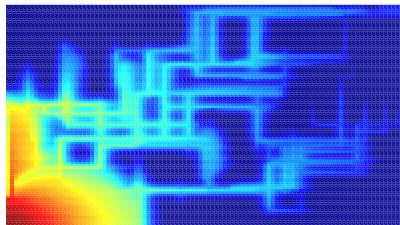
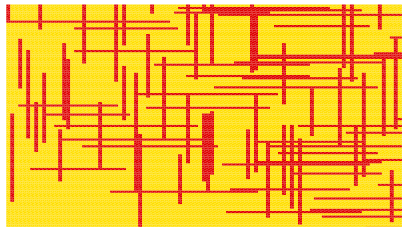


# Modeling of two-phase flow in fractured porous media on unstructured non-uniform coarse grids

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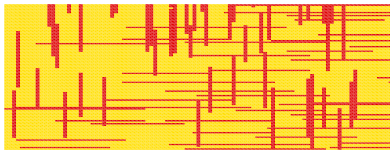
## Objective:

*Develop an algorithm for constructing coarse grids capable of resolving two-phase flow in fractured porous media accurately.*

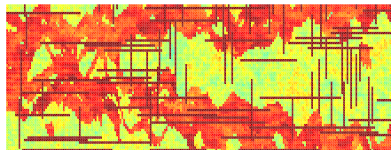
## Model assumptions:

- Statistically generated horizontal and vertical fractures with length between 20–40% of length of shortest side of reservoir.
- Velocity computed on a fine grid that resolves fractures.
- Saturation computed on an unstructured coarse grid.

Homogeneous model with 100 fractures



Heterogeneous model with 100 fractures



# Non-uniform coarsening algorithm

- 1 Compute the initial velocity field  $v$  on the fine grid and define

$$g(E) = \frac{1}{|E|} \int_E \log |v(x)| dx - \min(\log |v|) + 1, \quad E \subset \Omega.$$

- 2 Assign an integer from 1 to 10 to each cell  $c$  in the fine grid by

$$n(c) = \text{ceil} \left[ 10(g(c) - \min_c g) / (\max_c g - \min_c g) \right].$$

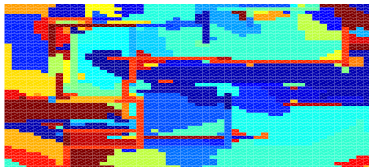
- 3 Initial blocks = connected groups of cells with the same  $n(c)$ .
- 4 Merge each block  $B$  with less volume than  $V_{\min}$  with

$$B' = \arg \min_{B'' \in \text{neighbors}} |g(B) - g(B'')|.$$

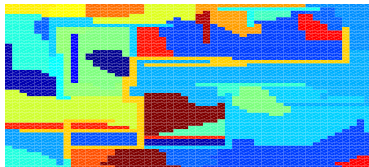
- 5 Refine each block  $B$  with  $|B|g(B) > G_{\max}$  as follows
  - 1 Pick an arbitrary cell  $c_0 \in B$  and locate the cell  $c_i \in B$  with center furthest away from the center of  $c_0$ .
  - 2 Define  $B' = c_i$  and progressively enlarge  $B'$  by adding cells from  $B$  adjacent to cells in  $B'$  until  $|B'|g(B') > G_{\max}$ .
  - 3 Define  $B = B \setminus B'$  and refine  $B$  further if  $|B|g(B) > G_{\max}$ .
- 6 Repeat step 2 and terminate.

# Non-uniform coarsening algorithm

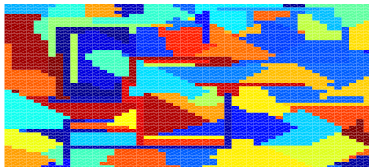
Coarse grid: Initial step, 152 cells



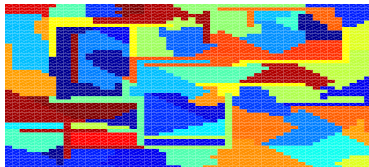
Coarse grid: Step 2, 47 cells



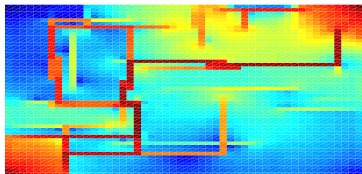
Coarse grid: Step 3, 95 cells



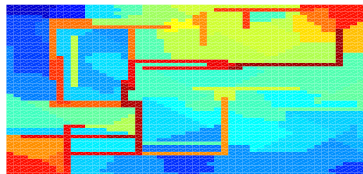
Coarse grid: Step 4, 69 cells



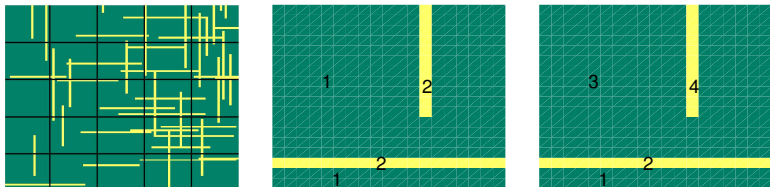
$\log |v|$  on fine grid (2500 cells)



$\log |v|$  on coarse grid (69 cells)



# Explicit Fracture-Matrix Separation (EFMS)



- Initial model: 100 x 100 grid with 50 fractures.
- Step 1: Introduce an initial coarse grid.
- Step 2: Separate fractures and matrix.
- Step 3: Split non-connected blocks.

# Operator splitting of saturation equation

Water saturation equation for a water-oil system:

$$\phi \frac{\partial S}{\partial t} + \nabla \cdot [f_w (v + \lambda_o \nabla p_{cow} + \lambda_o g(\rho_o - \rho_w) \nabla z)] = q_w,$$

Operator splitting of the water saturation equation

$$\begin{aligned} \phi \frac{\partial S}{\partial t} + \nabla \cdot (f_w v + f_w \lambda_o g(\rho_o - \rho_w) \nabla z) &= q_w, \\ \phi \frac{\partial S}{\partial t} + \nabla \cdot \left( f_w \lambda_o \frac{\partial p_{cow}}{\partial S} \nabla S \right) &= 0. \end{aligned}$$

# Numerical discretization of advection equation

Denote the non-degenerate fine grid interfaces by  $\gamma_{ij} = \partial T_i \cap \partial T_j$ .

$$S_m^{n+\frac{1}{2}} = S_m^n + \frac{\Delta t}{\int_{B_m} \phi \, dx} \left[ \int_{B_m} q_w \, dx - \sum_{\gamma_{ij} \subset \partial B_m} V_{ij}(S^{n+\frac{1}{2}}) - G_{ij}(S^{n+\frac{1}{2}}) \right].$$

Here

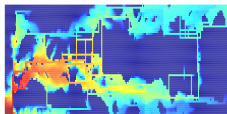
$$V_{ij}(S) = \max\{v_{ij} f_w(S|_{T_i}), -v_{ij} f_w(S|_{T_j})\},$$
$$G_{ij}(S) = g(\rho_o - \rho_w) |\gamma_{ij}| \frac{\lambda_w(S^+) \lambda_o(S^-)}{\lambda_w(S^+) + \lambda_o(S^-)} \nabla z \cdot n_{ij},$$

where  $v_{ij}$  = flux from  $T_i$  to  $T_j$ ,  $n_{ij}$  = unit normal on  $\gamma_{ij}$  from  $T_i$  to  $T_j$ , and  $S^+$  and  $S^-$  is the upstream saturation with respect to the gravity driven flow of oil and water, respectively.

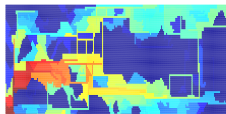
# Numerical example: Pure advective flow

Coarsening algorithm	# blocks	L2 water-cut error
Non-uniform coarsening	255	0.0240
EFMS	315	0.1428
Cartesian coarse grid	330	0.1838

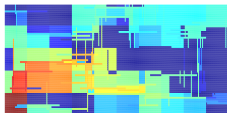
## Saturation profiles at 0.48 PVI



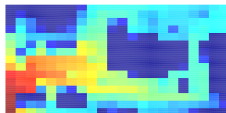
Reference solution



NUC solution

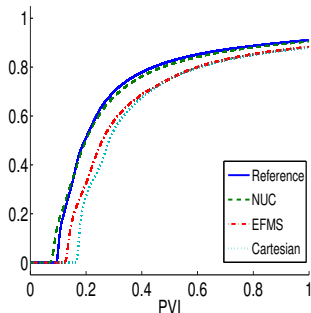


EFMS solution



Cartesian solution

## Water-cut curves

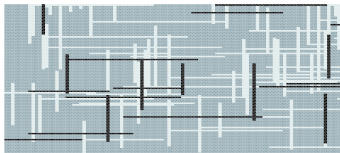




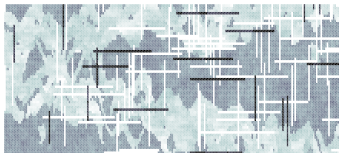
# Test-case I: No capillary diffusion

100 high permeable fractures and 20 low permeable fractures.  
Permeability of high permeable fractures:  $>$  matrix permeability.  
Permeability of low permeable fractures:  $\ll$  matrix permeability.  
25 simulations with different fracture distributions.

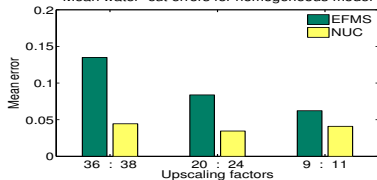
## Homogeneous model



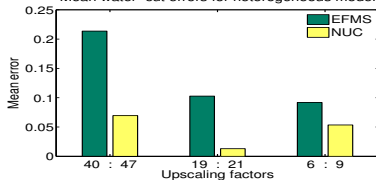
## Heterogeneous model



### Mean water-cut errors for homogeneous model



### Mean water-cut errors for heterogeneous model



Diffusion equation:

$$\phi \frac{\partial S}{\partial t} = \nabla \cdot d(S) \nabla S,$$

where  $d(S) = -f_w \lambda_o \frac{\partial p_{cow}}{\partial S}$  is a non-negative function.

Time discretization: Semi-implicit backward Euler

$$\phi S^{n+1} = \phi S^{n+1/2} + \Delta t \nabla \cdot d(S^{n+1/2}) \nabla S^{n+1}.$$

Spatial discretization: ???

How should  $\Delta t \nabla \cdot d(S^{n+1/2}) \nabla S^{n+1}$  be discretized on coarse grids with complex block geometries and strong subgrid heterogeneity?

Option 1: Fine grid discretization

$$(\Phi + \Delta t \mathbf{D}) S^{n+1} = \Phi S^{n+1/2},$$

where  $\Phi = \text{diag}(\phi)$  and  $\mathbf{D} = [d_{ij}(S)]$  stems from a fine grid discretization of the semi-elliptic operator  $L = \nabla \cdot d \nabla$ .

Option 1: Fine grid discretization

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Option 2: Coarse grid discretization by Galerkin projection

$$(\Phi_c + \Delta t \mathbf{D}_c) S_c^{n+1} = \Phi_c S_c^{n+1/2},$$

where  $\Phi_c = \mathbf{R}^t \Phi \mathbf{R}$  and  $\mathbf{D}_c = \mathbf{R}^t \mathbf{D} \mathbf{R}$ . Here  $\mathbf{R} = [r_{ij}]$  where

$$r_{ij} = \begin{cases} 1 & \text{if cell } i \text{ is contained in block } j, \\ 0 & \text{otherwise.} \end{cases}$$

Hence,  $\mathbf{R}$  maps coarse grid saturations onto the fine grid.

## Orthogonal projection property

If  $S^{n+1}$  solves the fine grid system with  $S^{n+1/2} = \mathbf{R}S_c^{n+1/2}$ , then

$$\|\mathbf{R}(S_c^{n+1} - S_c^{n+1/2})\| = \arg \min_{S_c} \|\mathbf{R}S_c - S^{n+1}\|,$$

where  $\|S\| = (S, (\Phi + \Delta t \mathbf{D})S)^{1/2}$ , i.e.,  $S_c^{n+1}$  is the optimal coarse grid approximation to  $S^{n+1}$  in the norm  $\|\cdot\|$ .

## Orthogonal projection property

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## The fine grid discretization gives too much diffusion!

Gradients of the blockwise constant saturation is computed at the fine grid level: Options 1 and 2 overestimate diffusion.

The overestimation factor scales with the ratio of the size of the coarse grid blocks relative to the size of the fine grid cells.

On average the diffusion should be damped by a factor  $(N_b/N_c)^{1/d}$ , where  $N_b$  = number of blocks and  $N_c$  = number of cells.

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Scaled Galerkin projection

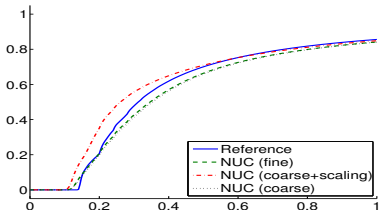
$$\left[ \Phi_c + \Delta t \left( \frac{N_b}{N_c} \right)^{1/d} \mathbf{D}_c \right] S_c^{n+1} = \Phi_c S_c^{n+1/2}.$$



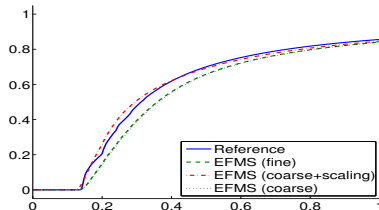
# Test-case II: With capillary diffusion

	Homogeneous model			Heterogeneous model		
	fine grid	G. proj.	scl. proj.	fine grid	G. proj.	scl. proj.
EFMS	0.071	0.077	0.028	0.091	0.107	0.051
NUC	0.047	0.055	0.079	0.063	0.109	0.033

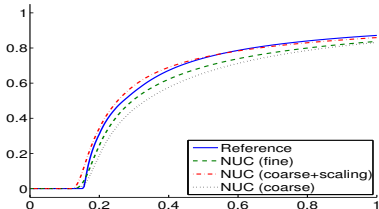
Water-cut curves for homogeneous model



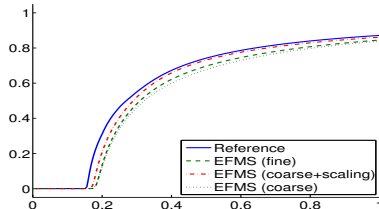
Water-cut curves for homogeneous model



Water-cut curves for heterogeneous model

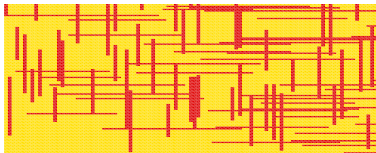


Water-cut curves for heterogeneous model

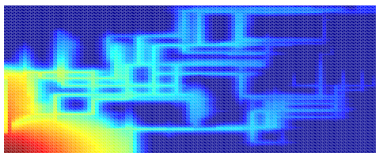


# Test-case II: With capillary diffusion

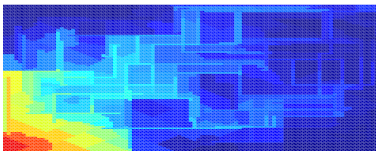
Homogeneous model with 100 high permeable fractures: Saturation profiles at 0.2 PVI



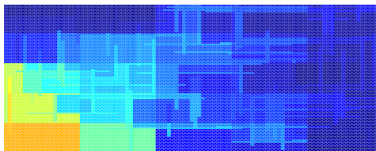
Fracture model



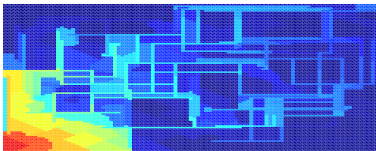
Reference solution



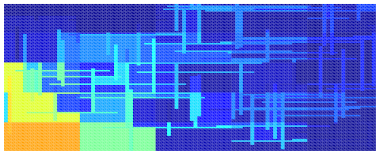
NUC: fine grid diffusion



EFMS: fine grid diffusion



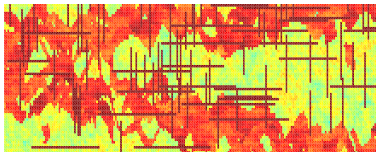
NUC: scaled G. projection



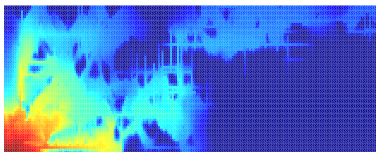
EFMS: scaled G. projection

# Test-case II: With capillary diffusion

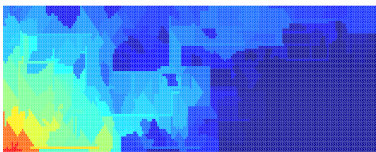
Heterogeneous model with 100 high permeable fractures: Saturation profiles at 0.2 PVI



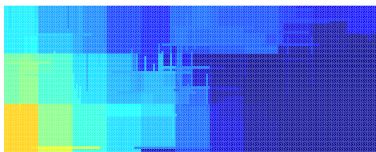
Fracture model



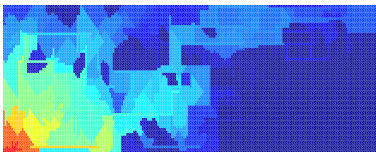
Reference solution



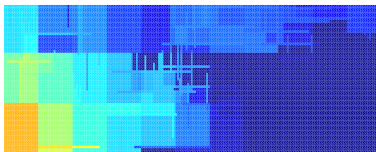
NUC: fine grid diffusion



EFMS: fine grid diffusion



NUC: scaled G. projection



EFMS: scaled G. projection

- For flows without capillary diffusion we consistently obtain significantly more accurate solutions on non-uniform coarse grids than on grids with explicit fracture-matrix separation.
- For flows with relatively strong capillary diffusion, the coarsening algorithms give comparable results for homogeneous fracture models, but non-uniform coarsening gives best results for heterogeneous fracture models.
- The scaled Galerkin projection generally models diffusion well on complex coarse grids, but more rigorous ways of damping the fine scale diffusion will be studied in further research.