Operator Splitting of Advection and Diffusion on Non-uniformly Coarsened Grids

Vera Louise Hauge   Jørg Espen Aarnes   Knut–Andreas Lie

Applied Mathematics, SINTEF ICT Oslo
Department of Mathematical Sciences, NTNU Trondheim

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Outline of presentation

- Objective and strategies
- Background and motivation
  - Non-uniform coarse grids
- Discretization of the saturation equation
  - Viscous part and diffusion part
- The two damping strategies
- Numerical examples
  - Pure capillary diffusion
  - Field scale example
  - Aspect ratio example
- Concluding remarks

Objectives and strategies

Overall objective:

- Fast flow simulations for high-resolution reservoir models.

Strategy:

- Reduce size of geomodel by using non-uniform grid coarsening.
  $\Longrightarrow$ Flow based grid: Keep important flow characteristics.
- Accompanied by multiscale pressure solvers.

Objective and strategies

Objective of this work:

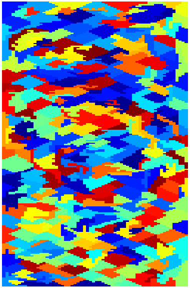
- Include capillary pressure effects in fast saturation simulations on non-uniform coarse grids.
- Operator splitting to discretize the capillary diffusion separately from the advective term.
  Assumption: Viscous flow dominant.
- Straightforward projection in the coarse-grid discretization
  $\Longrightarrow$ Overestimation of diffusion.

Strategy:

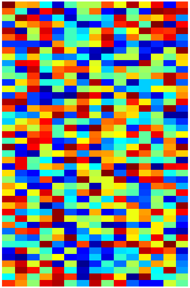
- Damping factors for the diffusion operator to correct for the overestimation of diffusion.

Background: Example of coarse grids

SPE10 model 2, layer 46. Original model 60×220 cells.
Random coloring: Shows shapes and sizes of coarse grid blocks.



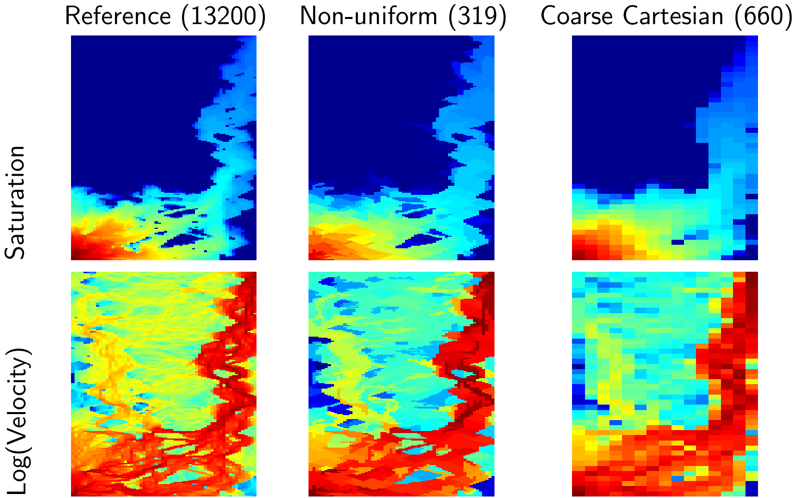
Non-uniform coarse grid
319 blocks



Cartesian coarse grid
660 blocks

Non-uniform coarse grid: Flow based, keeps important flow characteristics in the grid.

Simulation results on coarse grids



Note: Details of high-flow channels.

Numerical discretization

Splitting of the saturation equation:

$$\text{Viscous part:} \quad \phi\frac{\partial S}{\partial t} + \nabla \cdot (f_w v) = q_w$$

$$\text{Diffusion part:} \quad \phi\frac{\partial S}{\partial t} + \nabla \cdot d(S)\nabla S = 0$$

Viscous part:

- First-order finite volume method discretization.
- Fluxes are computed as upstream fluxes with respect to the *fine* grid fluxes on the coarse interfaces.

Numerical discretization

Diffusion part:

Time: Semi-implicit backward Euler method:

$$\phi S^{n+1} = \phi S^{n+1/2} - \Delta t \nabla \cdot d(S^{n+1/2}) \nabla S^{n+1}$$

Space: Cell-centered finite-difference discretization.

- Fine grid: Two-point flux approximation:

$$-\int_{\gamma_{ij}} d(S) \nabla S \cdot n_{ij} ds \approx -|\gamma_{ij}| \tilde{d}(S_i, S_j) \frac{S_i - S_j}{|x_i - x_j|}$$

- Coarse grid: Projection of the fine-grid discretization onto the coarse grid.

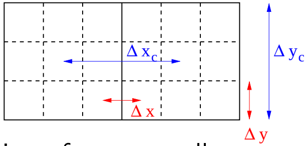
Damping of diffusion

Overestimation

- Projection of diffusion operator onto coarse grid $\Longrightarrow$ Overestimates diffusion.
- Reason: Saturation gradient computed on fine grid, whereas saturation values represent net saturations in the coarse blocks.

Damping of diffusion: Illustration

Coarse Cartesian grid



Each coarse block consists of $n_x \times n_y$ cells.

Considering a coarse interface in the x-direction

Coarse grid diffusion operator:

$$-\sum_{n_y} \Delta y \, d(\gamma_{ij}) \frac{S_i - S_j}{\Delta x} = -\Delta y_c \, d(\Gamma_{ij}) \frac{S_i - S_j}{\Delta x}$$

Desired operator:

$$-\Delta y_c \, d(\Gamma_{ij}) \frac{S_i - S_j}{\Delta x_c}$$

Damping factor of the diffusion term: $\Delta x / \Delta x_c = 1/n_x$

Damping of diffusion

Observation:

Capillary diffusion scales with the ratio in the size of coarse blocks relative to the size of fine cells.

Crude damping factor:

- (#coarse blocks / #fine cells)$^{1/d}$
- Correct factor for square coarse blocks.
- Not sufficient for non-uniform coarse grids with complex geometries.

Fine damping:

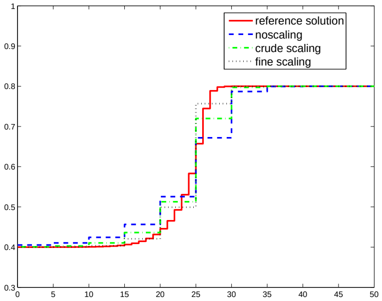
- Use directly the geometry information from the fine grid to correct the coarse-grid diffusion operator.
- One factor for each coarse interface $\Rightarrow$ More computation.

Numerical examples: Pure capillary diffusion

Transport only driven by capillary diffusion.

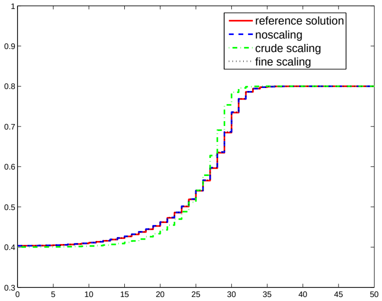
Fine grid:	50×1 cells	50×5 cells
Uniform coarse grid:	10×1 blocks	50×1 blocks
Crude damping factor:	$1/\sqrt{5}$	

$\Delta x/\Delta x_c = 0.2$



Overestimation

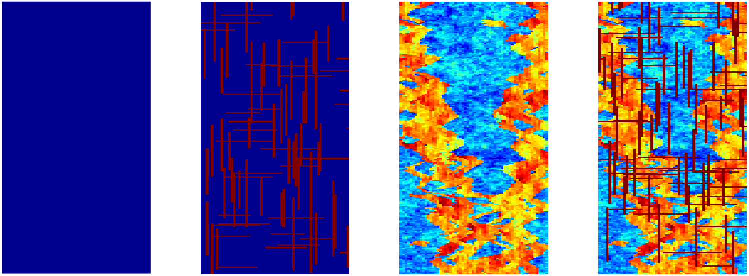
$\Delta x/\Delta x_c = 1$



Underestimation

Numerical examples: Field scale example

Quarter five-spot, strong capillary diffusion:

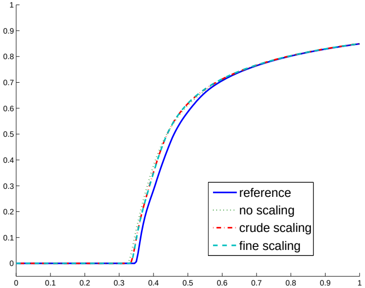


L^2 error of saturation in different reservoirs

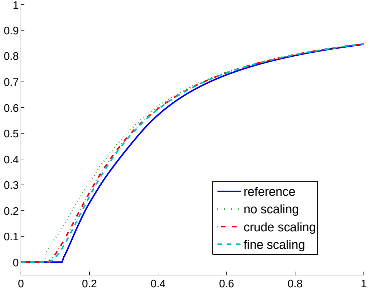
Model	Fractures	Upscaling	Damping		
			No	Crude	Fine
Homogeneous	no	23	0.0332	0.0295	0.0294
Homogeneous	yes	21	0.0387	0.0277	0.0270
SPE model	no	35	0.0608	0.0385	0.0316
SPE model	yes	30	0.0216	0.0162	0.0123

Numerical examples: Field scale example

Water-cut curves



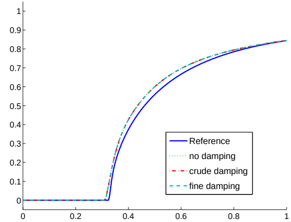
Homogeneous model with fractures



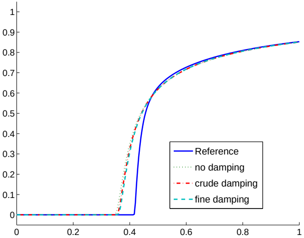
SPE model without fractures

Numerical examples: Aspect ratio

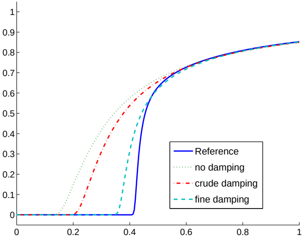
- Quarter five-spot models with homogeneous permeability field.
- Physical dimensions of 1, 100 and 1000 m in one direction and 1 m in the other (small to large aspect ratios).



Aspect ratio 1



Aspect ratio 100



Aspect ratio 1000

Concluding remarks

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Projection of the diffusion operator onto coarse grids overestimates the diffusion.

Crude damping sufficient:

- If coarse grid blocks are close to a square, with approximately the same number of fine cells in each direction and aspect ratio of order one.

Fine damping necessary:

- If the coarse grid blocks have large aspect ratios.
- Coarse blocks dissimilar in shape and size.

Thank you for your attention!

Questions?

http://www.sintef.no/GeoScale