Multiscale Simulation of Highly Heterogeneous and Fractured Reservoirs

Astrid F. Gulbransen Vera Louise Hauge
Jostein R. Natvig Bård Skaflestad

Applied Mathematics, SINTEF ICT
Oslo, Norway

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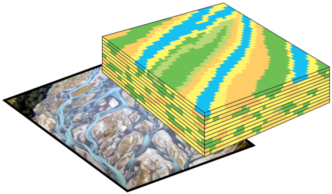
Research group

- 3 researchers
- 4 postdocs
- 1–2 PhD students
- 3 programmers



Collaboration with national and international partners in industry and academia

Research vision

Direct simulation of complex grid models of highly heterogeneous and fractured porous media — a technology that bypasses the need for upscaling.

http://www.math.sintef.no/GeoScale/

Applications:

- Validation during development of geomodels
- Fast simulations of multiple realizations
- Optimization of production, well placement, etc
- History matching
- Geological storage of CO_2

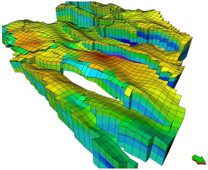
Funding:

- Strategic research grant and PhD/postdoc grants
- Research grants with end-user involvement
- Industry projects

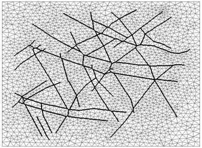
Challenges:

- Industry-standard grids are often nonconforming and contain skewed and degenerate cells
- There is a trend towards unstructured grids
- Standard discretization methods produce wrong results on skewed and rough cells
- The combination of high aspect and anisotropy ratios can give *very large* condition numbers

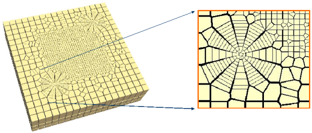
Corner point:



Tetrahedral:



PEBI:



Aim:
To develop a pressure solver with improved accuracy and flexibility.

Solution:

- use a mimetic finite difference method to improve accuracy and to reduce grid sensitivity
- use a multiscale method to balance speed and accuracy.

Model:

$$\lambda_t^{-1} K^{-1} v + \nabla p = 0, \ \text{(Darcy)},$$
$$\nabla \cdot v = q,$$

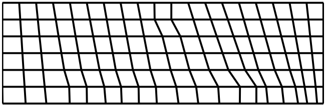
Seek discrete p and v that maintain

- mass balance
- a discrete form of Darcy's law

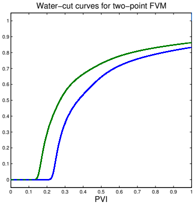
On polyhedral grids, the mimetic method yields exact solutions for linear pressure.

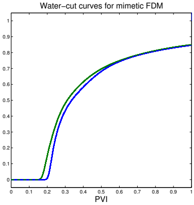
In fact, this is better that many commercial simulators!

Standard method + skew grids = grid-orientation effects

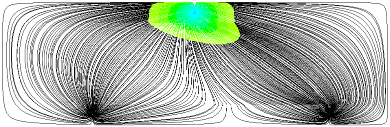


K: homogeneous and isotropic, symmetric well pattern
⟶ symmteric flow

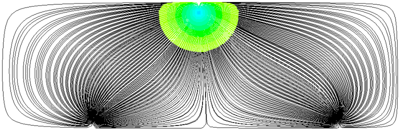




Streamlines with two-point method

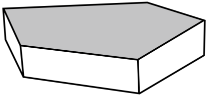


Streamlines with mimetic method



Mixed and mimetic formulation for one grid block:

$$\begin{bmatrix} B & C^{\mathsf{T}} \\ C & \end{bmatrix} \begin{bmatrix} v \\ p \end{bmatrix} = \begin{bmatrix} -a \\ Q \end{bmatrix}$$



By eliminating v we get

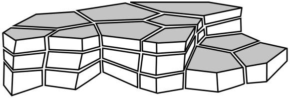
$$CB^{-1}C^{\mathsf{T}}p = Q + CB^{-1}a,$$

$$\text{MFEM:} \quad B = \int_K \phi_i \cdot \lambda^{-1} K^{-1} \phi_j \, \mathrm{d}\Omega$$

$$\text{Mimetic:} \quad B^{-1} = \lambda_t N K N^{\mathsf{T}} - \mathrm{tr}(K)(1 - UU^{\mathsf{T}})$$

Hybrid formulation:

$$\begin{bmatrix} B & C^\mathsf{T} & D^\mathsf{T} \\ C & & \\ D & & \end{bmatrix} \begin{bmatrix} v \\ p \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ Q \\ 0 \end{bmatrix}$$



Elimination of p and v yields a positive definite system for a.

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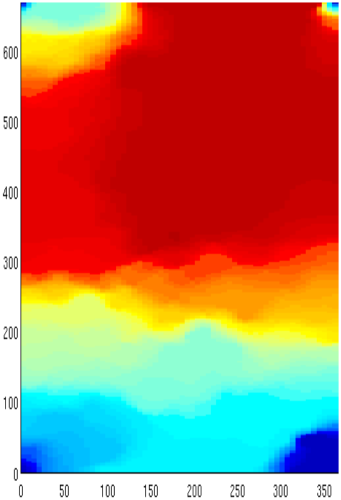
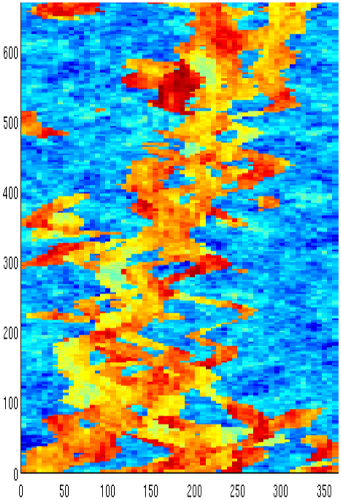
Pressure typically varies smoothly while velocity is largely determined by local heterogeneities





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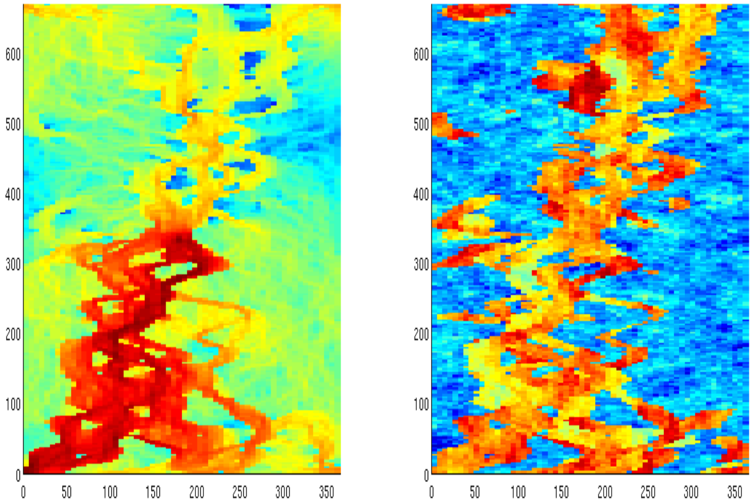
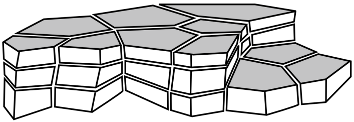
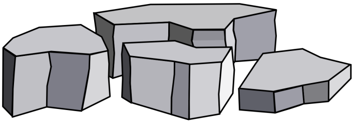




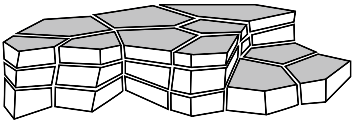
Original simulation grid



Partition into coarse grid

Key Idea

Express fluid flow in reservoir as a linear combination of local flow solutions on pairs of coarse grid blocks.



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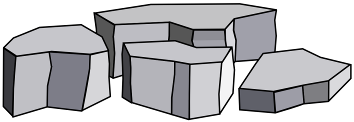

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Express fluid flow in reservoir as a linear combination of local flow solutions on pairs of coarse grid blocks.

- Local flows account for small-scale impact on global flow field
- Each localized flow field is obtained by resolving independent flow problems
- Any method may be used to discretize these problems

End Result

High-resolution velocity field computable with comparatively few degrees of freedom (local problems resolved once or infrequently)

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The multiscale/mixed pressure solver framework

An efficient alternative (cont'd)

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- Performance on compressible problems (i.e. with gas)
- Adapting coarse grid to placement of wells
- How to efficiently represent fractures on coarse grids
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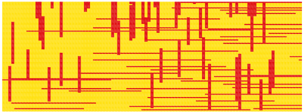
Modeling of two-phase flow in fractured porous media on unstructured non-uniformly coarsened grids

- We want to determine a coarse grid suitable for saturation simulations that preserves important characteristics of the flow.
- Investigate two coarsening strategies: Non-uniform coarsening and Explicit fracture-matrix separation

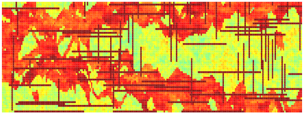
Key ideas:

- Velocity computed on a fine grid which resolves the fractures
- Saturation computed on the coarse grid

Homogeneous model with 100 fractures



Heterogeneous model with 100 fractures



Non-uniform coarsening algorithm

Two parameters:

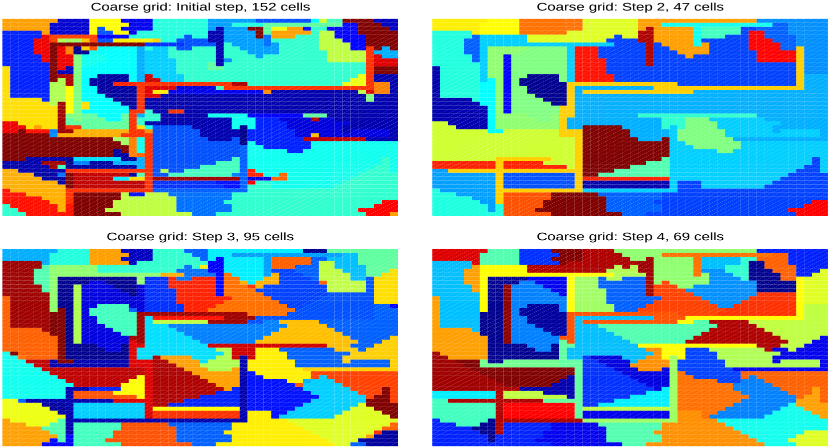
V_{min}: Minimum volume of a coarse block

G_{max}: Maximum flow through each coarse block

The most important points from the algorithm:

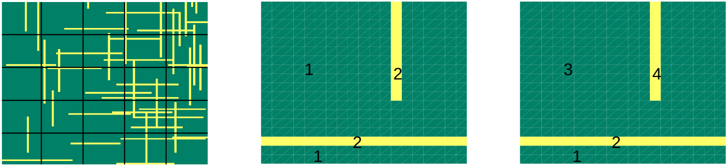
- Group cells of similar flow magnitude into coarse blocks
- Coarse blocks have to be connected
- Avoid too small blocks
- Avoid too large blocks

Non-uniform coarsening algorithm



Note: Random coloring of blocks

Explicit Fracture-Matrix Separation (EFMS)



Step 1 Step 2 Step 3

Initial model: 100 × 100 grid cells, 50 fracture lines

- Step 1: Introduce an initial coarse grid, here 5 × 5
- Step 2: Separate fracture and matrix part
- Step 3: Split non-connected blocks

Disadvantage: Upscaling factor difficult to tune.

Saturation equation and numerical discretization

Water saturation equation for a water-oil system:

S_m	$=$	S_m at previous time step	$+$	[Flux in	-	Flux out]

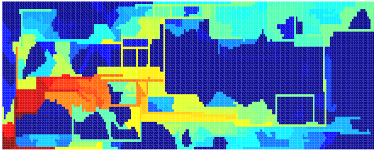
S_m = water saturation in coarse grid block *m*.

- First-order finite volume method discretization
- Fluxes are computed as upstream fluxes with respect to the *fine* grid fluxes on the coarse interfaces

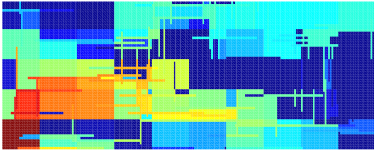
Comparison of coarse grids: NUC, EFMS and Cartesian.

Heterogeneous model with 100 fractures
Saturations solutions at 0.48 PVI.

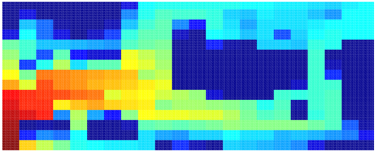
NUC grid with 206 blocks.



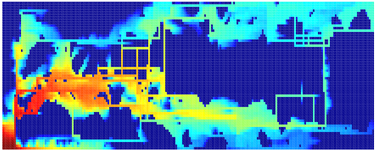
EFMS grid with 236 blocks.



20×20 Cartesian grid



Fine grid



Further research

- Capillary diffusion and gravity modeled on non-uniformly coarsened grids
- Compressible flow on non-uniformly coarsened grids

  $\Rightarrow$ Black-oil model on non-uniformly coarsened grids