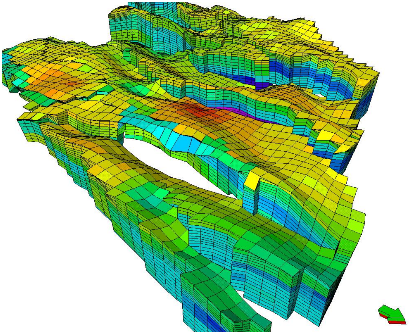
Generic multiscale framework for reservoir simulation that takes geological models as input



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NTNU:	Vegard Stenerud
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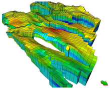
Motivation

Reservoir simulation workflow today:


Geomodel

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Upscaling

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Flow simulation

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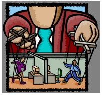

Management

Tomorrow:

- Earth Model shared between geologists and reservoir engineers
- Simulators take Earth Model as direct input
- Users allowed to specify grid-resolution at runtime to fit available computer resources and project requirements

Objective and implication

Main objective:

Build a generic framework for reservoir modeling and simulation capable of taking geomodels as input.

– *generic:* one implementation applicable to all types of models.

Value: Improved modeling and simulation workflows.

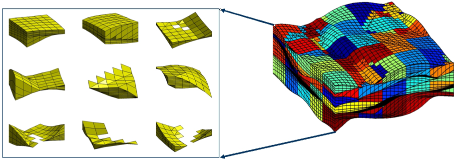
- Geologists may perform simulations to validate geomodel.
- Reservoir engineers gain understanding of geomodeling.
- Facilitate use of geomodels in reservoir management.

Discretization

Pressure equation:

- **Solution grid:** Geomodel — no effective parameters.
- **Discretization:** Multiscale mixed / mimetic method

Coarse grid: obtained by up-gridding in index space



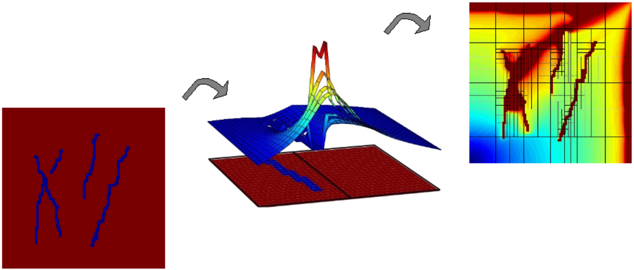
Mass balance equations:

- **Solution grid:** Non-uniform coarse grid.
- **Discretization:** Two-scale upstream weighted FV method — integrals evaluated on geomodel.
- **Pseudofunctions:** No.

Multiscale mixed/mimetic method (4M)

Generic two-scale approach to discretizing the pressure equation:

- Mixed FEM formulation on coarse grid.
- Flow patterns resolved on geomodel with mimetic FDM.



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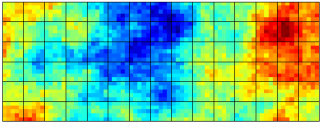

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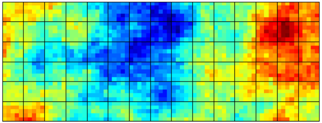

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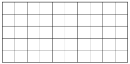
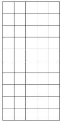
Coarse grid blocks:

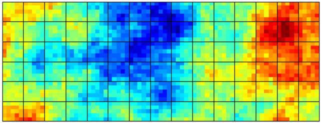




Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

Standard upscaling:



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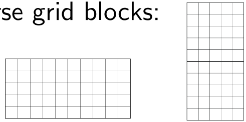

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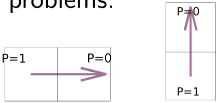

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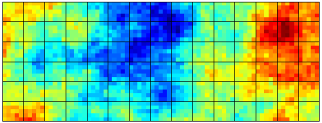

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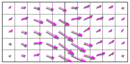

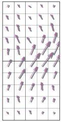

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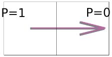

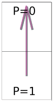

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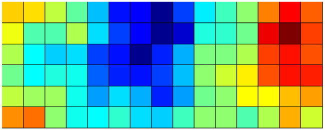

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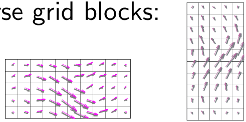

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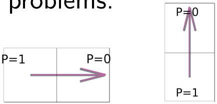

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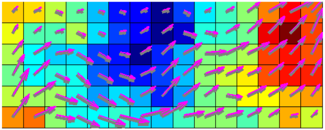

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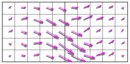

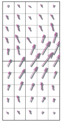

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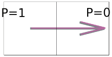

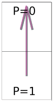

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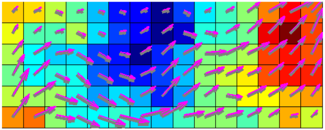

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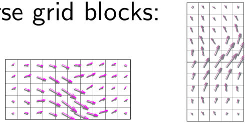

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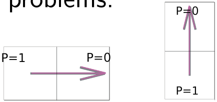
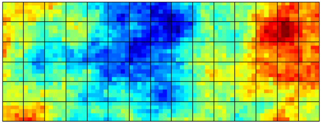


Multiscale method (4M):



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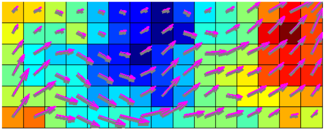

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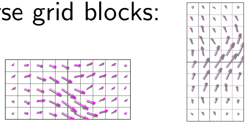

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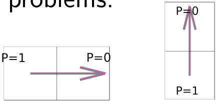

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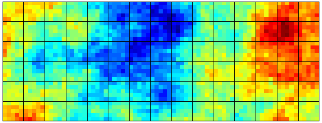

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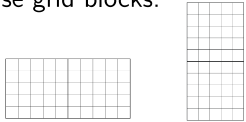

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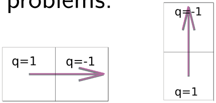

Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

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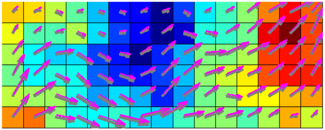

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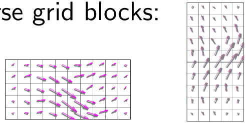

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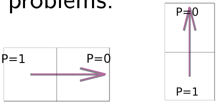

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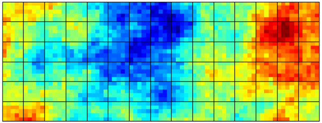

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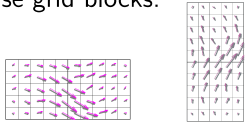

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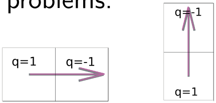
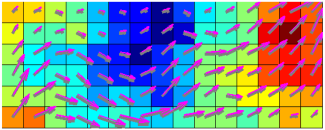


Standard upscaling:



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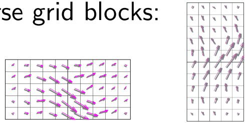

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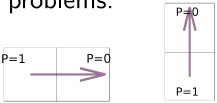

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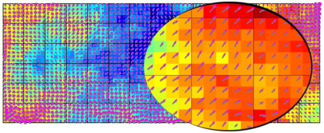

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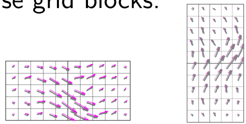

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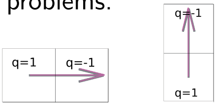


Discrete hybrid formulation: $(u,v)_m = \int_{T_m} u \cdot v \, dx$

Find $v \in V$, $p \in U$, $\pi \in \Pi$ such that for all blocks T_m we have

$$
\begin{aligned}
(\lambda^{-1}v, u)_m - (p, \nabla \cdot u)_m + \int_{\partial T_m} \pi u \cdot n \, ds &= (\omega g \nabla D, u)_m \\
(c_t \tfrac{\partial p_o}{dt}, l)_m + (\nabla \cdot v, l)_m + (\textstyle\sum_j c_j v_j \cdot \nabla p_o, l)_m &= (q, l)_m \\
\int_{\partial T_m} \mu v \cdot n \, ds &= 0.
\end{aligned}
$$

for all $u \in V$, $l \in U$ and $\mu \in \Pi$.

Solution spaces and variables: $\mathcal{T} = \{T_m\}$

$V \subset H^{\text{div}}(\mathcal{T})$, $U = \mathcal{P}_0(\mathcal{T})$, $\Pi = \mathcal{P}_0(\{\partial T_m \cap \partial T_n\})$.

v = velocity, p = block pressures, π = interface pressures.

Definition of approximation space for velocity:

The approximation space V is spanned by basis functions ψ_m^i that are designed to embody the impact of fine-scale structures.

Definition of basis functions:

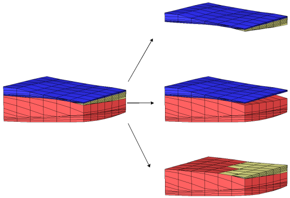
For each pair of adjacent blocks T_m and T_n, define ψ by

$$\psi = -K\nabla u \text{ in } T_m \cup T_n, \qquad \nabla \cdot \psi = \begin{cases} w_m & \text{in } T_m, \\ -w_n & \text{in } T_n, \end{cases}$$
$$\psi \cdot n = 0 \text{ on } \partial(T_m \cup T_n),$$

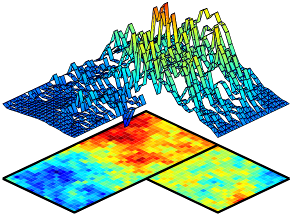
Split ψ: $\quad \psi_m^i = \psi|_{T_m}, \quad \psi_n^j = -\psi|_{T_n}.$

At initial time

Detect all adjacent blocks



Compute ψ for each domain



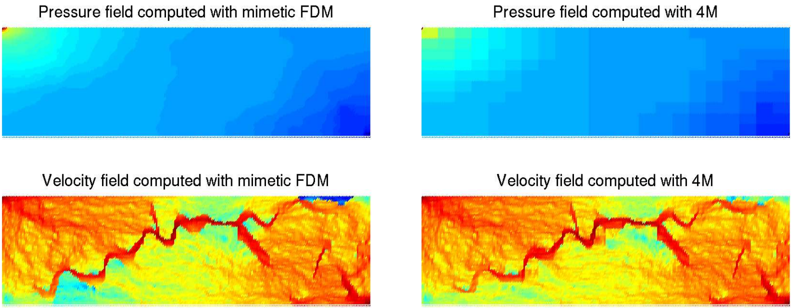
For each time-step:

- Assemble and solve coarse grid system.
- Recover fine grid velocity.
- Solve mass balance equations.

Multiscale mixed/mimetic method

Layer 36 from SPE10 model 2 (Christie and Blunt, 2001).

Example: Layer 36 from SPE10 (Christie and Blunt, 2001).



Primary features

- Coarse pressure solution, subgrid resolution at well locations.
- Coarse velocity solution with subgrid resolution everywhere.

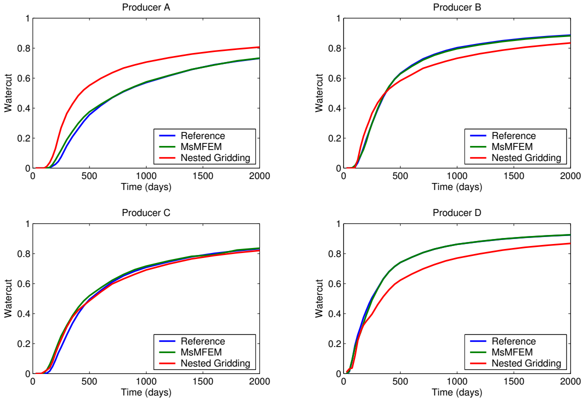
Model: SPE10 model 2, 1.1 M cells, 1 injector, 4 producers.

Coarse grid:
$5 \times 11 \times 17$

- Reference
- 4M
- Upscaling + downscaling

4M+streamlines: $\sim$ 2 minutes on desktop PC.

Water-cut curves at producers A–D



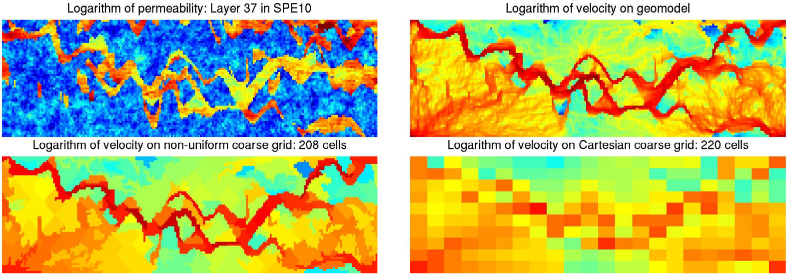
Modeling transport on the fine grid (e.g. geomodel) may be a bottle neck or infeasible.

Question: Can we derive a coarse grid formulation that exploits the key information in the high resolution velocity field?

Modeling transport on the fine grid (e.g. geomodel) may be a bottle neck or infeasible.

Question: Can we derive a coarse grid formulation that exploits the key information in the high resolution velocity field?

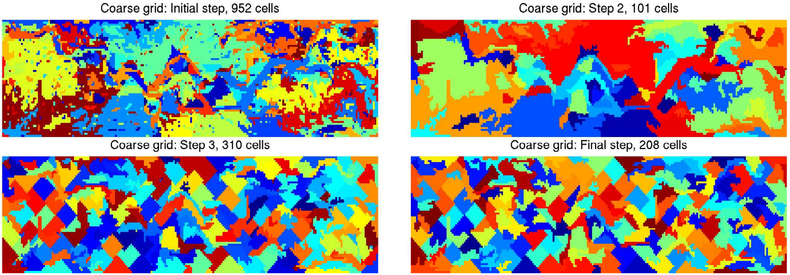
Yes, by using a coarse grid that resolves flow patterns.



Modeling transport on the fine grid (e.g. geomodel) may be a bottle neck or infeasible.

Question: Can we derive a coarse grid formulation that exploits the key information in the high resolution velocity field?

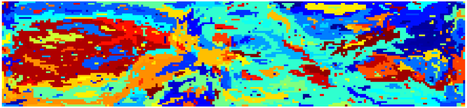
How: Separate, clean, refine, cleanup.



Separate: Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

$$\text{Region } i = \{c : \min(g) + (i-1)D < g(c) < \min(g) + iD\}.$$

Coarse grid: Initial step



Initial grid: connected subregions — 733 blocks

Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

Separate: Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

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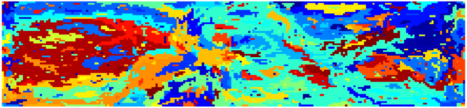
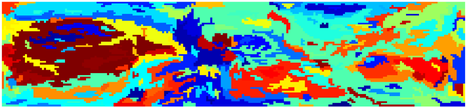


Initial grid: connected subregions — 733 blocks

Merge: If $|B| < c$, merge B with a neighboring block B' with

$$\frac{1}{|B|}\int_B \ln|v|dx \approx \frac{1}{|B'|}\int_{B'} \ln|v|\,dx$$

Coarse grid: Step 2

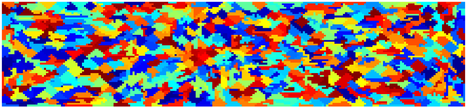


Step 2: 203 blocks

Refine: If criteria — $\int_B \ln |v| dx < C$ — is violated, do

- Start at ∂B and build new blocks B' that meet criteria.
- Define $B = B \backslash B'$ and progress inwards until B meets criteria.

Coarse grid: Step 3

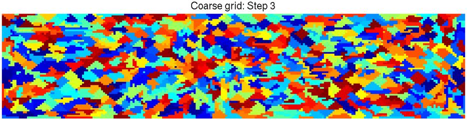


Step3: 914 blocks

Grid generation procedure
Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

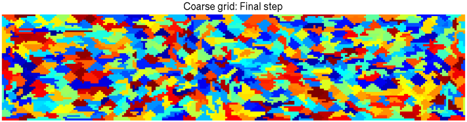
Refine: If criteria — $\int_B \ln|v|dx < C$ — is violated, do

- Start at ∂B and build new blocks B' that meet criteria.
- Define $B = B \backslash B'$ and progress inwards until B meets criteria.



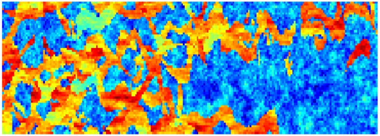
Step3: 914 blocks

Cleanup: Merge small blocks with adjacent block.



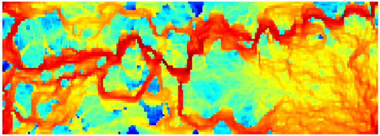
Final grid: 690 blocks

Logarithm of permeability: Layer 68

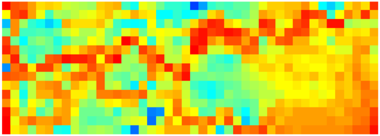


Geomodel: 13200 cells

Logarithm of velocity on geomodel

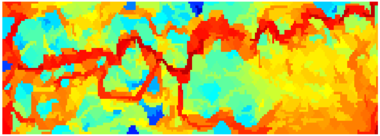


Logarithm of velocity on Cartesian coarse grid



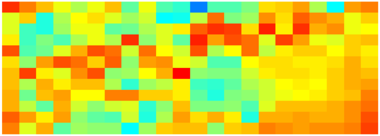
Coarse grid: 660 cells

Logarithm of velocity on non-uniform coarse grid



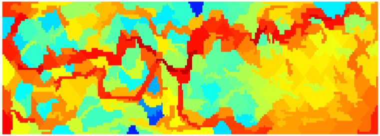
Coarse grid: 649 cells

Logarithm of velocity on Cartesian coarse grid



Coarse grid: 264 cells

Logarithm of velocity on non-uniform coarse grid



Coarse grid: 257 cells

Performance study

Model: Incompressible and immiscible two-phase flow (oil and water) without effects from gravity and capillary forces.

Initial state: Completely oil-saturated.

Parameters: $k_{rj} = s_j^2$, $0 \leq s_j \leq 1$, and $\mu_o/\mu_w = 10$.

Coarse grid formulation

Two-scale first order upstream-weighted finite volume method:

$$\Delta S_{w,i} = \frac{\triangle t}{\int_{V_i} \phi} \left(\int_{V_i} q_w \, dx - \int_{\partial V_i} f_w(S_w) v_w \cdot n \, ds \right)$$

Error measures: $t =$ PVI, $w =$ water-cut, r $=$ reference solution.

$e(S) = \int \left(\|S(\cdot,t) - S_\mathsf{r}(\cdot,t)\|_{L^1} / \|S_\mathsf{r}(\cdot,t)\|_{L^1} \right) dt.$

$e(w) = \|w - w_\mathsf{r}\|_{L^2} / \|w_\mathsf{r}\|_{L^2}.$

Performance study

Model: Incompressible and immiscible two-phase flow (oil and water) without effects from gravity and capillary forces.

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Coarse grid formulation

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$$\Delta S_{w,i} = \frac{\triangle t}{\int_{V_i} \phi} \left(\int_{V_i} q_w \, dx - \sum_{\gamma_{kl} \subset \partial V_i} f_w(S_w^{\text{upstream}}) v_{kl}^{\text{ms}} \right)$$

Error measures: $t =$ PVI, $w =$ water-cut, r $=$ reference solution.

$e(S) = \int \left(\|S(\cdot,t) - S_{\mathrm{r}}(\cdot,t)\|_{L^1} / \|S_{\mathrm{r}}(\cdot,t)\|_{L^1} \right) dt.$

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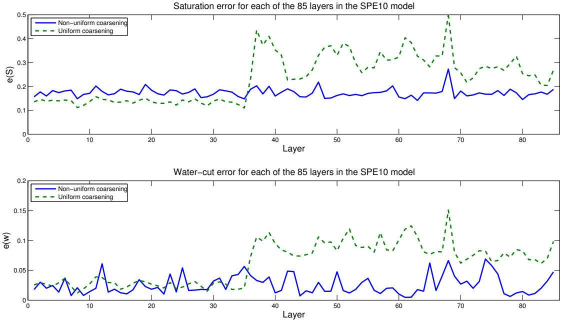


Geomodel:
$60 \times 220 \times 1$

Uniform grid:
$15 \times 44 \times 1$

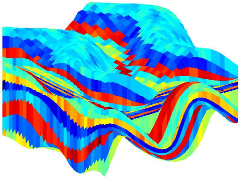
Non-uni. grid:
619–734 blocks

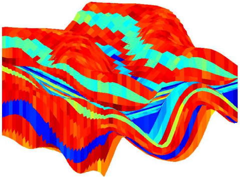
Observations:

- First 35 layers smooth ⇒ Uniform grid adequate.
- Last 50 layers fluvial ⇒ Uniform grid inadequate.
- Non-uniform grid gives consistent results for all layers.

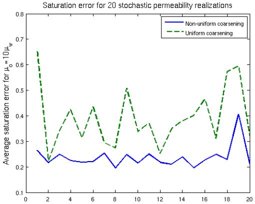
Example 2: Geomodel = unstructured corner-point grid

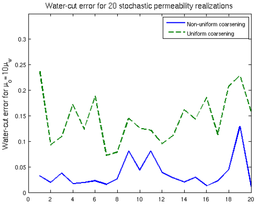
20 realizations from lognormal distribution, Q-of-5-spot well pattern, upsc. factor ∼ 25





⇐ 2 realizations.
Geomodel:
15206 cells





Uniform grid:
838 blocks

Non-uni. grid:
647–704 blocks

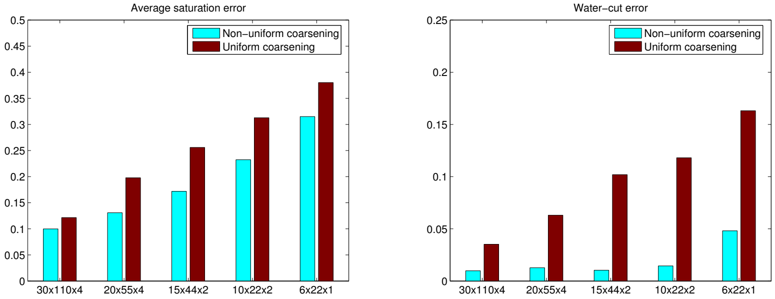
Observations:

- Coarsening algorithm applicable to unstructured grids
  — accuracy consistent with observations for SPE10 models.
- Results obtained with uniform grid (in index space) inaccurate.

Example 3: Geomodel = four bottom layers from SPE10

Robustness with respect to degree of coarsening, 5-spot well pattern

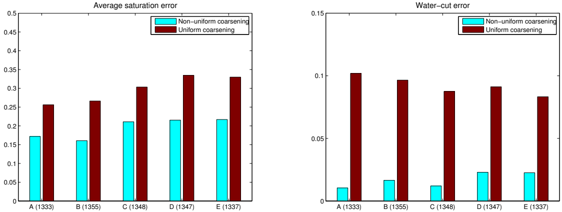
	Number of cells in grid (upscaling factor 4–400)				
Uniform grid	30x110x4 13200	20x55x4 4400	15x44x2 1320	10x22x2 440	6x22x1 132
Non-U. grid	7516	3251	1333	419	150



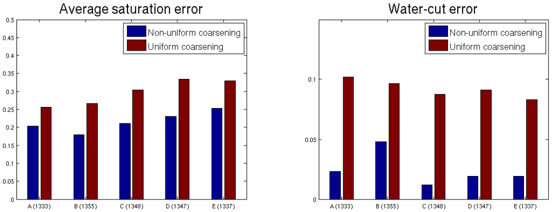
Observations:

- Non-uniform grid gives better accuracy than uniform grid.
- Water-cut error almost grid-independent for non-uniform grid.

Grid generated with respective well patterns.



Grid generated with pattern C



Observation:

Grid resolves high-permeable regions with good connectivity — Grid need *not* be regenerated if well pattern changes.

Multiscale mixed/mimetic method:

- Reservoir simulation tool that can take geomodels as input.
- Solutions in close correspondence with solutions obtained by solving the pressure equation directly.
- Computational cost comparable to flow based upscaling.

Applications:

- Reservoir simulation on geomodels
- Near-well modeling / Improved well models
- History matching on geomodels

Potential value for industry:

Improved modeling and simulation workflows.

Conclusions

Coarse grid for mass balance equations:

- A generic semi-automated algorithm for generating coarse grids that resolve flow patterns has been presented.
- Solutions are significantly more accurate than solutions obtained on uniform coarse grids with similar number of cells.
- Water-cut error: 1%–3% — pseudofunctions superfluous.
- Grid need **not** be regenerated when flow conditions change!

Potential application:

User-specified grid-resolution to fit available computer resources.

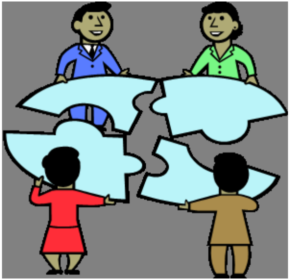
Relation to other methods:

Belongs to family of flow-based grids[a]: designed for flow scenarios where heterogeneity, rather than gravity, dominates flow patterns.

[a] Garcia, Journel, Aziz (1990,1992), Durlofsky, Jones, Milliken (1994,1997)

I have a dream ...

... that one day



geologists and reservoir engineers decide to communicate
and see their contributions as part of a larger picture,
and that multiscale methods are used for what they are worth.