Adaptive Multiscale Streamline Simulation and Inversion for High-Resolution Geomodels

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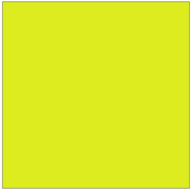
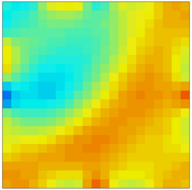
Introduction: History matching

History matching is the procedure of modifying the reservoir description to match measured reservoir responses.

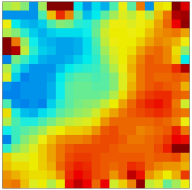
Initial:

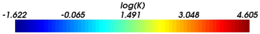


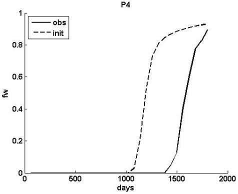
Matched:

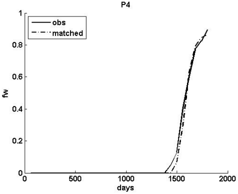


Reference:

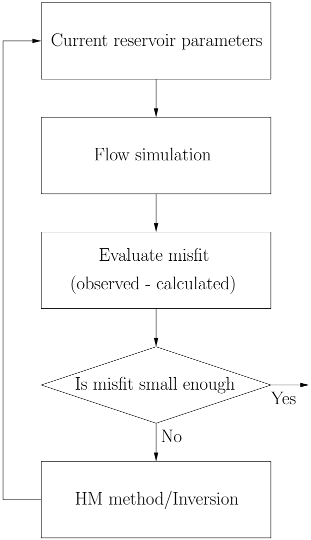




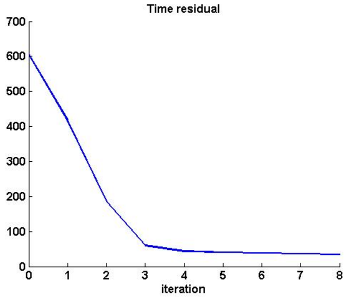




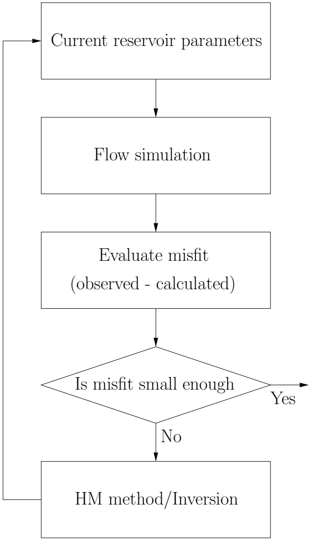
Introduction: History-matching loop



$$E = \sum (\mathbf{d}^{\text{obs}} - \mathbf{d}^{\text{cal}})^2, \quad \mathbf{d}^{\text{cal}} = g(\mathbf{m})$$



Challenges in history-matching loop



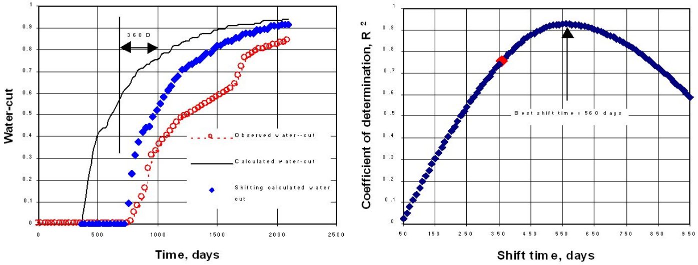
Problems:

- highly under-determined problem → non-uniqueness
- errors in model, data, and methods
- nonlinear forward model
- *non-convex misfit functions*
- *forward simulations are computationally demanding*

Inversion method: Generalized Travel-Time Inversion (GTTI) with analytic sensitivities [Vasco et al. (1999), He et al. (2002)]

The generalized travel time is defined as the 'optimal' time–shift that maximizes

$$R^2(\Delta t) = 1 - \frac{\sum[y^{\text{obs}}(t_i + \Delta t) - y^{\text{cal}}(t_i)]^2}{\sum[y^{\text{obs}}(t_i) - \bar{y}^{\text{obs}}(t_i)]^2}.$$



Basic underlying principles for the history–matching algorithm

- Minimize travel-time misfit for water–cut by iterative least-square minimization algorithm.
- Preserve geologic realism by keeping changes to prior geologic model minimal (if possible).
- Only allow smooth large-scale changes. Production data have low resolution and cannot be used to infer small-scale variations.

Minimization of functional:

$\Delta\tilde{\mathbf{t}}$: Travel–time shift
$\mathbf{S}$: Sensitivity matrix
$\mathbf{m}$: Reservoir parameters

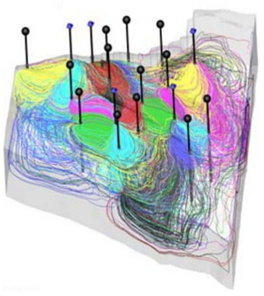
$$\|\Delta\tilde{\mathbf{t}} - \mathbf{S}\delta\mathbf{R}\| + \overbrace{\underbrace{\beta_1\|\delta\mathbf{R}\|}_{norm} + \underbrace{\beta_2\|\mathbf{L}\delta\mathbf{R}\|}_{smoothing}}^{Regularization}$$

$\mathbf{S}$ computed analytically along streamlines from a single flow simulation

Streamline-based history matching

Features of streamlines

- Very well suited for modeling large heterogeneous multi-well systems dominated by convection
- Generally fast flow simulation
- Delineate flow pattern (injector-producer pairs)
- Enables analytic sensitivities



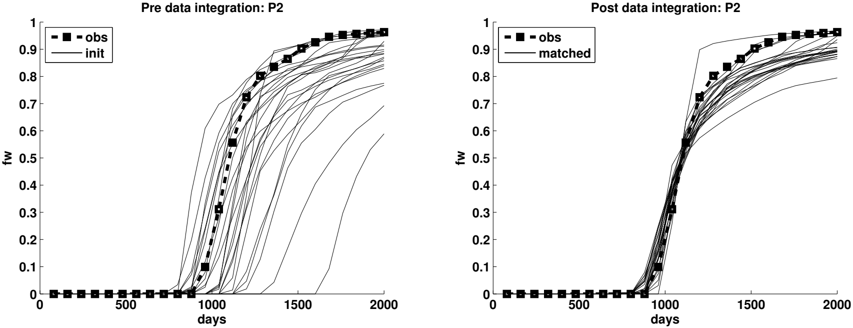
Source: www.techplot.com

Streamline-based history-matching methods

- Assisted history matching
- (Generalized) travel-time inversion methods
- Streamline-effective properties methods
- Miscellaneous

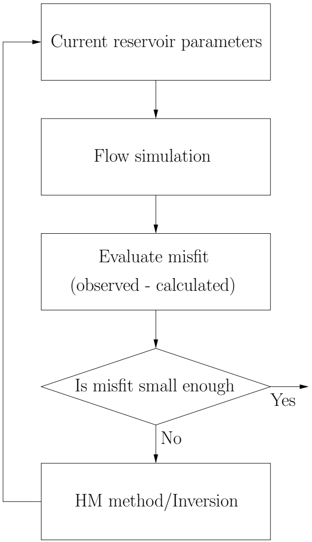
Example: Uncertainty quantification

Simple two-phase model (end-point mobility $M = 0.5$) on a 2D horizontal reservoir, lognormal permeability



Statistical analysis of mean and standard deviation

Challenge II: long runtime for forward simulations



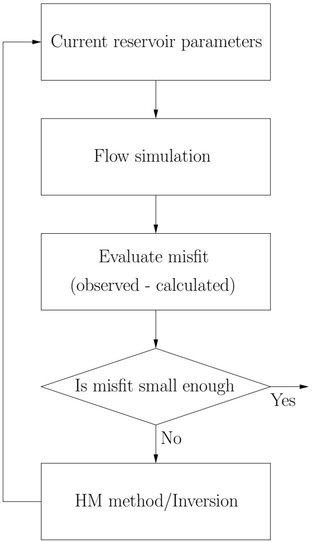
Streamline simulation much faster than conventional FD-methods.

Still, room for improvement.

Observations:

- pressure solver most expensive part of simulation
- data changes very little from one simulation to the next

Challenge II: long runtime for forward simulations



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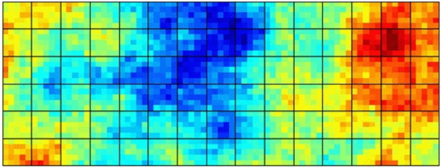
Reuse computations in areas with minor changes ⟶ multiscale methods

Multiscale pressure solver

Upscaling and downscaling in one step. Runtime like coarse-scale solver, resolution like fine-scale solver.

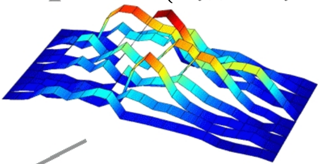
Fine grid: 75×30. Coarse grid: 15×6

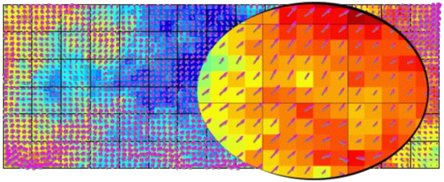


Basis functions for each pair of coarse blocks $T_i \cup T_j$:

$$\Psi_{ij} = -\lambda K \nabla \Phi_{ij}$$

$$\nabla \cdot \Psi_{ij} = \begin{cases} w_i(x), & x \in T_i \\ -w_j(x), & x \in T_j \end{cases}$$





Coarse grid: pressure and fluxes. Fine grid: fluxes

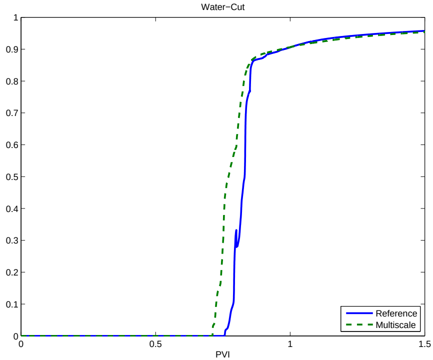
Global linear system with 249 unknowns:

$$\nabla \cdot v = q, \quad v = -\lambda K \nabla p$$

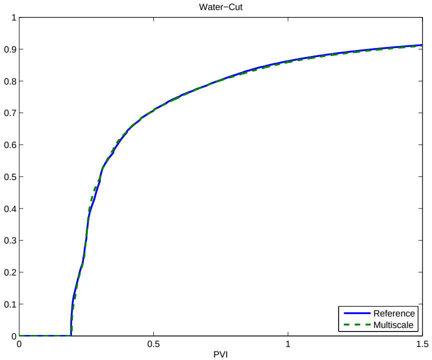
Multiscale methods: efficiency vs accuracy

Ex: q5-spot, SPE 10 (layer 85)[1], $60 \times 220 \rightarrow 10 \times 22$

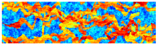
Water cuts obtained by *never* updating basis functions:



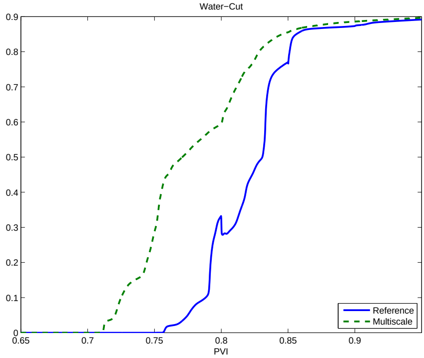
favorable $(M = 0.1)$



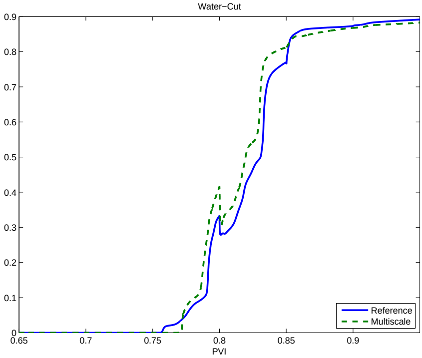
unfavorable $(M = 10.0)$

1: Fluvial permeability field, 

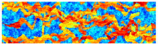
Improved accuracy by *adaptive* updating of basis functions:



no updating

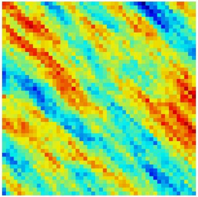


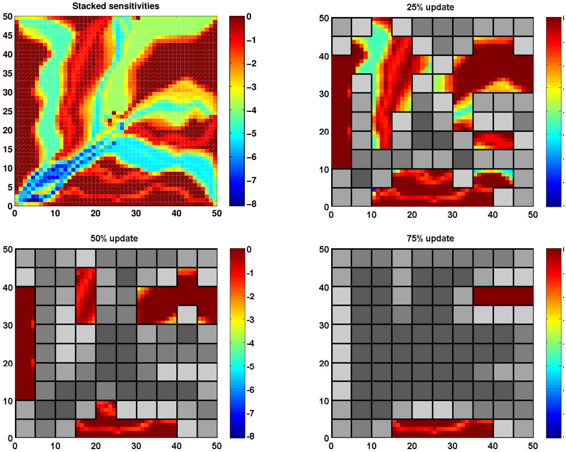
adaptive updating

1: Fluvial permeability field, 

Further computational savings

Can also reuse basis functions from previous forward simulation.
General idea: use sensitivity to steer updating

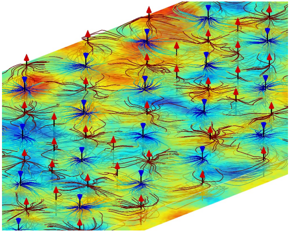


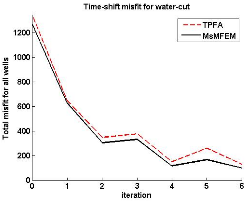


Assimilation of production data to calibrate model

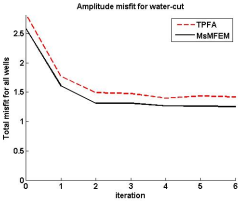
- 1 million cells, 32 injectors, and 69 producers
- 2475 days ≈ 7 years of water-cut data

Analytical sensitivities along streamlines + travel-time inversion (quasi-linearization of misfit functional)





Time-residual



Amplitude-residual

Computation time: ∼ 17 min on a desktop PC (6 iterations).

Solver	O/M	Misfit			CPU-time (wall clock)		
		T	A	$\Delta \ln k$	Total	Pres.	Transp.
Initial	—	100.0	100.0	0.821	—	—	—
Std. (7 pt.)	O	8.9	53.5	0.806	64 min	33 min	28 min
Std. (7 pt.)	M	9.6	50.4	0.806	39 min	30 min	5 min
Multiscale	O	11.2	47.3	0.812	43 min	7 min	32 min
Multiscale	M	10.4	45.4	0.828	17 min	7 min	6 min

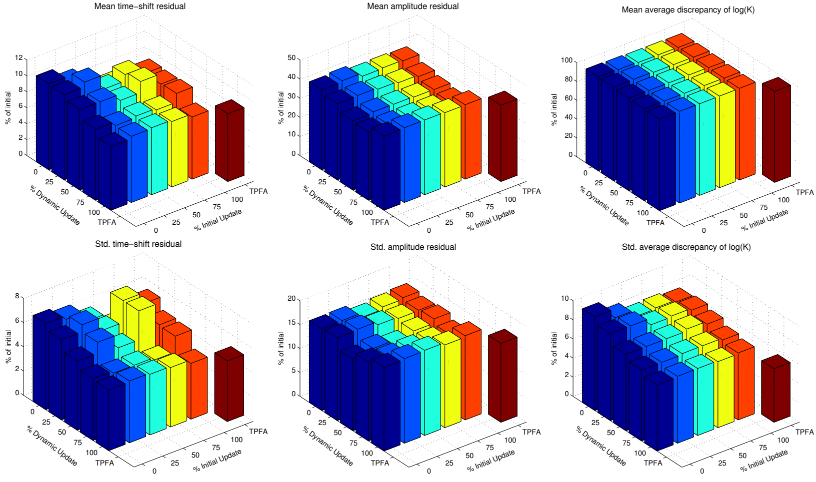
Misfit:

Time-shift misfit $\|\Delta \mathbf{t}\|_2$

Amplitude misfit $[\sum_k \sum_j (f_w^{\text{obs}} - f_w^{\text{cal}})^2]^{1/2}$

Permeability discrepancy $1/N \sum_{i=1}^{N} |\ln k_i^{\text{ref}} - \ln k_i^{\text{match}}|$

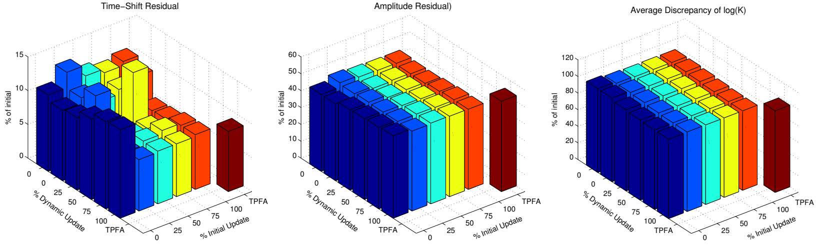
Robustness with respect to data reduction
Uncertainty quantification, revisited



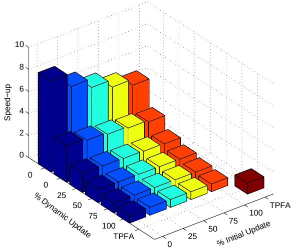
Robustness with respect to data reduction

Million-cell model, revisited

Reduction in residuals



Corresponding speedup:



Extentions

- Unstructured grids (done for inversion algorithm)
- Corner-point grids (testing to be done on Norne-model)
- Other types of data / more general flow
- Inclusion of seismics
- Use of sensitivities for other optimization workflows
- ...