Reservoir Simulation:
From Upscaling to Multiscale Methods

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Reservoir Simulation
What and why?

Reservoir simulation is the means by which a numerical model of the petrophysical characteristics of a hydrocarbon reservoir is used to analyze and predict fluid behavior in the reservoir over time.

Reservoir simulation is used as a basis for decisions regarding development of reservoirs and management during production. To this end, one needs to

- predict reservoir performance from geological descriptions and constraints,
- fit geological descriptions to static and dynamic data,
- assess uncertainty in predictions,
- optimize production strategies,

⋮

Reservoir Simulation

What are the challenges today?

Reservoir modelling is a true *multiscale* discipline:

- Measurements and models on a large number of scales
- Large number of models
- Complex grids with a large number of parameters
- High degree of uncertainty

⋮

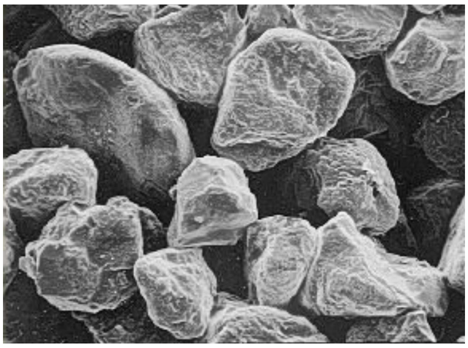
There is always a need for faster and more accurate simulators that use all available geological information

Physical Scales in Porous Media Flow

One cannot resolve them all at once

The scales that impact fluid flow in oil reservoirs range from

- the micrometer scale of pores and pore channels
- via dm–m scale of well bores and laminae sediments
- to sedimentary structures that stretch across entire reservoirs.



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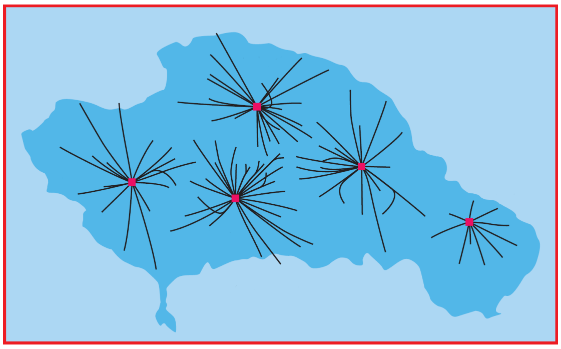
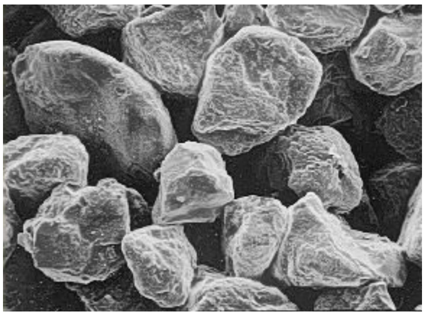
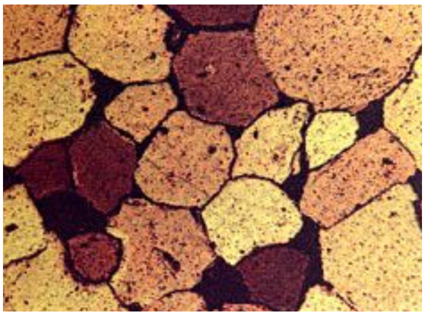
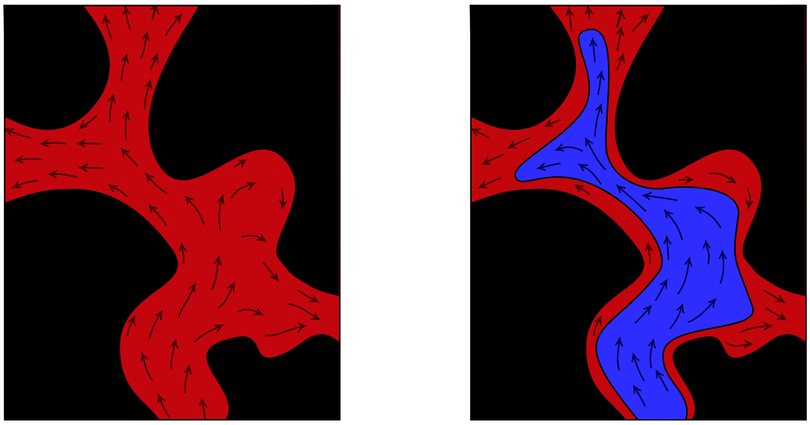


Physical Scales in Porous Media Flow

Microscopic: the scale of individual sand grains







Flow in individual pores between sand grains

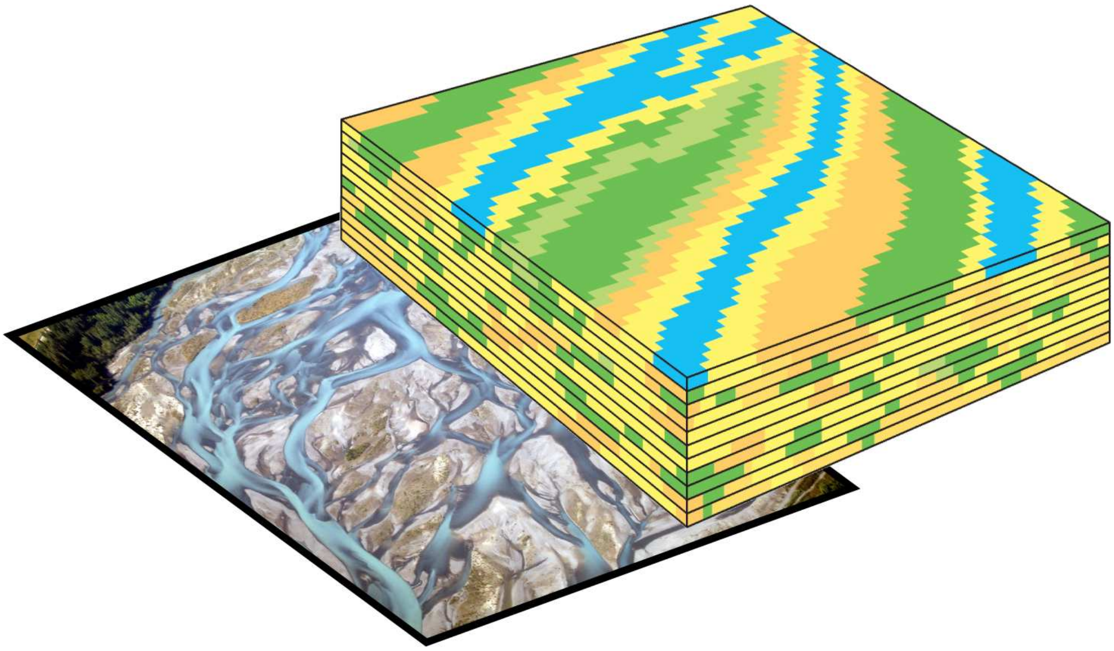
Physical Scales in Porous Media Flow

Geological: the meter scale of layers, depositional beds, etc

Porous sandstones often have repetitive layered structures, but faults and fractures caused by stresses in the rock disrupt flow patterns

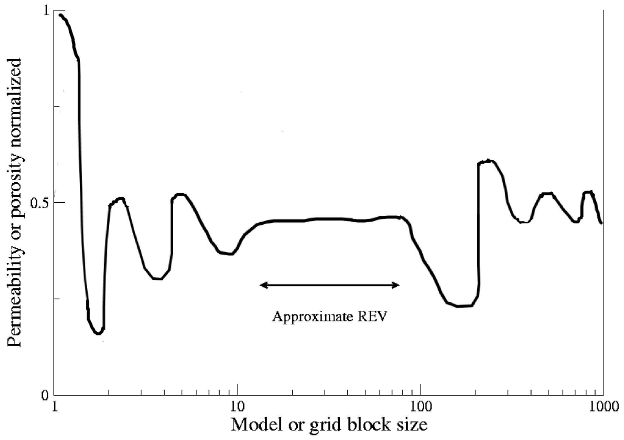
Physical Scales in Porous Media Flow

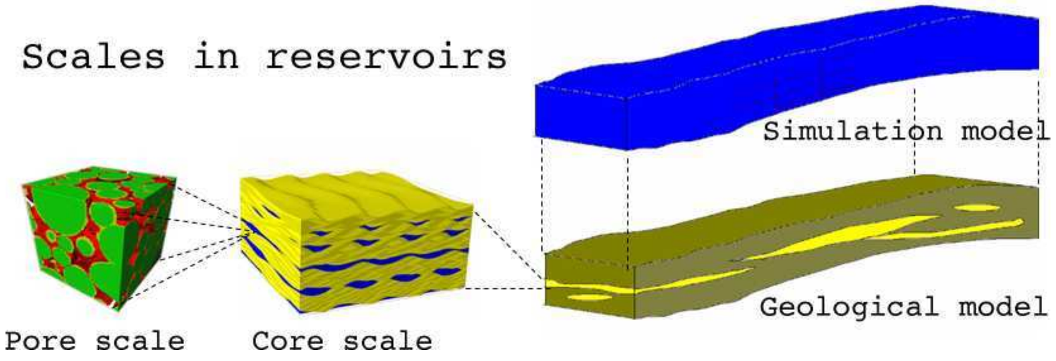
Reservoir: the kilometer scale of sedimentary structures



Physical Scales in Porous Media Flow

Choosing a scale for modelling



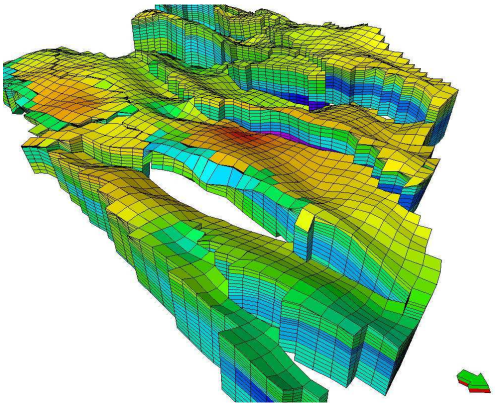


Geological Models

The knowledge database in the oil company

Geomodels:

- are articulations of the experts' perception of the reservoir
- describe the reservoir geometry (horizons, faults, etc)
- give rock parameters (e.g., permeability $\mathbf{K}$ and porosity ϕ) that determine the flow



In the following: the term "geomodel" will designate a grid model where rock properties have been assigned to each cell

Flow Simulation

Model problem: incompressible, single phase

Consider the following model problem

$$\begin{aligned} \text{Darcy's law:} \quad & v = -K\left(\nabla p - \rho g \nabla D\right), \\ \text{Mass balance:} \quad & \nabla \cdot v = q \quad \text{in } \Omega, \\ \text{Boundary conditions:} \quad & v \cdot n = 0 \quad \text{on } \partial\Omega. \end{aligned}$$

The multiscale structure of porous media enters the equations through the absolute permeability K, which is a symmetric and positive definite tensor with uniform upper and lower bounds.

We will refer to p as pressure and v as velocity.

Flow Simulation

The impact of rock properties

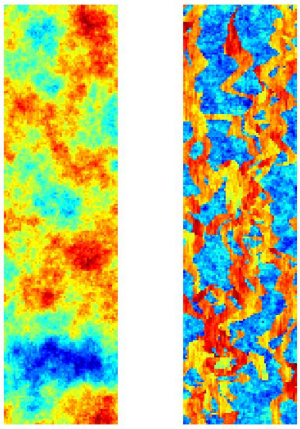
Rock properties are used as parameters in flow models

- Permeability $\mathbf{K}$ spans many length scales and have multiscale structure

$$\max \mathbf{K} / \min \mathbf{K} \sim 10^3\text{–}10^{10}$$

- Details on all scales impact flow

Ex: Brent sequence



Tarbert Upper Ness

Challenges:

- How much details should one use?
- Need for good linear solvers, preconditioners, etc.

Flow Simulation

Gap in resolution and model sizes

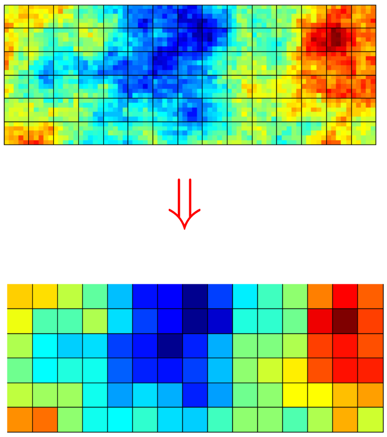
Gap in resolution:

- High-resolution geomodels may have $10^6 - 10^{10}$ cells
- Conventional simulators are capable of about $10^5 - 10^6$ cells

Traditional solution: upscaling of parameters

Assume that u satisfies the elliptic PDE:

$$-\nabla\big(K(x)\nabla u\big) = f.$$



Upscaling amounts to finding a new field $K^*(\bar{x})$ on a coarser grid such that

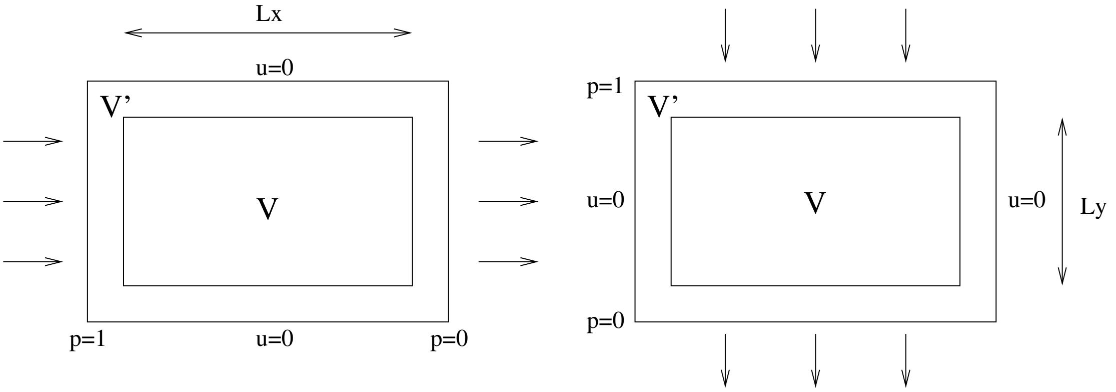
$$-\nabla\big(K^*(\bar{x})\nabla u^*\big) = \bar{f},$$

$$u^* \sim \bar{u}, \qquad q^* \sim \bar{q}\ .$$

Upscaling Geological Models
Industry-standard methods

How do we represent fine-scale heterogeneities on a coarse scale?

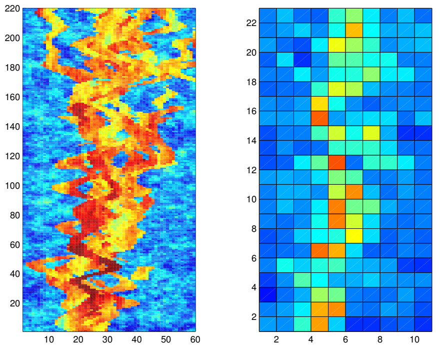
- Combinations of arithmetic, geometric, harmonic averaging
- Power averaging $\left(\frac{1}{|V|}\int_V a(x)^p\,dx\right)^{1/p}$
- Equivalent permeabilities ($a^*_{xx} = -Q_x L_x/\Delta P_x$)



Upscaling Geological Models
Is it necessary and does one want to do it?

There are many difficulties associated with upscaling

- Bottleneck in the workflow
- Loss of details
- Lack of robustness
- Need for resampling for complex grid models
- Not obvious how to extend the ideas to 3-phase flows



Need for fine-scale computations?

In the future: need for multiphysics on multiple scales?

Fluid Simulations Directly on Geomodels

Research vision:

Direct simulation of complex grid models of highly heterogeneous and fractured porous media - a technology that bypasses the need for upscaling.

Applications:

Huge models, multiple realizations, prescreening, validation, optimization, data integration, ..

To this end, we seek a methodology that

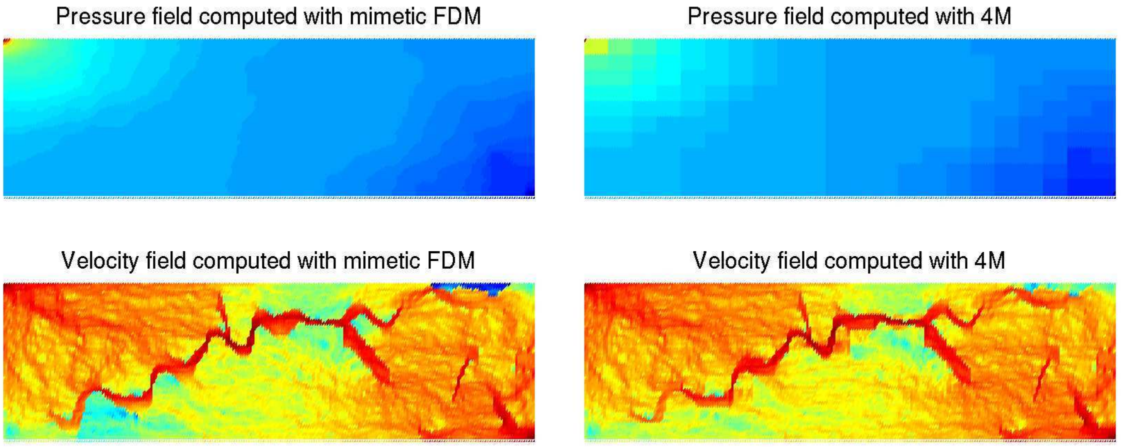
- incorporates small-scale effects into coarse-scale system;
- gives a detailed image of the flow pattern on the fine scale, without having to solve the full fine-scale system;
- is robust, conservative, accurate, and efficient.

Multiscale Pressure Solvers

Efficient flow solution on complex grids – without upscaling

Basic idea:

- Upscaling and downscaling in one step
- Pressure varies smoothly and can be resolved on coarse grid
- Velocity with subgrid resolution



Pressure field computed with mimetic FDM

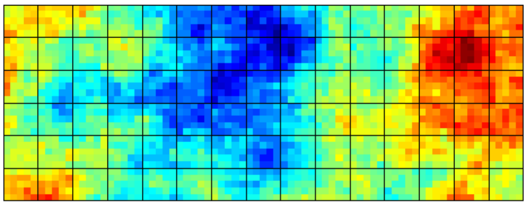
Pressure field computed with 4M

Velocity field computed with mimetic FDM

Velocity field computed with 4M

From Upscaling to Multiscale Methods

Standard upscaling:



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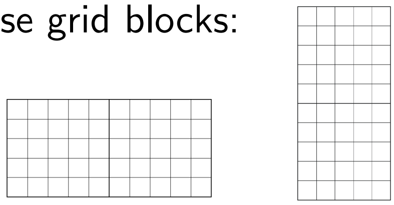
Coarse grid blocks:



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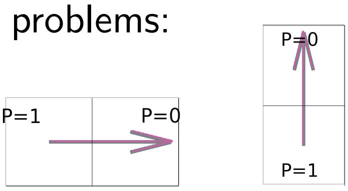
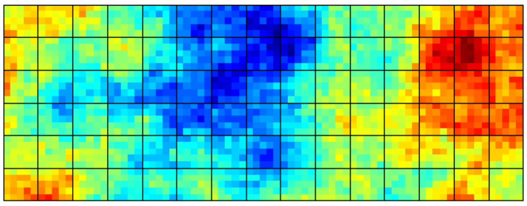
Flow problems:



Multiscale method:



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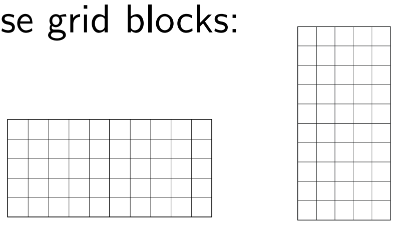
Coarse grid blocks:



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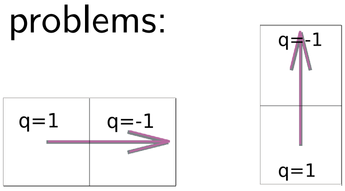
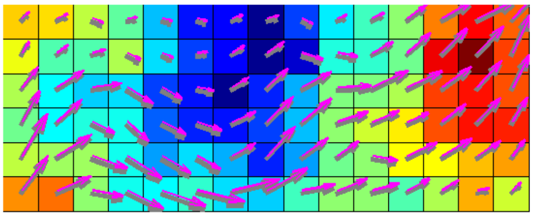
Flow problems:



From Upscaling to Multiscale Methods

Standard upscaling:



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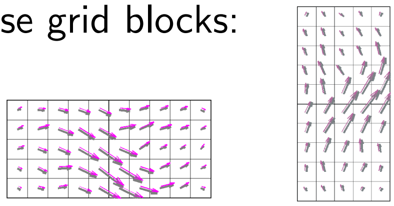
Coarse grid blocks:



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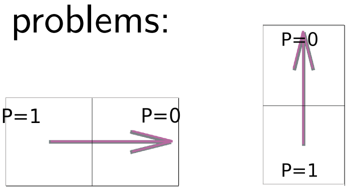
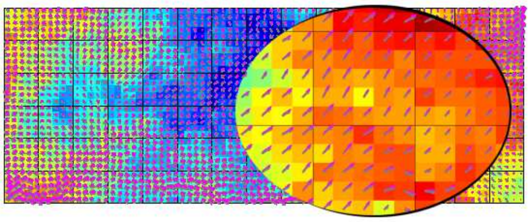
Flow problems:



Multiscale method:



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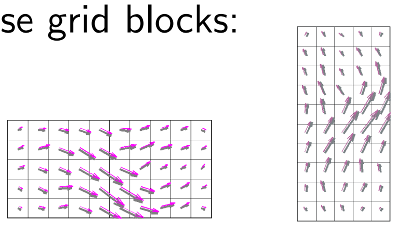
Coarse grid blocks:



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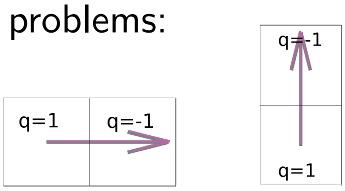
Flow problems:



The Multiscale Mixed Finite-Element Method

Standard finite-element method (FEM):

Piecewise polynomial approximation to pressure, $\int l \nabla K \nabla p \, dx = \int lq \, dx$

Mixed finite-element methods (MFEM):

Piecewise polynomial approximations to pressure *and* velocity

$$\begin{aligned} \int_\Omega k^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= \int_\Omega k^{-1} \rho g \nabla D \cdot u \, dx \quad && \forall u \in U, \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega ql \, dx && \forall l \in V. \end{aligned}$$

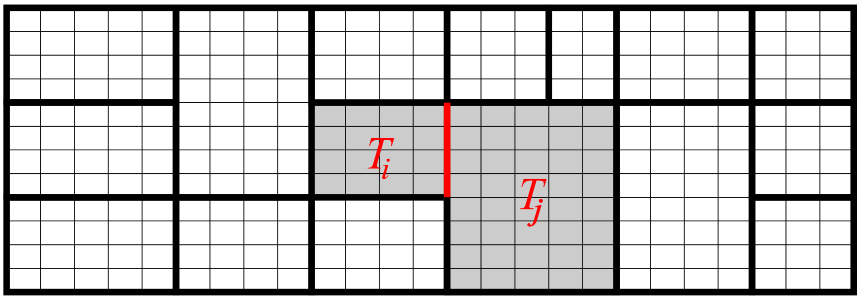
Multiscale mixed finite-element method (MsMFEM):

Velocity approximated in a (low-dimensional) space V^{ms} designed to embody the impact of fine-scale structures.

Multiscale Mixed Finite Elements

Grids and basis functions

Assume we are given a *fine* grid with permeability and porosity attached to each fine-grid block:



We construct a *coarse* grid, and choose the discretisation spaces U and V^{ms} such that:

- For each coarse block T_i, there is a basis function $\phi_i \in U$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in V^{\text{ms}}$.

(Multiscale) Mixed Finite Elements

Discretisation matrices (without hybridization)

Saddle-point problem:

$$\begin{pmatrix} B & C \\ C^T & 0 \end{pmatrix} \begin{pmatrix} v \\ p \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix},$$

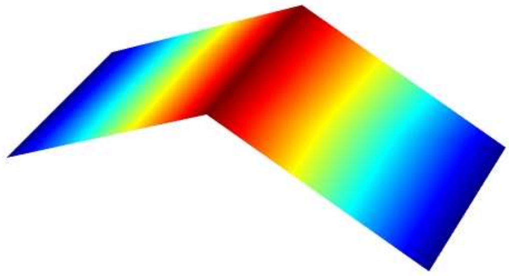
$$b_{ij} = \int_\Omega \psi_i k^{-1} \psi_j \, dx,$$

$$c_{ij} = \int_\Omega \phi_j \nabla \cdot \psi_i \, dx$$

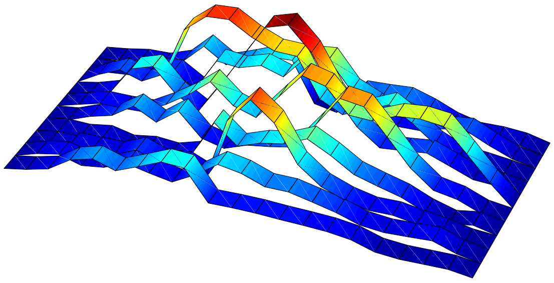
Basis ϕ_j for pressure: equal one in cell j, zero otherwise

Basis ψ_i for velocity:

1.order Raviart–Thomas:



Multiscale:



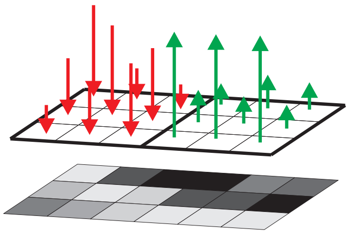
Multiscale Mixed Finite Elements

Basis for the velocity field

Velocity basis function ψ_{ij}: unit flow through Γ_{ij} defined as

$$\nabla \cdot \psi_{ij} = \begin{cases} w_i(x), & \text{for } x \in T_i, \\ -w_j(x), & \text{for } x \in T_j, \end{cases}$$

and no flow $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$.

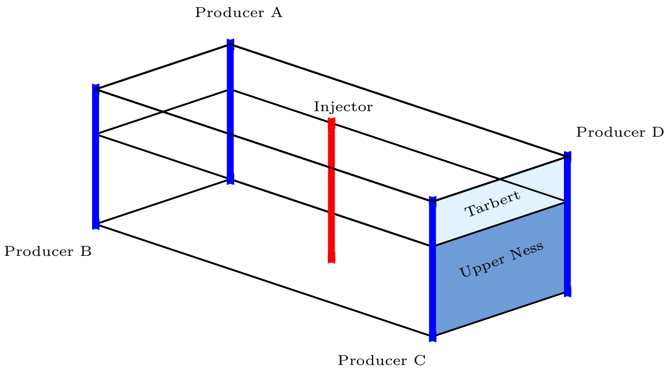


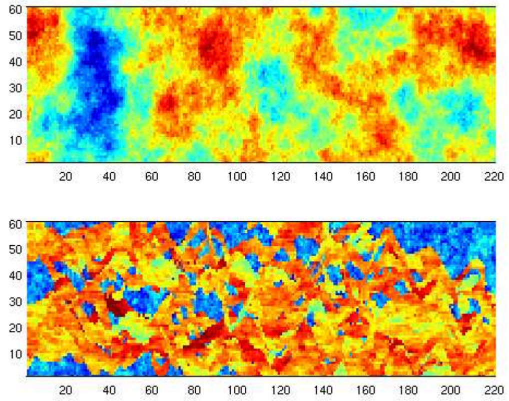
Global velocity:

$v = \sum_{ij} v_{ij} \psi_{ij}$, where v_{ij} are (coarse-scale) coefficients.

Multiscale Simulation versus Upscaling

10^{th} SPE Comparative Solution Project





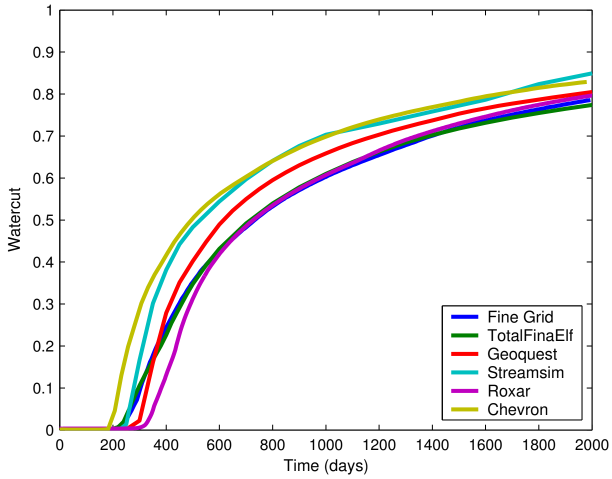
- Geomodel: $60 \times 220 \times 85 \approx 1,1$ million grid cells, $\max K_x / \min K_x \approx 10^7$, $\max K_z / \min K_z \approx 10^{11}$
- Simulation: 2000 days of production (2-phase flow)

Commercial (finite-difference) solvers: incapable of running the whole model

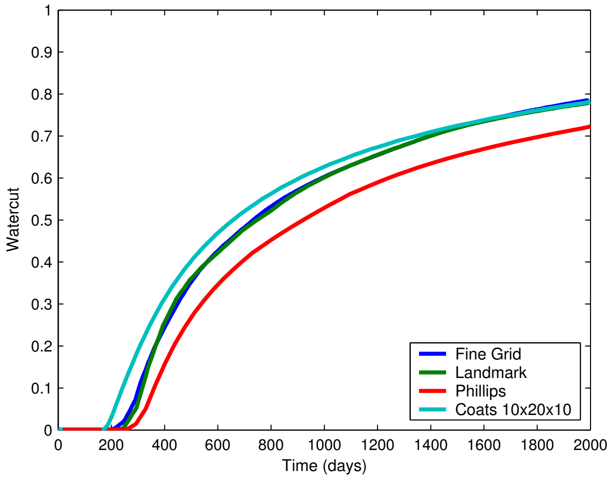
Multiscale Simulation versus Upscaling

10^{th} SPE Comparative Solution Project

Upscaling results reported by industry



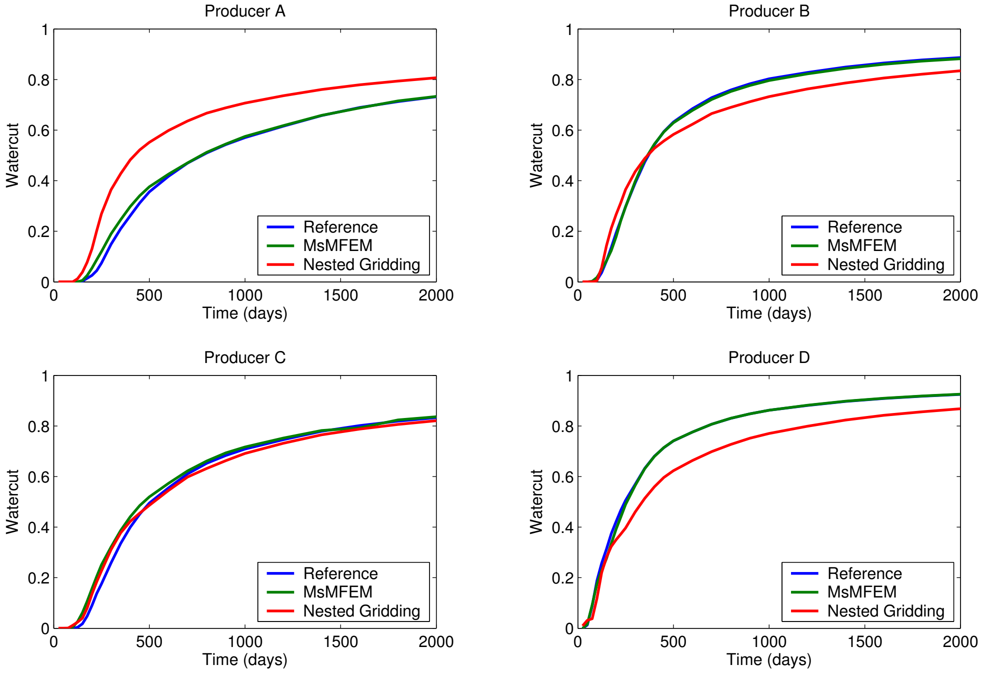
single-phase upscaling



two-phase upscaling

Multiscale Simulation versus Upscaling

10th SPE Comparative Solution Project

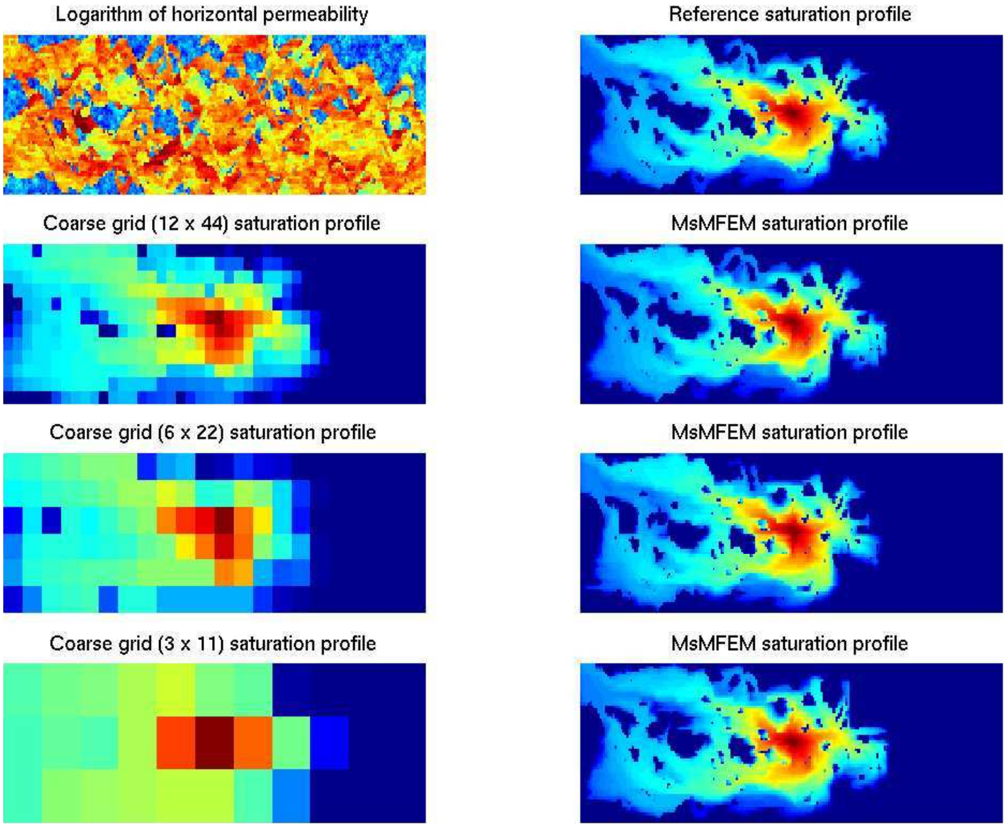


🟥 upscaling/downscaling, 🟩 MsMFEM/streamlines, 🟦 fine grid

Runtime: 2 min 22 sec on 2.4 GHz desktop PC

Robustness

SPE10, Layer 85 (60×220 Grid)



Logarithm of horizontal permeability

Reference saturation profile

Coarse grid (12 x 44) saturation profile

MsMFEM saturation profile

Coarse grid (6 x 22) saturation profile

MsMFEM saturation profile

Coarse grid (3 x 11) saturation profile

MsMFEM saturation profile

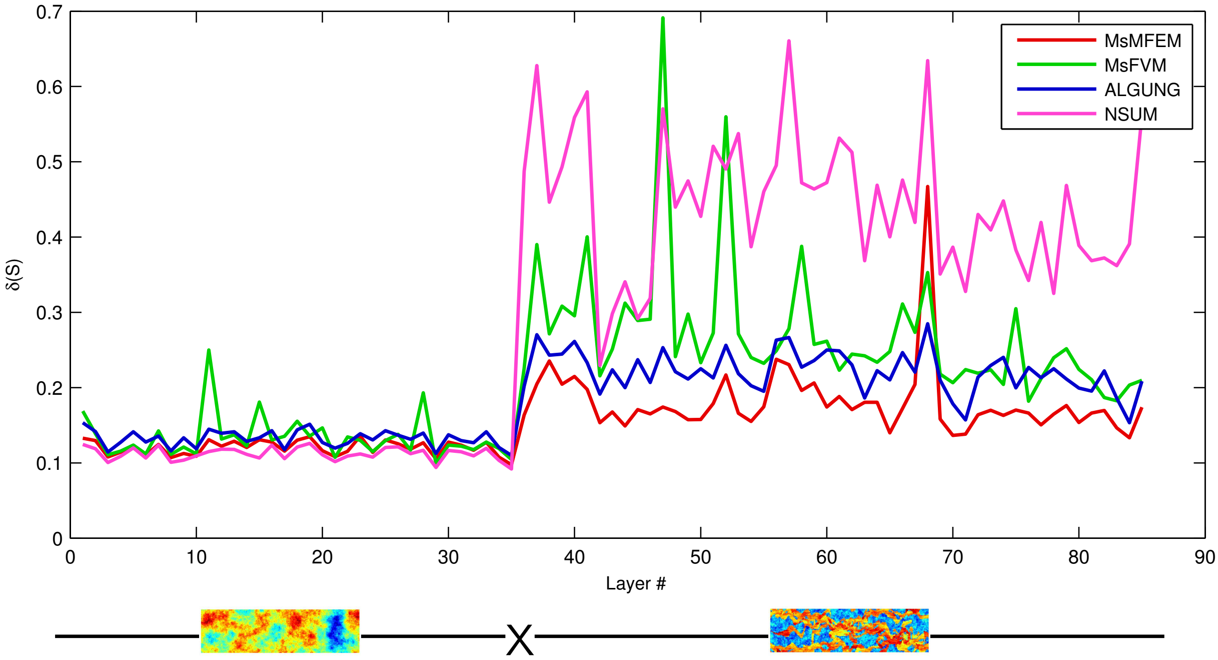
Comparison of Multiscale and Upscaling Methods

1. Local-global upscaling (Durlofsky et al)
   - global boundary conditions, iterative improvement (bootstrap)
   - reconstruction of fine-grid velocities
2. Multiscale mixed finite elements (Chen & Hou, . . .)
   - multiscale basis functions for velocity
   - coarse-scale pressure
3. Multiscale finite-volume method (Jenny, Tchelepi, Lee,. . .)
   - multiscale basis functions for pressure
   - reconstruction of velocity on fine grid
4. Numerical subgrid upscaling (Arbogast, . . .)
   - direct decomposition of the solution, $V = V_c \oplus V_f$
   - RT0 on fine scale, BDM1 on coarse

Comparison of Multiscale and Upscaling Methods

SPE 10, individual layers

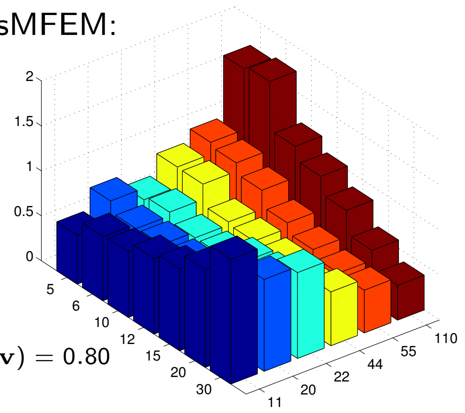
Saturation errors at 0.3 PVI on 15 × 55 coarse grid



Comparison of Multiscale and Upscaling Methods

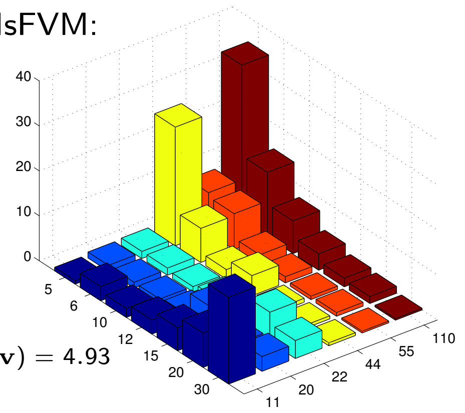
Velocity errors for Layer 85

MsMFEM:



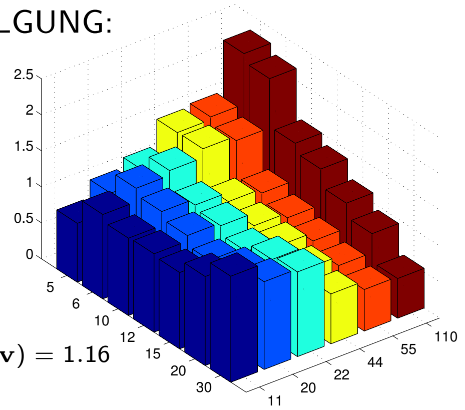
$\bar{\delta}(\mathbf{v}) = 0.80$

MsFVM:



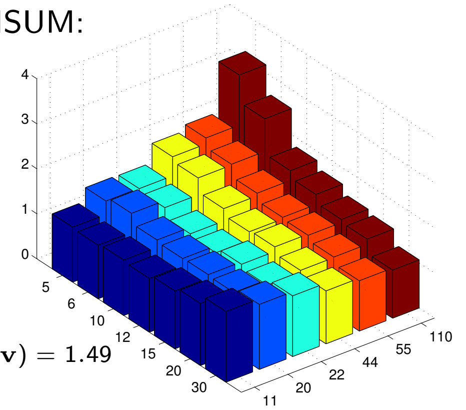
$\bar{\delta}(\mathbf{v}) = 4.93$

ALGUNG:



$\bar{\delta}(\mathbf{v}) = 1.16$

NSUM:



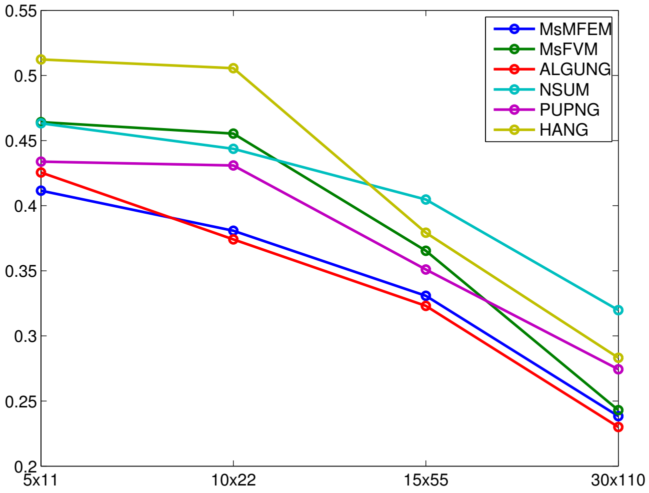
$\bar{\delta}(\mathbf{v}) = 1.49$

Comparison of Multiscale and Upscaling Methods

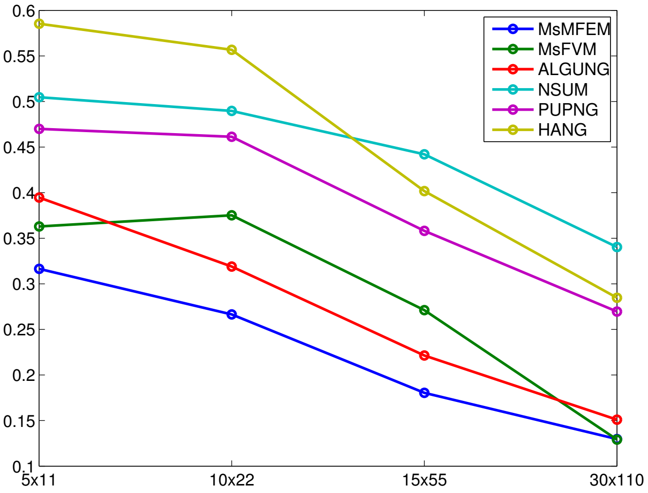
Average saturation errors on Upper Næss formation (Layers 36–85)

Cartesian coarse grids:

Multiscale methods give enhanced accuracy when subgrid information is exploited.



Fluid transport: coarse grid



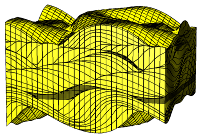
Fluid transport: fine grid

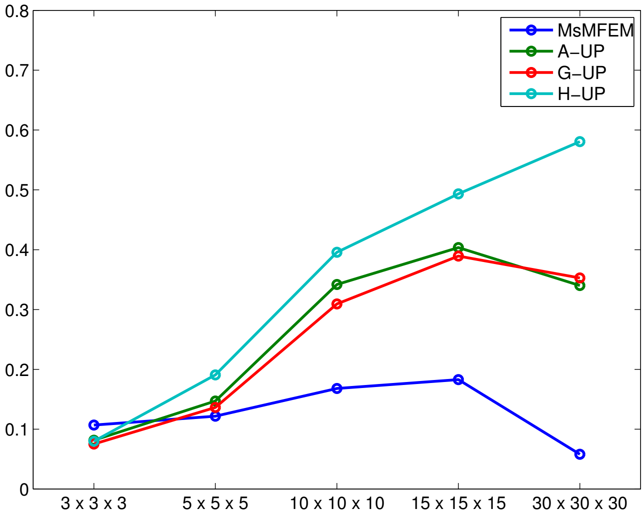
Comparison of Multiscale and Upscaling Methods

MsMFEM versus upscaling on complex coarse grids

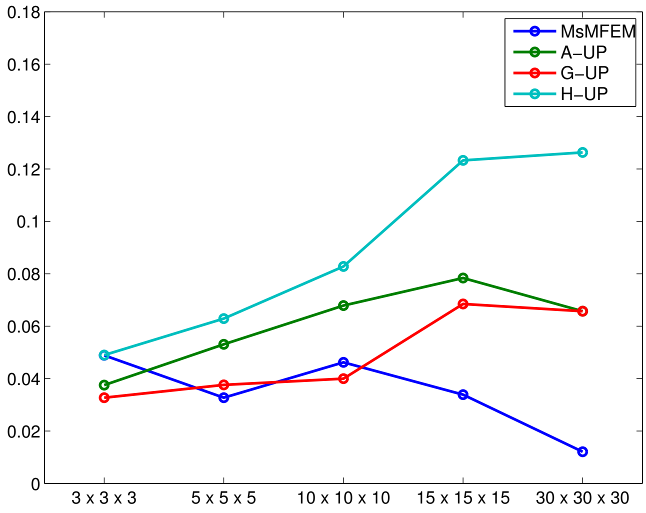
Complex coarse grid-block geometries:

MsMFEM is more accurate than upscaling, also for coarse-grid simulation.





Coarse-grid velocity errors



Coarse-grid saturation errors

Up-gridded 30 × 30 × 333 corner-point grid with layered log-normal permeability

Computational Complexity

Order-of-magnitude argument

Assume:

- Grid model with $N = N_s * N_c$ cells:
  - N_c number of coarse cells
  - N_s number of fine cells in each coarse cell
- Linear solver of complexity $\mathcal{O}(m^\alpha)$ for $m \times m$ system
- Negligible work for determining local b.c., numerical quadrature, and assembly (can be important, especially for NSUM)

Direct solution

N^α operations for a two-point finite volume method

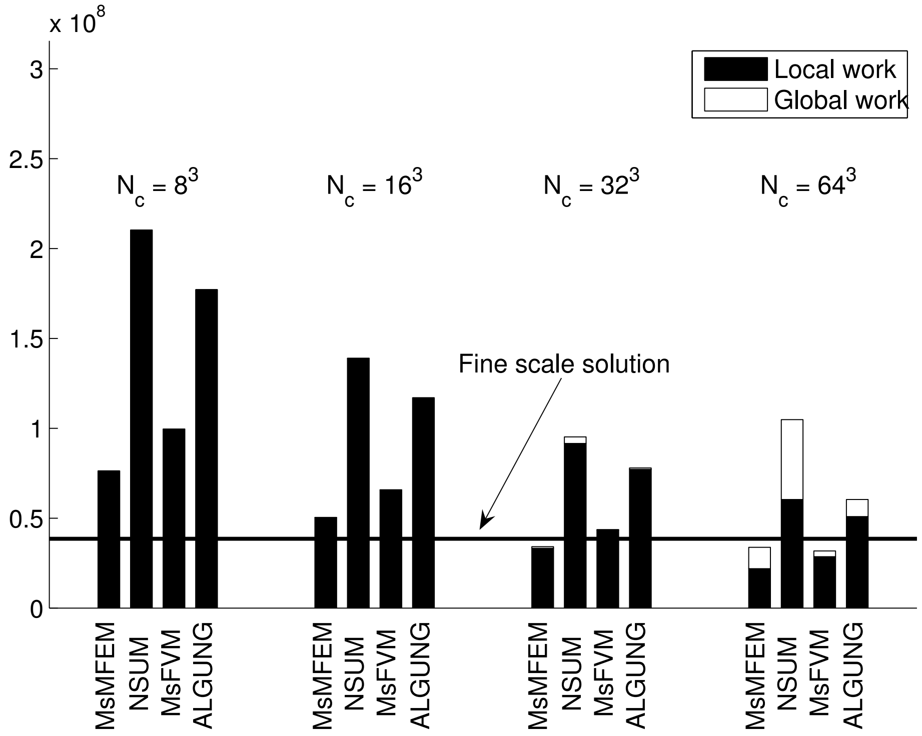
MsMFEM

Computing basis functions: $D \cdot N_c \cdot (2N_s)^\alpha$ operations

Solving coarse-scale system: $(D \cdot N_c)^\alpha$ operations

Computational Complexity

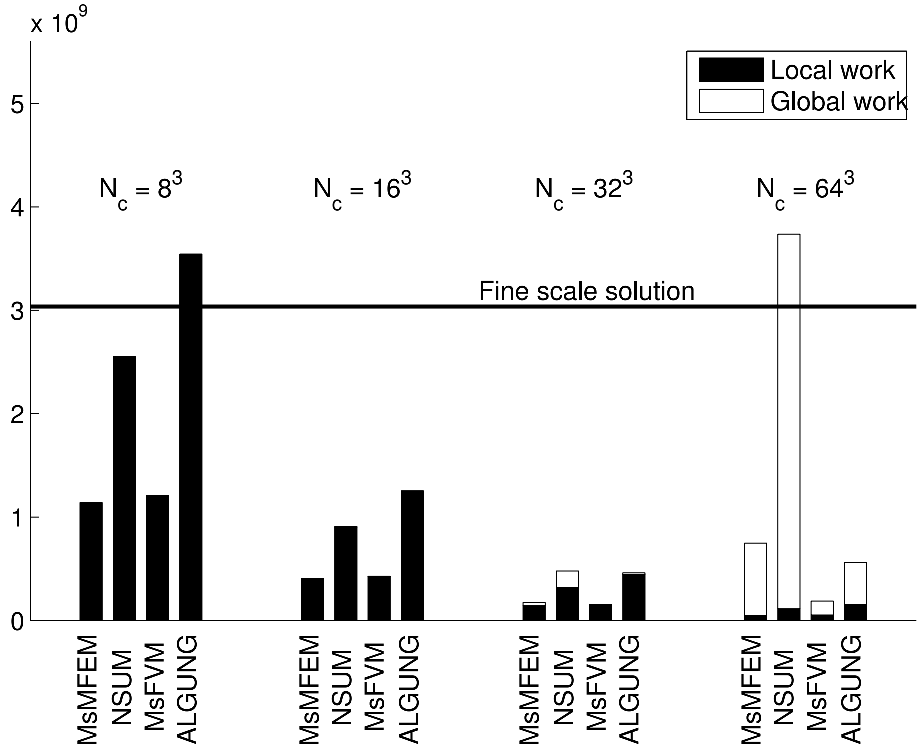
Example: $128 \times 128 \times 128$ fine grid



Comparison with algebraic multigrid (AMG), $\alpha = 1.2$

Computational Complexity

Example: $128 \times 128 \times 128$ fine grid



Comparison with less efficient solver, $\alpha = 1.5$

Multiphase Flow

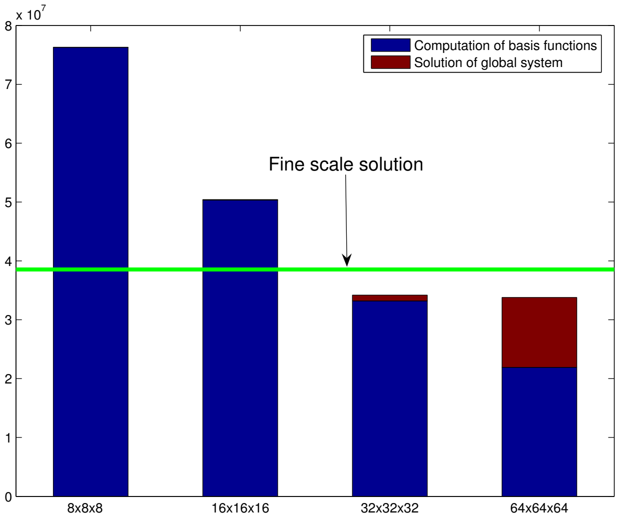
Time-dependent problems: $\nabla(K(x)\lambda(S)\nabla p) = q(S)$

Direct solution may be more efficient, so why bother with multiscale?

- Full simulation: $\mathcal{O}(10^2)$ time steps.
- Basis functions need not always be recomputed

Also:

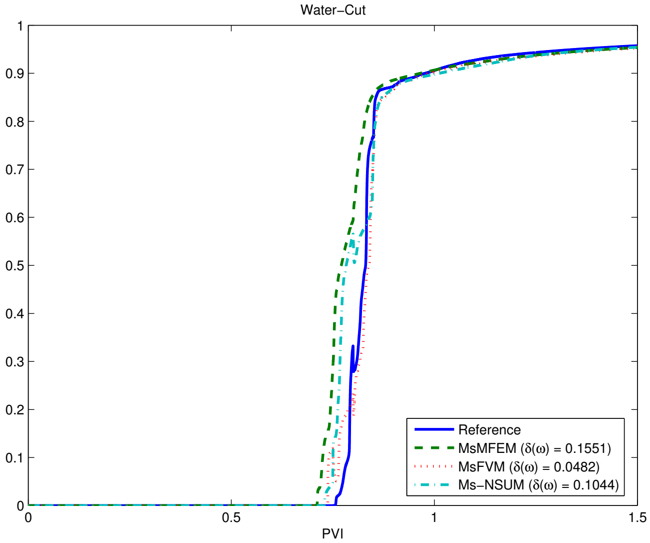
- Possible to solve very large problems
- Easy parallelization



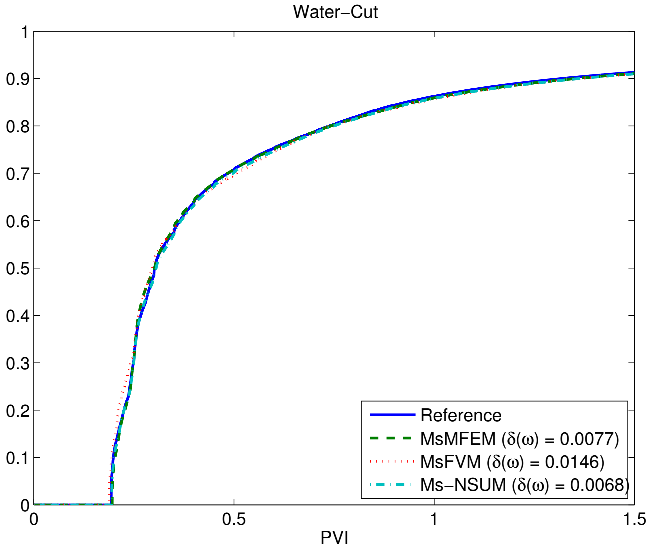
Two-Phase Flow

Example: quarter five-spot, Layer 85 from SPE 10, coarse grid: 10 × 22

Water cuts obtained by never updating basis functions:



favorable $(M = 0.1)$

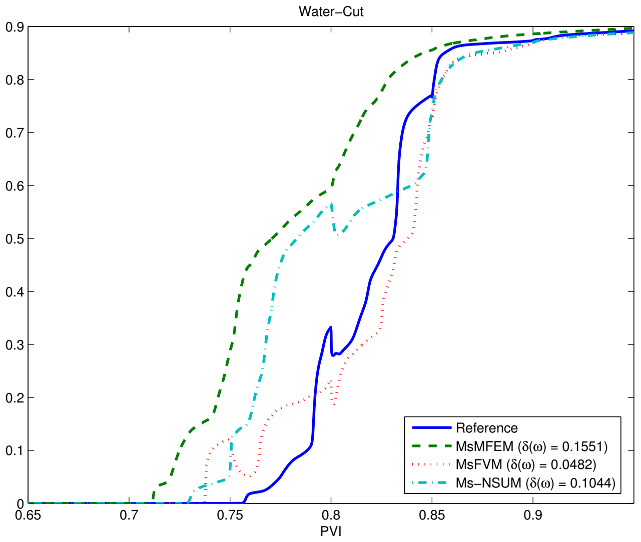


unfavorable $(M = 10.0)$

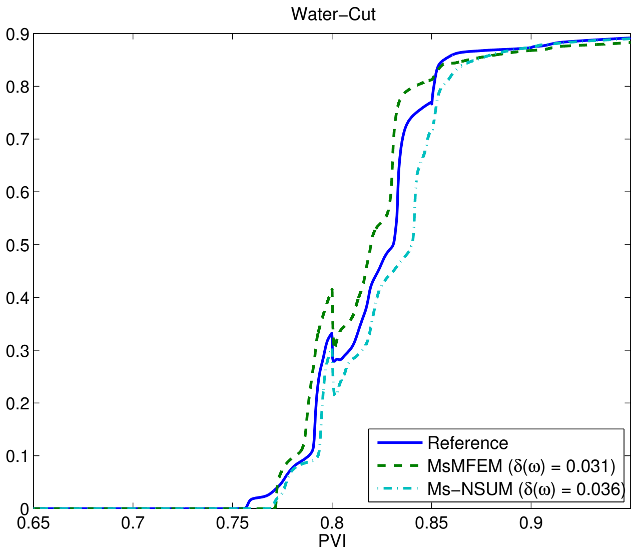
Two-Phase Flow

Example: quarter five-spot, Layer 85 from SPE 10, coarse grid: 10 × 22

Improved accuracy by adaptive updating of basis functions:



no updating

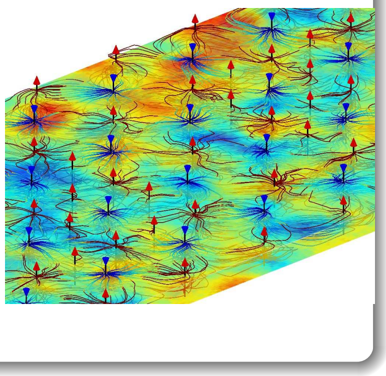


adaptive updating

Application: History Matching on Geological Models

Assimilation of production data to calibrate model

- 1 million cells, 32 injectors, and 69 producers
- 2475 days ≈ 7 years of water-cut data
- 6 iterations in data integration method
- 7 forward simulations, 15 pressure updates each



Computation time (on desktop PC):

- Original method: ∼ 40 min (pressure solver: 30 min)
- Multiscale method: ∼ 17 min (pressure solver: 7 min)

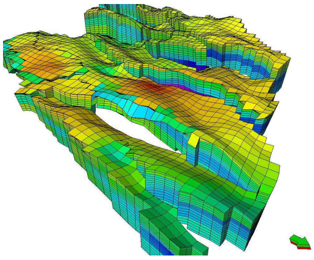
Geological Models as Direct Input to Simulation

‘Medium-fitted’ grids to model complex reservoir geometries

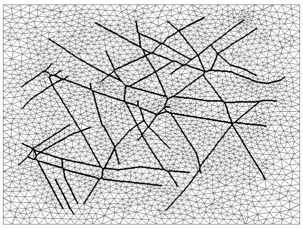
Another challenge:

- Industry-standard grids are often nonconforming and contain skewed and degenerate cells
- There is a trend towards unstructured grids
- Standard discretization methods produce wrong results on skewed and rough cells

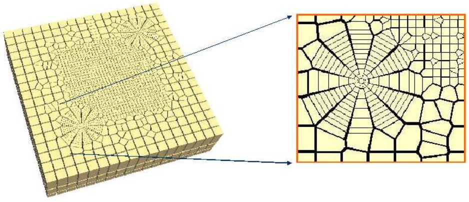
Corner point:



Tetrahedral:



PEBI:

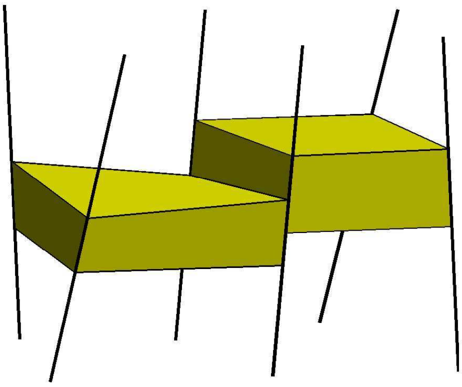


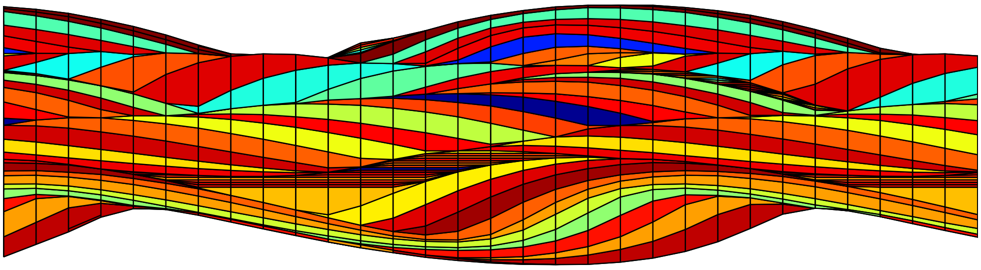
Corner-Point Grids

Industry standard for modelling complex reservoir geology

Specified in terms of:

- areal 2D mesh of vertical or inclined pillars
- each volumetric cell is restriced by four pillars
- each cell is defined by eight corner points, two on each pillar

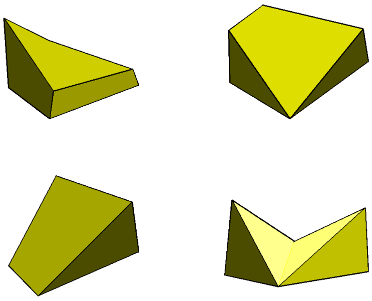




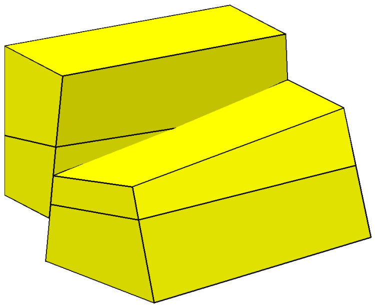
Discretisation on Corner-Point Grids

Exotic cell geometries from a simulation point-of-view

Skew and deformed grid blocks:



Non-matching cells:



- Can use standard MFEM provided that one has mappings and reference elements
- Can subdivide corner-point cells into tetrahedra
- We use mimetic finite differences (recent work by Brezzi, Lipnikov, Shashkov, Simoncini)

Discretisation on Corner-Point Grids

Mimetic finite differences, hybrid of MFEM and multipoint FVM

Let u, v be piecewise linear vector functions and $\mathbf{u}$, $\mathbf{v}$ be the corresponding vectors of discrete velocities over faces in the grid, i.e.,

$$\mathbf{v}_k = \frac{1}{|e_k|} \int_{e_k} v(s) \cdot n \, ds$$

Then the block $\mathbf{B}$ in the mixed system satisfies

$$\int_\Omega v^T K^{-1} u = \mathbf{v}^T \mathbf{B} \mathbf{u} \qquad \left(= \sum_{E \in \Omega} \mathbf{v}_E^T \mathbf{B}_E \mathbf{u}_E \right)$$

The matrices $\mathbf{B}_E$ define *discrete inner products*

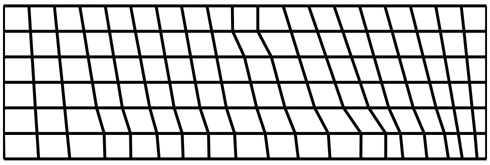
Mimetic idea:

Replace $\mathbf{B}_E$ with a matrix $\mathbf{M}_E$ that mimics some properties of the continuous inner product (SPD, globally bounded, Gauss-Green for linear pressure)

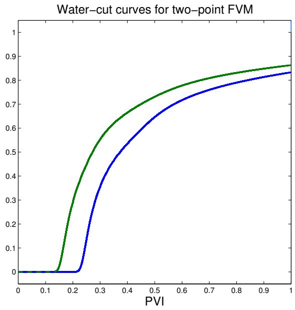
Mimetic Finite Difference Methods

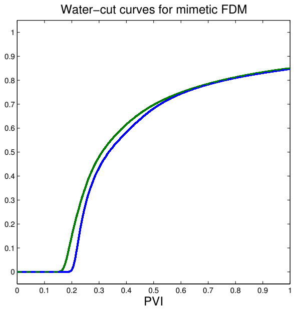
General method applicable to general polyhedral cells

Standard method + skew grids = grid-orientation effects

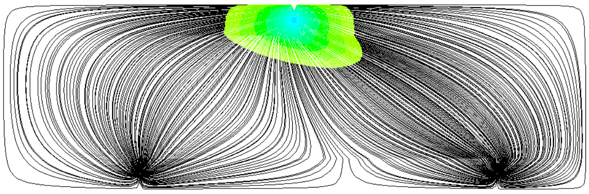


$\mathbf{K}$: homogeneous and isotropic,
symmetric well pattern
$\longrightarrow$ symmteric flow

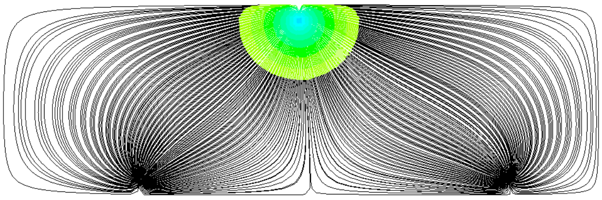




Streamlines with standard method



Streamlines with mimetic method

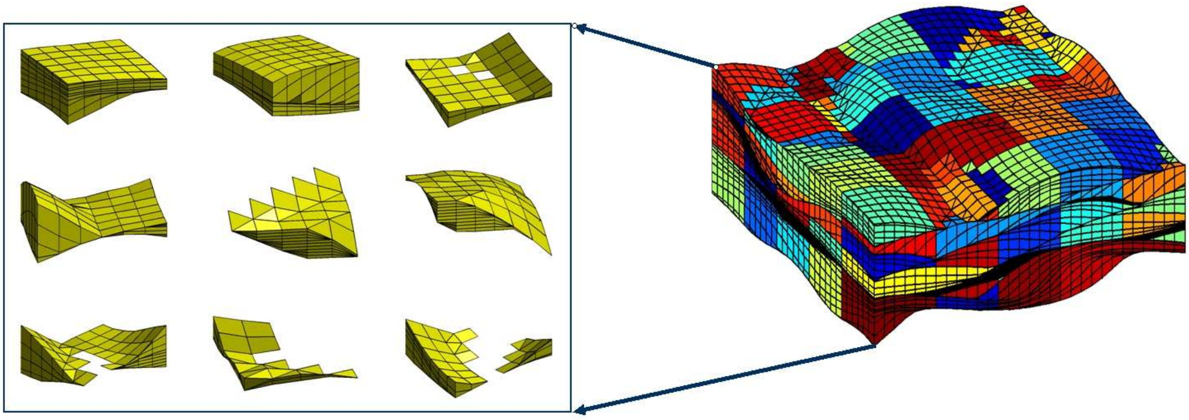


Multiscale Mixed Finite Elements

An automated alternative to upscaling?

Coase grid = union of cells from fine grid

MsMFEMs allow fully automated coarse gridding strategies: grid blocks need to be connected, but can have arbitrary shapes.

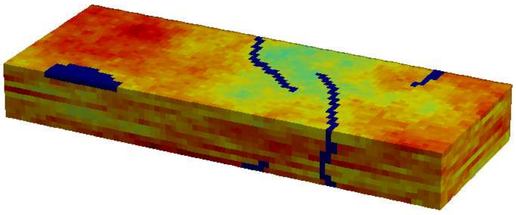


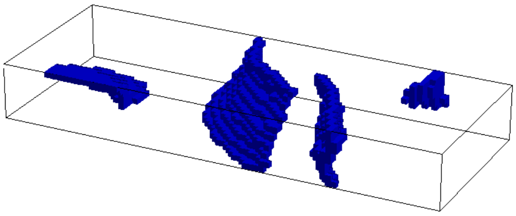
Uniform up-gridding: grid blocks are shoe-boxes in index space.

Model is courtesy of Alf B. Rustad, Statoil

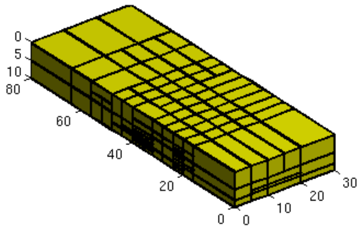
Multiscale Mixed Finite Elements

Examples of exotic grids

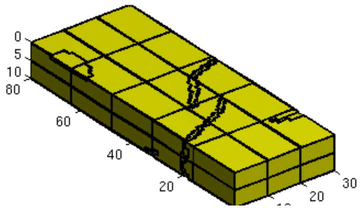




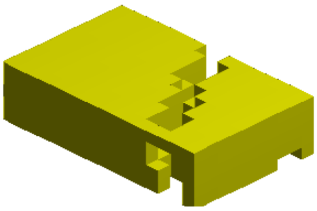
Non-uniform grid, hexahedral cells

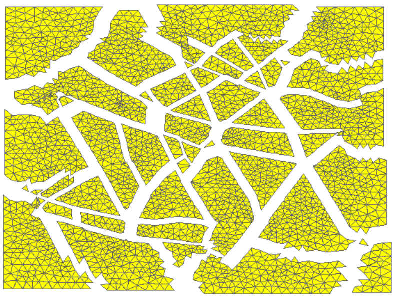


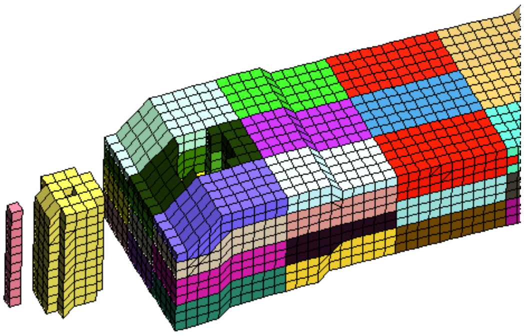
Non-uniform grid, general cells



General grid-cell

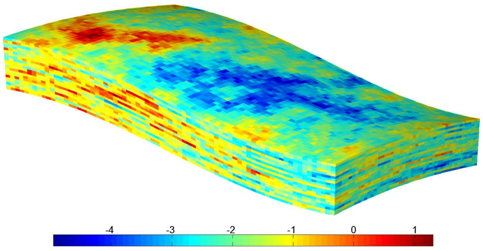


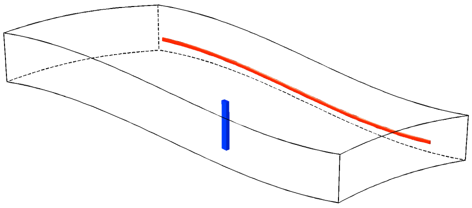


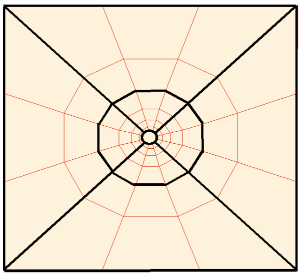


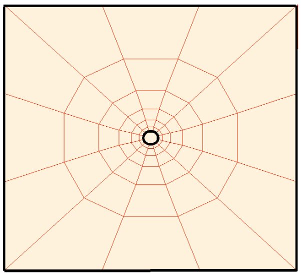
Multiscale Mixed Finite Elements

Ideal for coupling with well models









Fine grid to annulus, one coarse block for each well segment $\Longrightarrow$ no well model needed.

Summary

Advantages of multiscale mixed/mimetic pressure solvers

Ability to handle industry-standard grids

- highly skewed and degenerate cells
- non-matching cells and unstructured connectivities

Compatible with current solvers

- can be built on top of commercial/inhouse solvers
- can utilize existing linear solvers

More efficient than standard solvers

- faster and requires less memory than fine-grid solvers
- automated generation of coarse simulation grids
- easy to parallelize