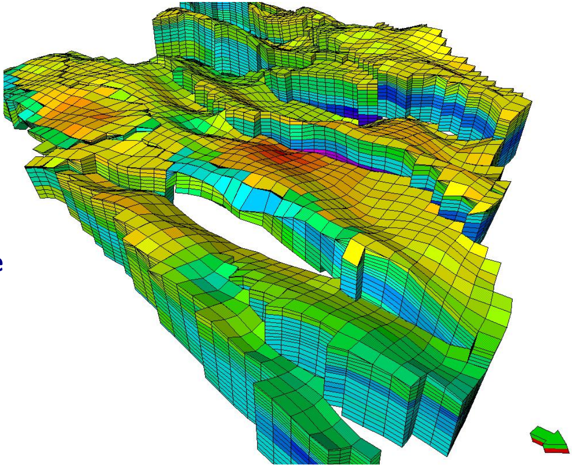
Are multiscale methods becoming mature for reservoir simulation studies on real-field models?



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Multiscale methods for reservoir simulation:

- Multiscale finite volume method
- Multiscale mixed finite element method
- Upscaling-downscaling approaches based on nested gridding

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Capture subgrid effects on coarse grids, and allow reconstruction of velocity fields on underlying fine grids

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Primary applications:

- Perform simulations on non-upscaled geological models
- Perform simulations on coarse grid models with complex geometrical features and/or complex grid block geometries

Prerequisites for real-field simulation studies

Prerequisite I: Applicable

Ability to handle unstructured industry standard geomodel grids.

Prerequisite II: Efficient

More efficient / more easily parallelizable / less memory requirements than fine grid solvers.

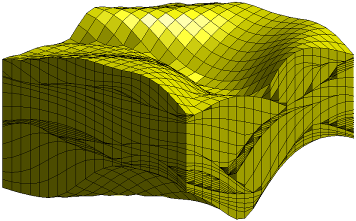
Prerequisite III: Better than upscaling

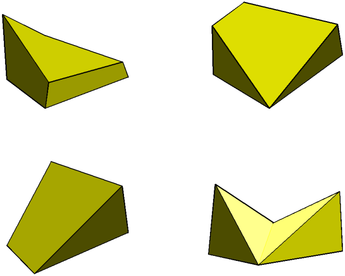
More accurate / less complex than upscaling based strategies.

Industry standard geomodel grids I: Corner-point grid

- Grid-cell corner-points lie on inclined pillars (lines).
- Layers may collapse to a hyperplane in certain regions.
- Non-collapsed cells have polyhedral shape with 5 – 8 corners.

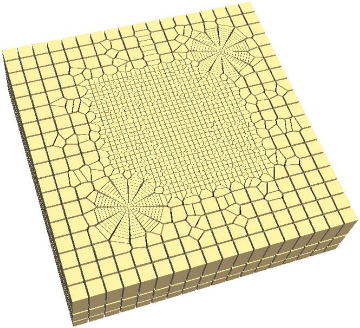
In physical space, corner-point grids are unstructured!

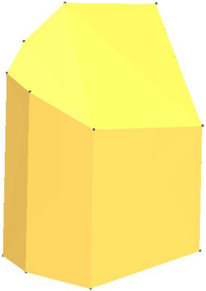




Industry standard geomodel grids II: PEBI grid

- **Voronoi grid:** Each cell is a convex polyhedron P associated with a point $p \in \mathcal{P}$ such that if $x \in P$, then x is closer to p than any other point in $\mathcal{P}$.
- **PEBI grid:** Voronoi grid where the line that connects two neighboring points is perpendicular to the interface between the two corresponding Voronoi cells.





Consider the following model problem

Darcy's law:	$v = -k\left(\nabla p - \rho g \nabla D\right),$
Mass balance:	$\nabla \cdot v = q \quad \text{in } \Omega,$
Boundary conditions:	$v \cdot n = 0 \quad \text{on } \partial\Omega.$

The multiscale structure of porous media enters the equations through the absolute permeability k, which is a symmetric and positive definite tensor with uniform upper and lower bounds.

We will refer to p as pressure and v as velocity.

Mixed finite element methods

In mixed FEMs one seeks $v \in V$ and $p \in U$ such that

$$\begin{aligned} \int_\Omega k^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= \int_\Omega k^{-1} \rho g \nabla D \cdot u \, dx \quad \forall u \in V, \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx \quad \forall l \in U. \end{aligned}$$

Here $V \subset \{v \in (L^2)^d : \nabla \cdot v \in L^2, \; v \cdot n = 0 \text{ on } \partial\Omega\}$ and $U \subset L^2$.

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Multiscale mixed finite element method (MsMFEM)

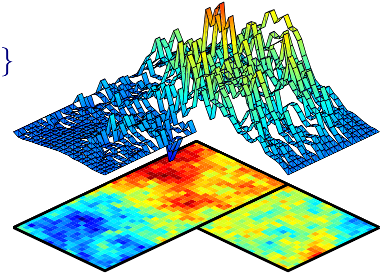
V designed to embody the impact of fine scale structures.

Associate a basis function χ_m for **pressure** with each grid block K:

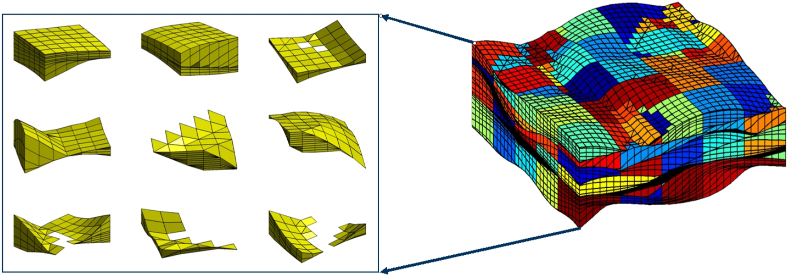
$$U = \text{span}\{\chi_m : K_m \in \mathcal{K}\} \quad \text{where} \quad \chi_m = \begin{cases} 1 & \text{if } x \in K_m, \\ 0 & \text{else,} \end{cases}$$

and a **velocity** basis function ψ_{ij} with each interface $\partial K_i \cap \partial K_j$:

$$\begin{aligned} V &= \text{span}\{\psi_{ij} = -k\nabla\phi_{ij}\} \\ \psi_{ij} \cdot n &= 0 \text{ on } \partial(K_i \cup K_j) \\ \nabla \cdot \psi_{ij} &= \begin{cases} q(K_i) & \text{in } K_i, \\ -q(K_j) & \text{in } K_j. \end{cases} \end{aligned}$$



MsMFEMs allows fully automated coarse gridding strategies: grid blocks need to be connected, but can have arbitrary shapes.



Uniform up-gridding: grid blocks are shoe-boxes in index space.

MsMFEM requires that a conservative numerical method is used to compute velocity basis functions.

Corner-point grid:

- TPFA or MPFA finite volume methods
- MFEM on tetrahedral subgrid of corner-point grid
- Mimetic finite difference method

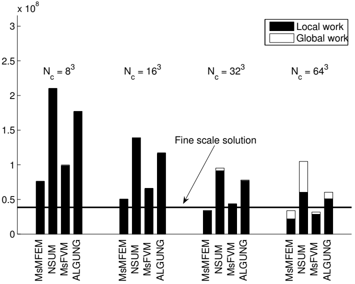
PEBI grid:

- TPFA finite volume method
- Mimetic finite difference method

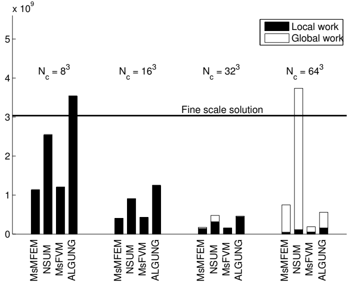
Given a subgrid discretization technique, MsMFEM applies!

Prerequisite II: Reduced computational complexity

Time $t(n)$ to solve a linear system of dimension n: $t(n) \sim O(n^{\alpha})$.



$\alpha = 1.2$



$\alpha = 1.5$

Cost of subgrid computations vs. coarse grid computations

MsFVM = Multiscale finite volume method (Jenny et al.)
NSUM = Numerical subgrid upscaling method (Arbogast et al.)
ALGUNG = Adaptive local-global upscaling + Nested gridding downscaling (Chen and Durlofsky)

Why MsMs offer significant savings in computation time

Efficiency

The pressure equation generally needs to be solved multiple times.

- Basis functions are computed only *once*.

Parallelization

Multiscale methods are easy to parallelize.

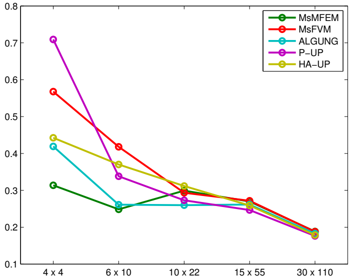
- Basis functions can be computed and processed independently.

Memory requirements

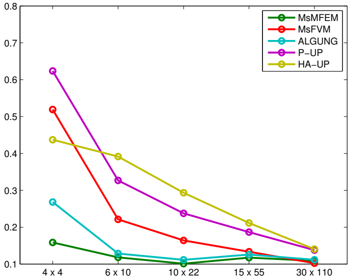
No need to store global fine-grid geomodel in memory.

- Fine-grid data can be distributed or loaded in patches.
- Solution of coarse grid system is requires significantly less memory than solution of global fine-grid system.

Cartesian coarse grids: MsMs tend to give enhanced accuracy only if simulations are performed on a subgrid of the coarse grid.



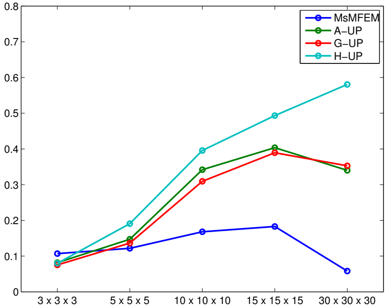
Coarse grid simulation



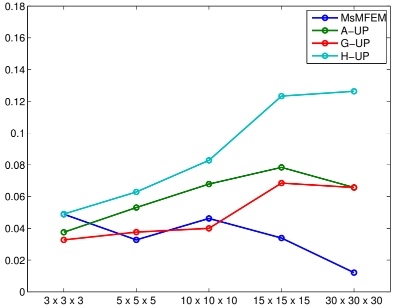
Fine grid simulation

Saturation errors relative to a reference solution.

Complex coarse grid-block geometries: MsMFEM is more accurate than upscaling, also for coarse grid simulation.



Coarse grid velocity errors



Coarse grid saturation errors

Corner-point grid model with layered log-normal geostatistics.

- √ Handles industry standard geological models.
- √ Offers significant savings in computation time, is easier to parallelize, and requires less memory than fine grid solvers.
- √ Provides a more robust and flexible alternative to upscaling.
- √ Provides a tool to perform reservoir simulation studies directly on large geological models.

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- $\surd$ Provides a more robust and flexible alternative to upscaling.
- $\surd$ Provides a tool to perform reservoir simulation studies directly on large geological models. **Valid option?**

High-resolution: Geomodels with multi-million cells and more.

Current high-resolution simulation applications for MsMFEM:

- A validation tool for geomodeling.
- Visualization of flow patterns and injector-producer pairs.
- History matching on moderate sized geomodels ($\sim 10^6$ cells).

  MsMFEM + streamline methods have been used to history match a geomodel with 32 injectors, 69 producers, and approximately 1 million cells.

Applications that are currently out of bound:

- History matching on very large geomodels.
- Flows strongly influenced by capillary forces.

High-resolution simulations require simplifying assumptions:

- Capillary forces are negligible.
- Gravity forces can be handled by operator splitting.

Options for modeling transport:

- **Streamline methods** – efficiency decays for compressible flows, and when frequent pressure updates are needed.
- **Implicit upstream schemes** – can be made as efficient as streamline methods, and are more generic.

How to make implicit upstream schemes efficient

Topological sort

By arranging cells in a directed graph, systems that arise from implicit upstream schemes can be solved using a sequential cell-by-cell Newton iteration.

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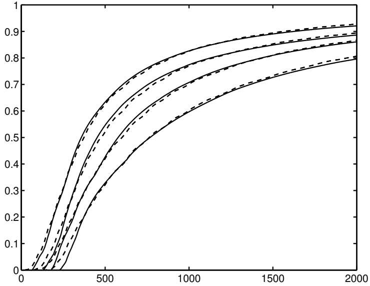
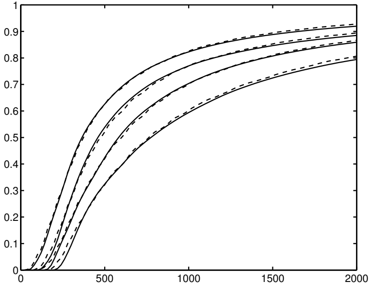
Key features

- System with n unknowns solved in $\mathcal{O}(n)$ operations.
- Cell-wise newton iterations.
- “No” grid restrictions.
- Low memory requirements.
- Easy to parallelize.

Water-cut curves for each producer computed with first-order upwind scheme (solid line) and FrontSim (dashed line).



Δt=1 day: 40 minutes.



Δt=20 days: 2 minutes

Computer: AMD Athlon 64 X2 dual core processor (2×2.2 GHz).

*Christie and Blunt: *Tenth SPE Comparative Solution Project: A Comparison of Upscaling Techniques*

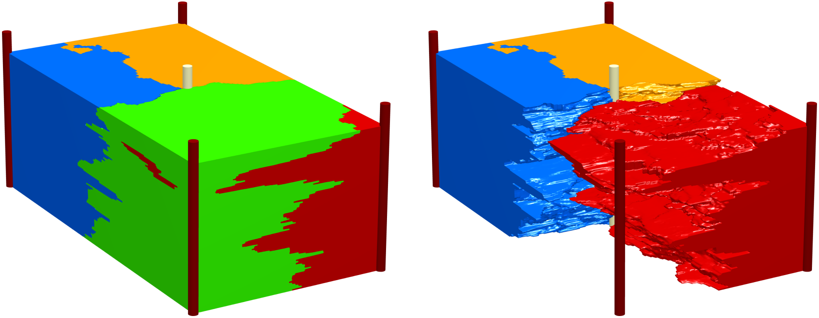
Example II: Compartmentalization of reservoir

MsMFEM and topological sorting routine

– also applicable to compartmentalize reservoirs:

$$v \cdot \nabla C_i = \begin{cases} 1 & \text{for each cell perforated by injector } i, \\ 0 & \text{otherwise.} \end{cases}$$

Model 2 of the Tenth SPE Comparative Solution Project



Conclusions and outlook

Status:

MsMFEM is a robust and versatile tool for reservoir simulation.

Aim:

An efficient and *seamless* methodology for oil reservoir simulation:

- Simulations with user-defined resolution and accuracy.

Road Ahead:

Further validation, more complex physics, and history matching.