Multiscale mixed/mimetic methods – Generic tools for reservoir modeling and simulation

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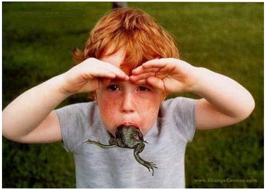
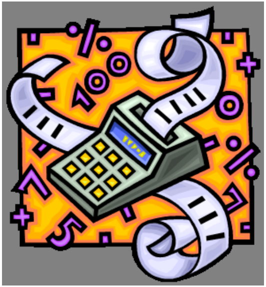


Nature's input

Multiscale simulation



Predicted production

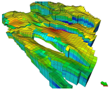
Motivation

Today:

Geomodels too large and complex for flow simulation:
Upscaling performed to obtain

- Simulation grid(s).
- Effective parameters and pseudofunctions.

Reservoir simulation workflow



Geomodel

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Upscaling

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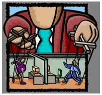
Flow simulation

⟶



Management

Tomorrow:

Earth Model shared between geologists and reservoir engineers —
Simulators take Earth Model as input.

Objective and implication

Main objective:

Build a generic multiscale pressure solver for reservoir modeling and simulation capable of taking geomodels as input.

– *generic:* one implementation applicable to all types of models.

Value: Improved modeling and simulation workflows.

- Geologists may perform simulations to validate geomodel.
- Reservoir engineers gain understanding of geomodeling.
- Facilitate use of geomodels in reservoir management.

Equations:

- Pressure equation

  $c_t \frac{\partial p_o}{dt} + \nabla \cdot v + \sum_j c_j v_j \cdot \nabla p_o = q$

- Mass balance equation for each component

Primary variables:

- Darcy velocity v
- Liquid pressure p_o
- Phase saturations s_j, aqueous, liquid, vapor.

Solution strategy: Iterative sequential

$$\begin{aligned} v_{\nu+1} &= v(s_{j,\nu}), \\ p_{o,\nu+1} &= p_o(s_{j,\nu}), \end{aligned} \qquad s_{j,\nu+1} = s_j(p_{o,\nu+1}, v_{\nu+1}).$$

(Fully implicit with fixed point rather than Newton iteration).

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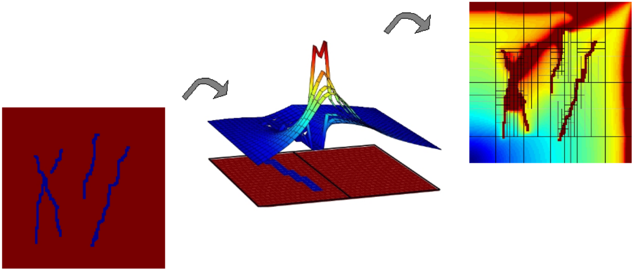
Advantages with sequential solution strategy:

- Grid for pressure and mass balance equations may be different.
- Multiscale methods may be used to solve pressure equation.
- Pressure eq. allows larger time-steps than mass balance eqs.

Multiscale mixed/mimetic method (4M)

Generic two-scale approach to discretizing the pressure equation:

- Mixed FEM formulation on coarse grid.
- Flow patterns resolved on geomodel with mimetic FDM.



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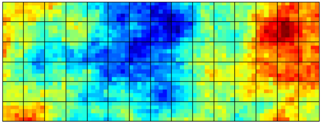

Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

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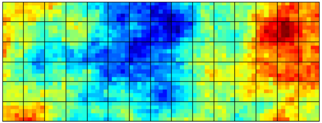

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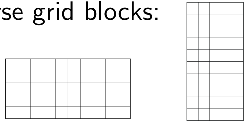
Coarse grid blocks:



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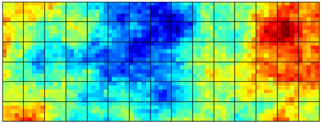

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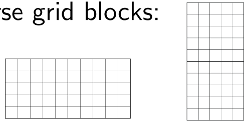

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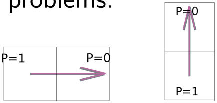
Flow problems:



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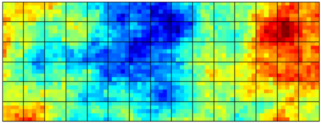

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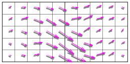

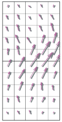

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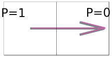

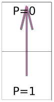

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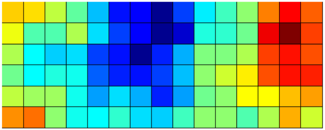

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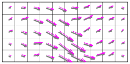

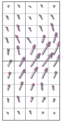

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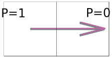

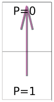

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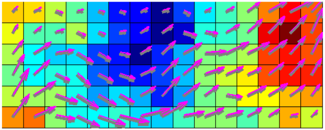

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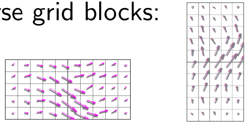

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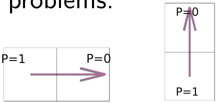

Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

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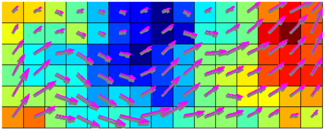

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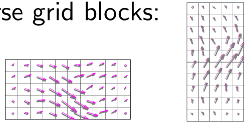

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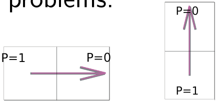
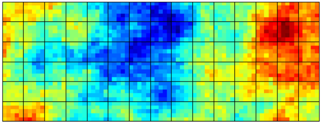


Multiscale method (4M):



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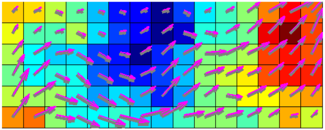

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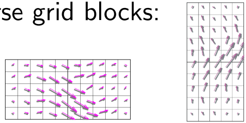

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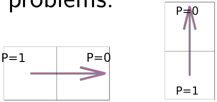

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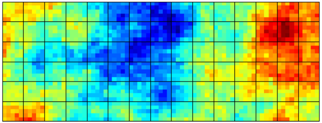

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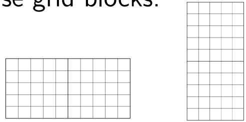

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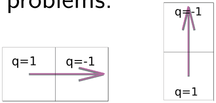

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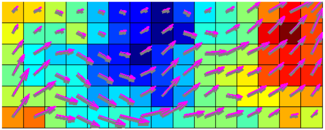

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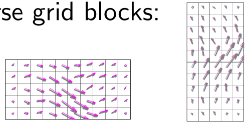

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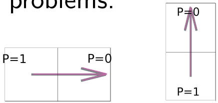

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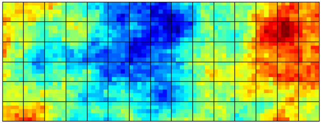

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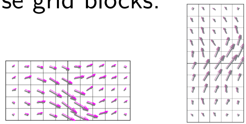

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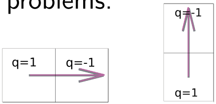

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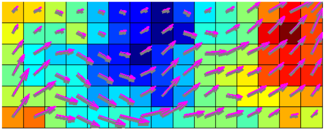

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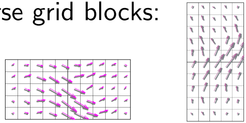

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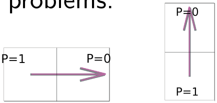

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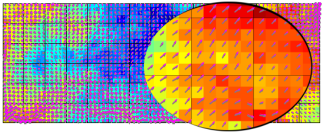

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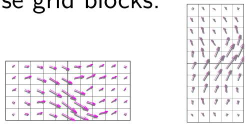

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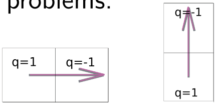


Discrete hybrid formulation: $(u,v)_m = \int_{T_m} u \cdot v \, dx$

Find $v \in V$, $p \in U$, $\pi \in \Pi$ such that for all blocks T_m we have

$$
\begin{aligned}
(\lambda^{-1} v, u)_m - (p, \nabla \cdot u)_m + \int_{\partial T_m} \pi u \cdot n \, ds &= (\omega g \nabla D, u)_m \\
(c_t \tfrac{\partial p_o}{dt}, l)_m + (\nabla \cdot v, l)_m + (\textstyle\sum_j c_j v_j \cdot \nabla p_o, l)_m &= (q, l)_m \\
\int_{\partial T_m} \mu v \cdot n \, ds &= 0.
\end{aligned}
$$

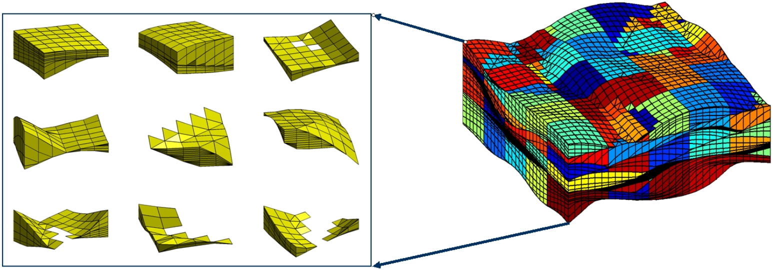
for all $u \in V$, $l \in U$ and $\mu \in \Pi$.

Solution spaces and variables: $\mathcal{T} = \{T_m\}$

$V \subset H^{\text{div}}(\mathcal{T})$,	$U = \mathcal{P}_0(\mathcal{T})$,	$\Pi = \mathcal{P}_0(\{\partial T_m \cap \partial T_n\})$.
v = velocity,	p = block pressures,	π = interface pressures.

Each coarse grid block is a connected set of cells from geomodel.

Example: Coarse grid obtained with uniform coarsening in index space.



Grid adaptivity at well locations:

One block assigned to each cell in geomodel with well perforation.

Definition of approximation space for velocity:

The approximation space V is spanned by basis functions ψ_m^i that are designed to embody the impact of fine-scale structures.

Definition of basis functions:

For each pair of adjacent blocks T_m and T_n, define ψ by

$$\psi = -K\nabla u \text{ in } T_m \cup T_n, \qquad \nabla \cdot \psi = \begin{cases} w_m & \text{in } T_m, \\ -w_n & \text{in } T_n, \end{cases}$$
$$\psi \cdot n = 0 \text{ on } \partial(T_m \cup T_n),$$

Split ψ: $\quad \psi_m^i = \psi|_{T_m}, \quad \psi_n^j = -\psi|_{T_n}.$

Basis functions time-independent if w_m is time-independent.

Role of weight functions

Let $(w_m, 1)_m = 1$ and let v_m^i be coarse-scale coefficients.

$$v = \sum_{m,i} v_m^i \psi_m^i \quad \Rightarrow \quad (\nabla \cdot v)|_{T_m} = w_m \sum_i v_m^i.$$

$\longrightarrow$ w_m gives distribution of $\nabla \cdot v$ among cells in geomodel.

Choice of weight functions

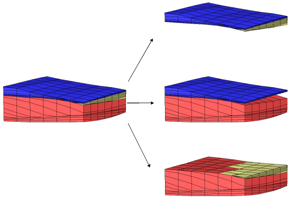
$$\nabla \cdot v \sim c_t \frac{\partial p_o}{dt} + \sum_j c_j v_j \cdot \nabla p_o$$

- Use adaptive criteria to decide when to redefine w_m.
- Use $w_m = \phi$ ($c_t \sim \phi$ when saturation is smooth).

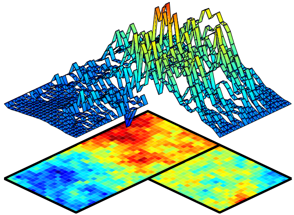
$\longrightarrow$ **Basis functions computed once, or updated infrequently.**

At initial time

Detect all adjacent blocks



Compute ψ for each domain



For each time-step:

- Assemble and solve coarse grid system.
- Recover fine grid velocity.
- Solve mass balance equations.

Velocity basis functions computed using mimetic FDM

Mixed FEM for which the inner product $(u, \sigma v)$ is replaced with an approximate explicit form ($u, v \in H^{\mathrm{div}}$ and σ SPD),
— no integration, no reference elements, no Piola mappings.

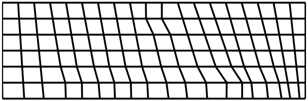
May also be interpreted as a multipoint finite volume method.

Properties:

- Exact for linear pressure.
- Same implementation applies to all grids.
- Mimetic inner product *needed* to evaluate terms in multiscale formulation, e.g., $(\psi_m^i, \lambda^{-1}\psi_m^j)$ and $(\omega g \nabla D, \psi_{m,j})$.

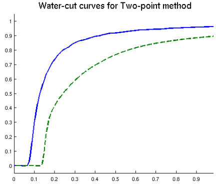
Two-point FD method is "generic", but ...

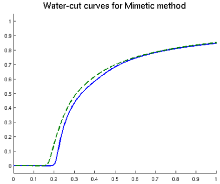
Example:



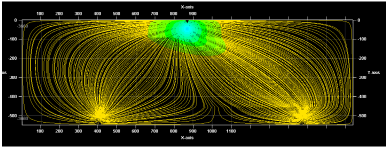
Homogeneous+isotropic, symmetric well pattern $\longrightarrow$ equal water-cut.

Two-point method + skewed grids
= grid orientation effects.

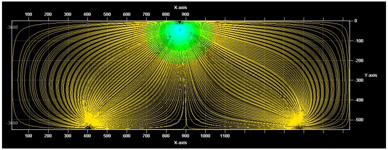




Two-point FV method

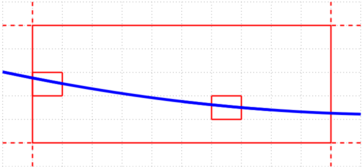


Mimetic FD method



Grid block for cells with a well

- correct well-block pressure
- no near well upscaling
- free choice of well model.



Alternative well models

1. Peaceman model:

   $$q_{\text{perforation}} = -W_{\text{block}}(p_{\text{block}} - p_{\text{perforation}}).$$

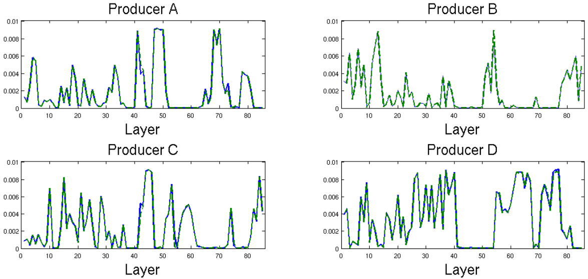
   Calculation of well-index grid dependent.

2. Exploit pressures on grid interfaces:

   $$q_{\text{perforation}} = -\sum_i W_{\text{face}i}(p_{\text{face}i} - p_{\text{perforation}}).$$

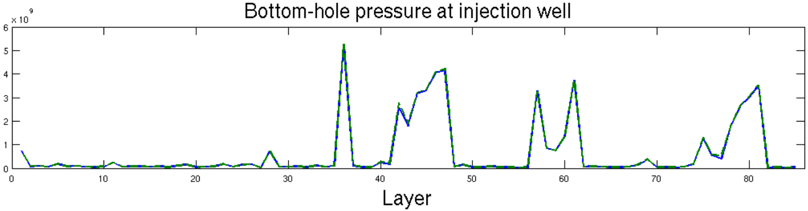
   Generic calculation of $W_{\text{face}i}$.

5-spot: 1 rate constr. injector, 4 pressure constr. producers
Well model: Interface pressures employed.

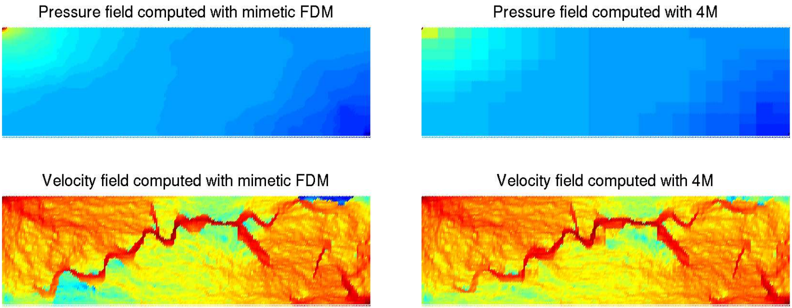


Distribution of production rates

- Reference (60×220)
- Multiscale (10×22)



Example: Layer 36 from SPE10 (Christie and Blunt, 2001).



Primary features

- Coarse pressure solution, subgrid resolution at well locations.
- Coarse velocity solution with subgrid resolution everywhere.

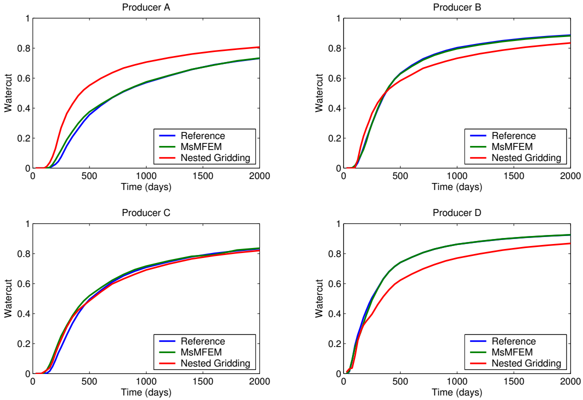
Model: SPE10 model 2, 1.1 M cells, 1 injector, 4 producers.

Coarse grid:
$5 \times 11 \times 17$

- Reference
- 4M
- Upscaling + downscaling

4M+streamlines: $\sim$ 2 minutes on desktop PC.

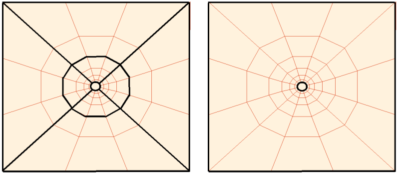
Water-cut curves at producers A–D

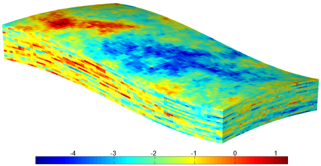


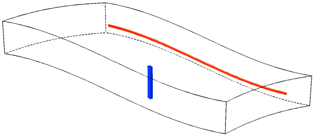
Krogstad and Durlofsky, 2007:

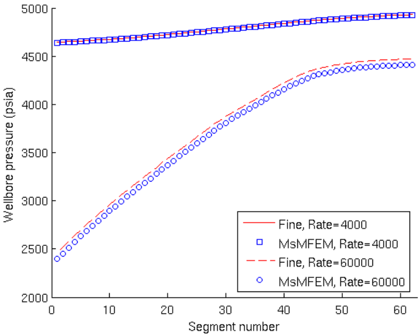
Fine grid to annulus,
block for each well segment

- No well model needed.
- Drift-flux wellbore flow.



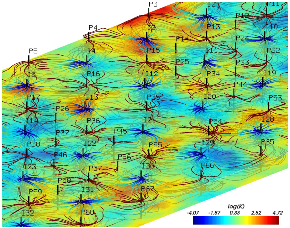


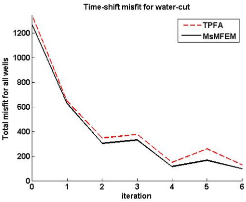




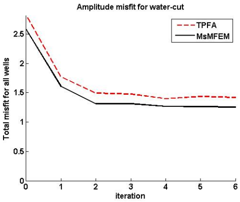
Stenerud, Kippe, Datta-Gupta, and Lie, RSS 2007:

- 1 million cells, 32 injectors, and 69 producers
- Matching travel-time and water-cut amplitude at producers
- Permeability updated in blocks with high average sensitivity
  $\longrightarrow$ Only few multiscale basis functions updated.





Time-residual



Amplitude-residual

Computation time: $\sim$ 17 min. on desktop PC. (6 iterations).

Conclusions

Multiscale mixed/mimetic method:

- Reservoir simulation tool that can take geomodels as input.
- Solutions in close correspondence with solutions obtained by solving the pressure equation directly.
- Computational cost comparable to flow based upscaling.

Applications:

- Reservoir simulation on geomodels
- Near-well modeling / Improved well models
- History matching on geomodels

Potential value for industry:

Improved modeling and simulation workflows.