)^d : \nabla \cdot v \in L^2\}$ and $U \subset L^2$.

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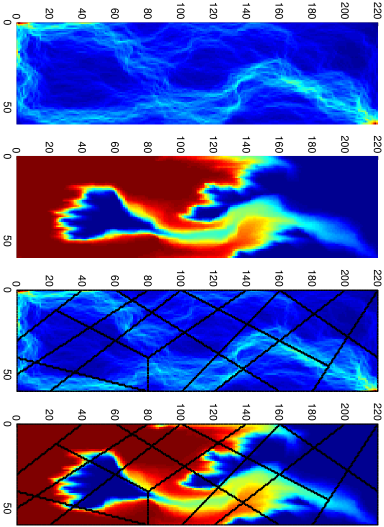
In MsMFEM the approximation space V for velocity v is designed so that it embodies the impact of fine scale structures.



Accuracy: flow scenarios match closely fine grid simulations.

Mass conservation: conserves mass on coarse and fine grids.

Efficiency: basis functions can be computed in parallel and need not be recomputed.

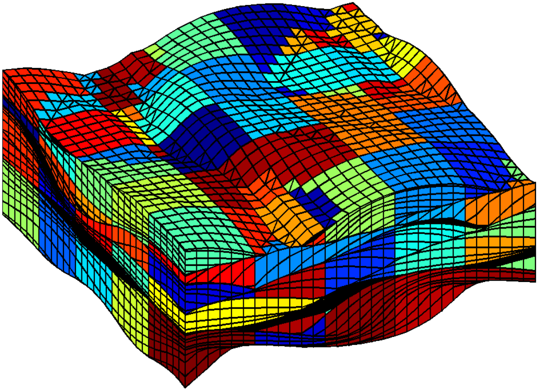
Flexibility: unstructured and irregular grids are handled easily.

Robustness: suitable for models with highly oscillatory coefficients and large grid-cell aspect ratios.

MsMFEM simplifies "upscaling" and coarse grid generation

No need for resampling procedures

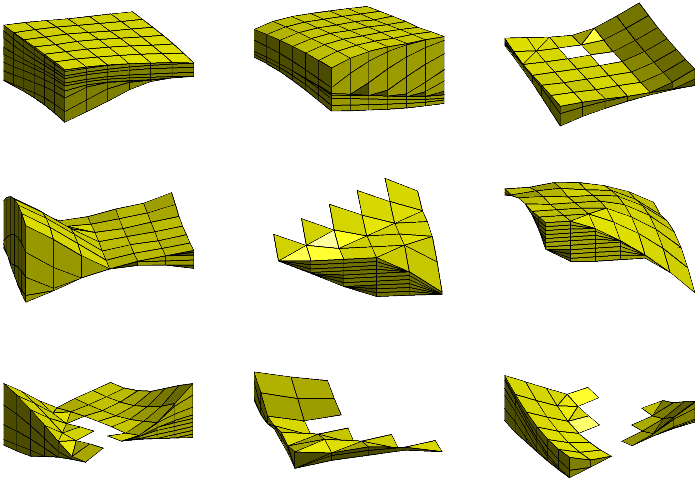
MsMFEM may utilize any coarse grid with blocks that consists of a connected collection of cells in an underlying fine grid



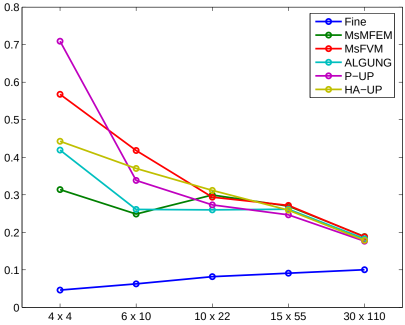
Multiscale mixed finite element methods

Examples of blocks that arise when partitioning corner-point grids in index space

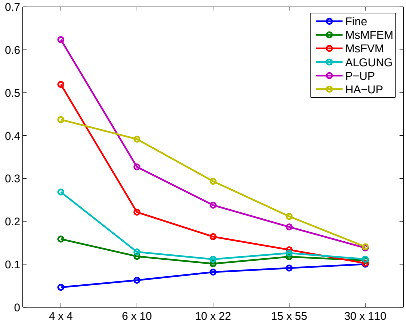
Disconnected blocks are split into a family of connected subblocks.



Multiscale methods versus upscaling methods



Coarse grid simulation



Fine grid simulation

To capitalize on the enhanced velocity resolution provided by multiscale methods we need to exploit subgrid details.

MsMFEM for porous media flow pressure equations

Conclusion I: MsMFEM provides a flexible, robust and efficient tool to get accurate velocity fields on fine grids. The saturation equation becomes the bottle-neck in large flow simulations.

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Challenge: Can we develop a similar multiscale methodology for solving the saturation equation more efficiently?

Assume that S^n is a saturation field on a fine grid $\{T\}$ at $t = t_n$, and denote non-degenerate fine grid interfaces by $\gamma_{ij} = \partial T_i \cap \partial T_j$.

1: For each block K in a coarse grid (not necessarily the same coarse grid as for MsMFEM), do

$$\bar{S}^{n+1}|_K = \bar{S}^n|_K + \frac{\triangle t}{\int_K \phi\, dx}\left[\int_K q_w\, dx - \sum_{\gamma_{ij}\subset\partial K} F_{ij}(S^n)\right],$$

where $F_{ij}(S) = \max\{f_w(S_i)v_{ij}, -f_w(S_j)v_{ij}\}$.

2: Map $\bar{S}^{n+1}|_K$ onto the fine grid: $S^{n+1}|_K = I_K(\bar{S}^{n+1})$.

Coarse-to-fine grid interpolation:

Basic idea: Given a saturation value $\bar{S}$ in a coarse grid block K, define χ_K so that $I_K(\bar{S}) = \chi_K(x, t(\bar{S}))$ gives a plausible saturation field inside K. Here $t(\bar{S})$ is defined so that mass is preserved:

$$\int_K I_K(\bar{S})\phi \, dx = \bar{S}|_K \int_K \phi \, dx.$$

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Local approach: χ_K is defined by

$$\phi \frac{\partial \chi_K}{\partial t} + \nabla \cdot (f(\chi_K) v(x, t_0)) = q_w \text{ in } K,$$

$\chi_K(x, 0) = S(x, t_0)$, and $f(\chi_K) = 1$ on the inflow part of ∂K.

Global approach: $\chi_K(x, t) = S^0(x, t)|_K$, where S^0 is a solution of the global saturation on an arbitrary subgrid of the coarse grid.

Multiscale method with local approach

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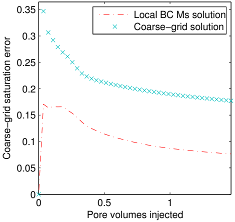
Model 2 from 10th SPE comparative solution project (Cartesian grid).

- **Fine grid:** 1.122 $\cdot 10^6$ cells.
- **Coarse grid:** 2244 blocks.



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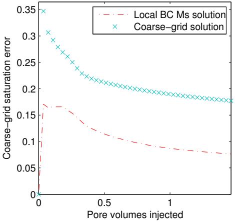
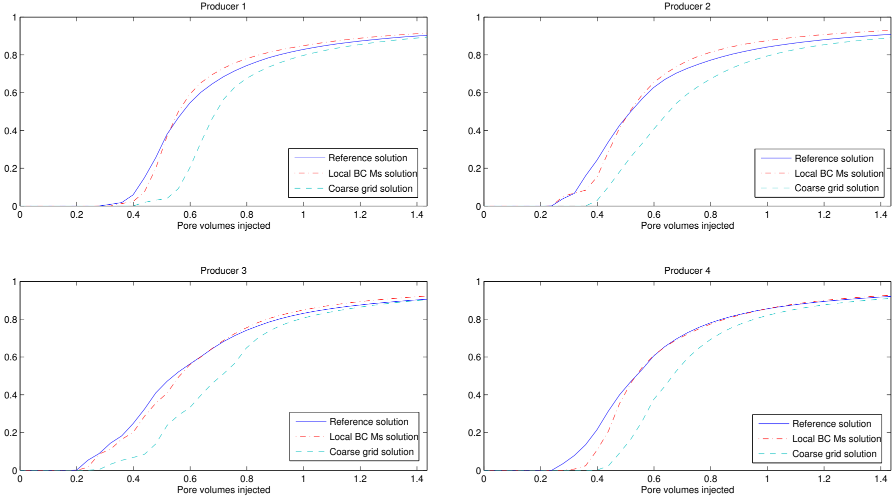


+ Very flexible w.r.t. grids (same constraint as MsMFEM).
+ Flow physics handled according to fine-scale model.
– Limited applicability: Assumes that flow patterns do not change significantly throughout simulation.

Local approach: Prediction of oil-production

Water-cut curves (fraction of water in produced fluid) for producers in SPE benchmark.



Multiscale method with global approach

The global approach has similarities with upscaled simulation models that employ pseudo-relative permeability functions.

- **Application:** Run multiple simulations using one set of interpolation operators. (History matching and/or quantify uncertainty in oil-production forecasts).

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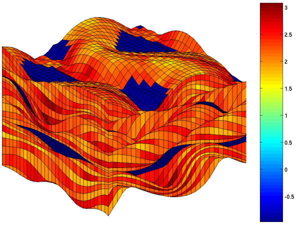
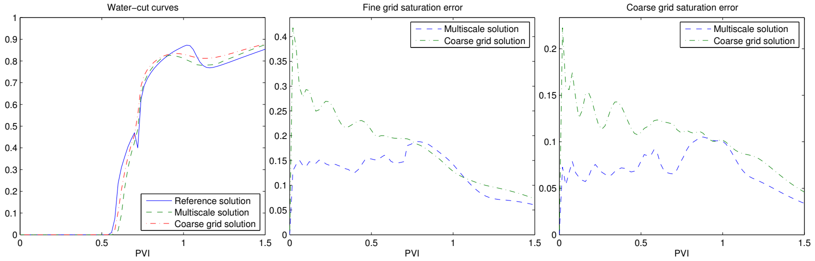
+ Flexible w.r.t. grids and physics modeled on fine-scale.

+/− Allows flow patterns to change during initial simulation, but flow patterns in subsequent simulations should not deviate a lot from flow patterns in initial simulation.

- **Velocity:** MsMFEM with a single set of basis functions.
- **History:** Well configuration changed at 0.7 PVI.
- **Permeability:** $K = 10^{\delta} K_0$, $\delta(x) \sim \text{Random}([-2, 2])$.



Logarithm of K_0



Why multiscale?

Development of multiscale methodologies for flow in porous media is not necessarily driven by a desire for higher resolution. Robustness and flexibility w.r.t grids is probably more important.

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Multiscale methods tend to have the following features:

- Easily parallelizable.
- Low memory requirements.
- Fine-scale computations part of a preprocessing step.
- Easy grid generation: No need for resampling procedures.
- More accurate and versatile than standard methods.

Conclusions

Status: Multiscale methods do not currently provide a full-fledged alternative to multi-phase flow upscaling, but it is our belief that they can help create more robust and flexible simulation tools.

The proposed methodology seems to be quite robust, and is straightforward to implement for complex unstructured grids.

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Next: More physics: miscible and compressible flow that can be dominated by gravity and/or capillary forces.